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PREFACE

This issue of the QUARTERLY TRANSACTIONS, Number 3, Volume 51, contains the last two papers from the Winter Convention; also those papers presented at the Great Lakes District Meeting, Milwaukee, Wisconsin, and the Northeastern District Meeting, Providence, Rhode Island, which were approved for publication. The papers are, as usual, arranged chronologically. This is the only publication of the Providence Meeting papers in full, as no separate copies have been printed.

Surge-Proof Transformers

BY H. V. PUTMAN*
Member, A.I.E.E.

A short report of artificial lightning tests made on a 42,000-kva., 220-kv. shell type transformer was published in the October issue of ELECTRICAL ENGINEERING.[1] A complete report of these tests was published in the *Electric Journal*.[2] In these reports three points of particular interest in connection with these tests are mentioned: (1) the transformers tested were the largest on which lightning tests have been made; (2) the tests made were more severe than previous tests;† and (3) the transformers tested were of a new surge-proof construction involving fundamental improvements in the design of the insulation. An illustration is shown in Fig. 1 of a lightning test during which the transformer bushing was flashed over at approximately one and one-half million volts.

It is the purpose of this paper to describe the novel features of this improved transformer construction.

Two features are of special interest:

1. Without the use of shields to neutralize ground capacity, it has been possible to build shell type transformers in which the distribution of surge voltages is substantially uniform, and which are practically non-oscillating even for the longest surges.

2. A major improvement in the impulse strength of the insulation has been made through the elimination of creepage surfaces from the entire insulation structure.

Appendix I discusses the merits and limitations of uniform voltage distribution. It is pointed out that uniform distribution of voltage within transformer windings has for its object the uniform distribution of the voltage stress throughout the insulation. Even perfect distribution of the surge voltages throughout the windings equalizes the voltage stress primarily on only part of the insulation structure, which is the insulation between turns and coils. There still remain at the ends of the coil columns or groups, high stress concentrations in the major insulation between the winding and ground, which are in no way mitigated by uniform distribution within the winding. Furthermore, in shielded transformers the introduction of the large metal surfaces at line potential in close proximity to the entire high-voltage winding requires the introduction of an additional insulation structure between the shields and winding in which high stress concentrations exist. In the new surge-proof transformers the initial distribution is not only substantially uniform within the winding, but the voltage stress is fairly uniformly distributed throughout the entire insulation structure.

These results have been obtained by simply employing proper design proportions and coil arrangements—a surprisingly simple explanation of such an important achievement. A static plate has been used across the face of the first coil to assist the surge in entering the winding and to eliminate the necessity of padding the line coil. No shields have been employed.

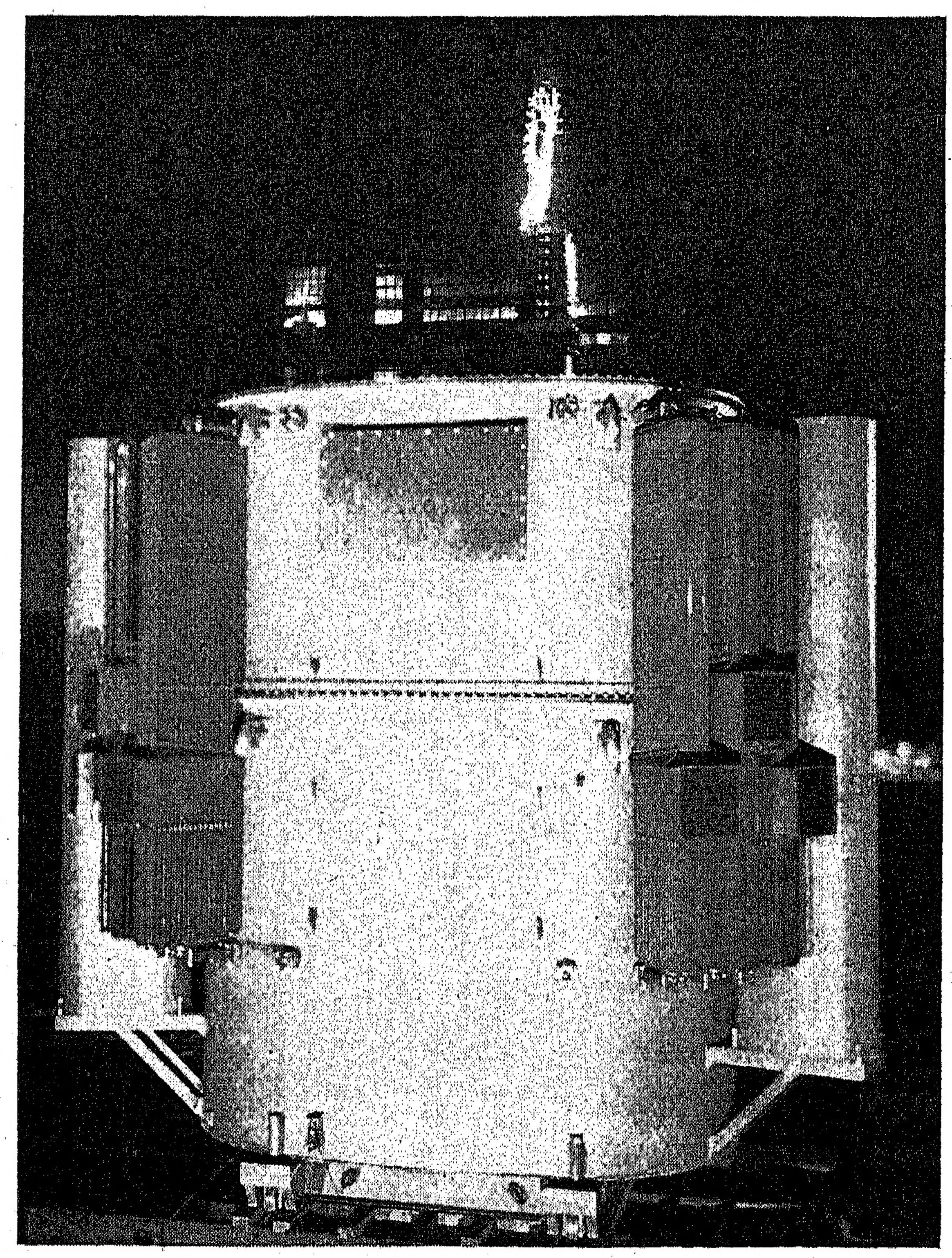

FIG. 1—LIGHTNING TEST ON 42,000-KVA., 220-KV. TRANSFORMER, SHOWING BUSHING FLASHOVER AT APPROXIMATELY 1,500,000 VOLTS AT A TIME LAG OF 2½ MICROSECONDS

Two distinct arcs are seen, because of the metal flange at the middle of the bushing

As pointed out on previous occasions, the initial distribution of surges within individual shell type groups is practically uniform because of the relatively high ratio of coil-to-coil capacity compared to the capacity to ground.[3] It is this inherent characteristic of shell type groups which has been utilized to produce an initial distribution that is substantially uniform, and to eliminate internal oscillations.

*Mgr. Transformer Engg. Department, Westinghouse E. & M. Co., Sharon, Pa.

†Both long and short wave surges, representative of lightning surges coming in over the line, were applied, and, in addition, a final surge having a wave front of 11,000,000 volts per microsecond was applied to simulate a direct stroke of lightning to the line close to the apparatus. This final surge was probably much more severe than any likely to be produced by real natural lightning.

1. For references see Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

Appendix II discusses the research work done on transformer insulation which pointed the way to the elimination of creepage surfaces.

It was reported last year by the Transformer Subcommittee that there was at least an approximate relation between the ultimate impulse strength of insulation structures as used in transformers and their low-frequency dielectric strength, and therefore the ordinary low-frequency induced voltage tests could be considered a measure of impulse strength.[4] The results of researches now indicate that this conclusion is questionable; that there is no necessary relation between impulse strength and 60-cycle strength; that the impulse ratio of a complete transformer depends on how much of the low-frequency dielectric strength is obtained by the use of creepage surfaces.

Experiments indicate that creepage surfaces have low impulse ratios compared to puncture barriers, and hence to build an insulation structure with the highest impulse ratio and therefore with the highest ultimate impulse strength requires the complete elimination of creepage surfaces.

It was found possible to apply this principle to practical designs with only very simple changes in the insulation structure. In the new surge-proof transformers the insulation barriers are arranged substantially coincident with the equipotential surfaces of the electrostatic field as explained in the appendix. Any insulation failure either between coils, or lengthwise of the coil group, or from any point in the winding to ground, must result from a puncture of a number of barriers proportional to the normal difference in potential between those points. In these transformers, "volts per inch," the design engineer's measure of the insulation strength of creepage surfaces, has become a thing of the past.

In appearance, these new surge-proof shell type transformers are not different from their predecessors. The casual observer would probably notice no difference between the core and coil structure of the old and the new. Yet the simple changes which have been made have set a new standard of performance in withstanding lightning surges.

The improvements in insulation design which have made this possible are the final achievement of several years of fundamental research. That these important results have been obtained in such simple fashion is a tribute to the skill of those engineers who participated in this development.

Appendix I

Merits and Limitations of Uniform Voltage Distribution

A fundamental problem to which consideration must be given in designing surge-proof transformers is that of reducing the dielectric stress, in the insulation, resulting from surge voltages penetrating the winding.

It is clear that nothing within the transformer winding itself can in any way affect the magnitude of surge voltages to which the high-voltage end of the winding is subjected. Whatever voltage is impressed upon the terminal of the transformer bushing is also impressed on the high-voltage end of the winding. Therefore, it is necessary to insulate between the high-voltage end of the winding and ground for the very maximum surge voltage which may be impressed on the transformer bushing by the most violent lightning surge. This voltage is, of course, limited by the flashover value of the transformer bushing, but this may be very high for surges having extremely steep wave fronts.

However, even if nothing can be done in the design to reduce the surge voltage at the high-voltage end of the winding, it is desirable to have that voltage uniformly distributed throughout the winding in order to equalize

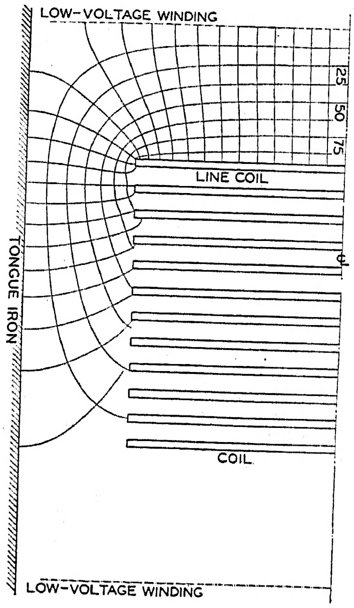

Fig. 2—Electrostatic Field and Equipotential Surfaces in Shell Type Transformer Winding with Uniform Distribution of Voltage

This plot shows the voltage stress concentration at the high-voltage end of the coil group which occurs even with uniform distribution

the voltage stress on the insulation between the turns and coils.* It is clear that if the surge voltage piled upon the end of the winding more insulation would be required between turns and coils at the end of the winding, and this extra insulation would be unnecessary and ineffective for normal voltage. If, however,

*Engineers have long recognized the merits of uniform voltage distribution and given consideration to methods of obtaining it. The condenser bushing is perhaps the best known example of an insulation structure in which means are provided to give a uniform distribution of voltage stress throughout the insulation. As early as 1912, Fortescue invented and patented (see U.S. patent 1,129,463) means of shielding core type transformers to obtain uniform distribution. The development of the cathode ray oscillograph stimulated investigations of transient phenomena in transformer windings generally, and in the case of core type transformers, led to the work of Palueff in developing means of shielding.[5]

surge voltages are uniformly distributed like the normal voltage, then the insulation which would be placed uniformly between turns and coils for the normal voltage is properly located to withstand the surge voltages also. Therefore, uniform distribution does make it possible to make the most effective use of a given amount of turn-to-turn and coil-to-coil insulation.

But uniform distribution, even though perfect, does not equalize the voltage stress in any marked degree in the major insulation between the high-voltage winding

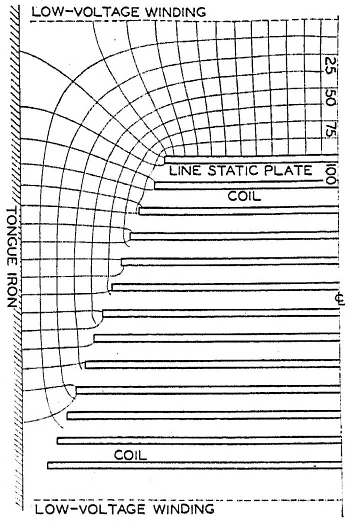

Fig. 3—Electrostatic Field and Equipotential Surfaces in Surge-Proof Shell Type Transformer Winding of the Single Group Type with Uniform Distribution of Voltage

It will be noted that the voltage stress throughout the insulation between the winding and tongue iron has been made substantially uniform and the stress concentration at the corner of the group has been materially reduced

and the low-voltage winding or ground. At the ends of the coil columns or groups there still remain points of high stress concentration which are practically unaffected by the distribution of voltage within the winding. In Fig. 2 is shown a plot of the electrostatic field with uniform distribution of voltage showing these points of concentration at the end of a shell type group. Similar points of concentration occur in core type transformers at the ends of the coil columns. Figs. 3 and 4 show similar plots for two of the winding arrangements used in the new surge-proof transformers. It will be noted in Figs. 3 and 4 that points of stress concentration have been greatly reduced throughout the entire insulation structure and the insulation is stressed substantially uniformly at all points. This is a very important result. Uniform distribution, desirable as it is from the standpoint of the turn and coil insulation, is only a partial solution to this major problem of equalizing the voltage stress throughout the entire insulation structure. But this major problem has been solved directly in these new surge-proof transformers and by the simple expedient of employing proper design proportions. In solving it, a distribution of surge voltages which is substantially uniform within the winding follows as a secondary result.

There is nothing in this simple method of solution which in any way limits its application to practical transformer designs. Fig. 5 shows a distribution curve plotted from test results on a sample transformer having a single shell type group. Fig. 6 shows a comparison between calculated and tested results on another sample transformer. Fig. 7 shows the same comparison for the 42,000-kva., 220-kv. transformer mentioned in the paper. Fig. 8 shows a calculated initial distribution curve for a 20,000-kva., single-phase, fully insulated transformer to be used for single-phase railway distribution.

No discussion of this subject would be complete without touching on the subject of internal oscillations produced by surges. Engineers are agreed on the following facts pertaining to internal oscillations:

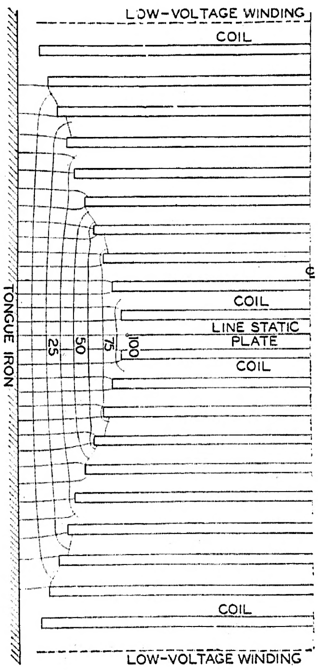

Fig. 4—Electrostatic Field and Equipotential Surfaces in Surge-Proof Shell Type Transformer Winding of the Twin Group Type with Uniform Distribution of Voltage

Since the high-voltage lead enters the middle of the group, voltage stress concentrations at the corners are inherently eliminated, and due to the construction, a uniform stress distribution throughout the entire insulation structure is obtained

1. Surges cannot produce severe internal oscillations unless the duration of the surges is longer than the natural period of the transformer.

2. Surges cannot produce severe internal oscillations regardless of their duration if they are uniformly distributed throughout the winding.

3. In certain types of transformers where natural periods are such that violent internal oscillations occur due to ordinary long wave lightning surges, such

oscillations may overstress and cause a failure in the major insulation between winding and ground, unless these stresses have been taken into consideration in the design of the insulation.

The shell type transformers with graded insulation previously manufactured by the company, with which the writer is connected, have not been subject to violent internal oscillations because their natural periods have been much longer than the wavelength of surges of any appreciable magnitude produced by lightning. On this account, no failure of major insulation has ever occurred

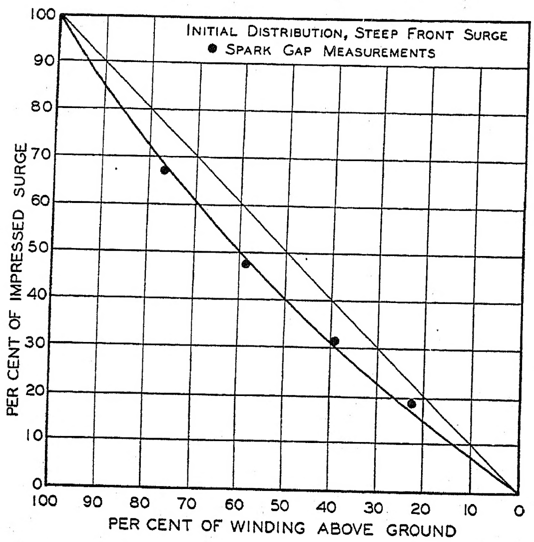

FIG. 5—INITIAL SURGE-VOLTAGE DISTRIBUTION IN A SAMPLE SHELL TYPE TRANSFORMER HAVING ONE HIGH-VOLTAGE GROUP

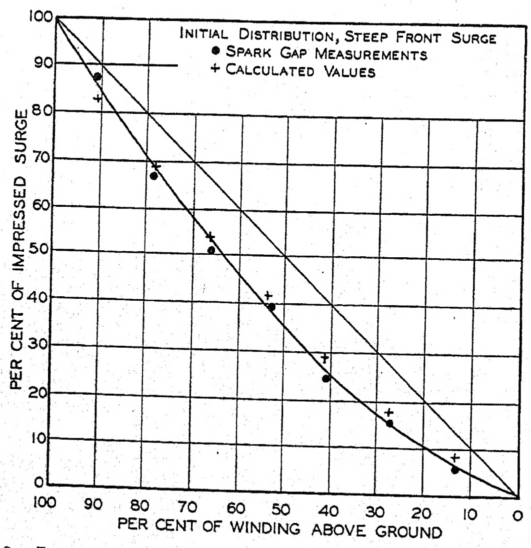

FIG. 6—INITIAL SURGE-VOLTAGE DISTRIBUTION IN A SAMPLE SURGE-PROOF SHELL TYPE TRANSFORMER

in these transformers due to lightning surges. While the new transformers have much shorter natural periods than their predecessors, they are practically non-oscillating because surges are distributed substantially uniformly throughout the winding. Fig. 7 shows both the initial and maximum surge voltage distributions.

In all these tests surges having a steep front and a very long tail (60 microseconds to half value) were used. Test points taken both by sphere gap measurements and by cathode ray oscillograph are shown. These indicate close agreement between calculated and tested results. As would be expected, the results show no appreciable internal oscillations.

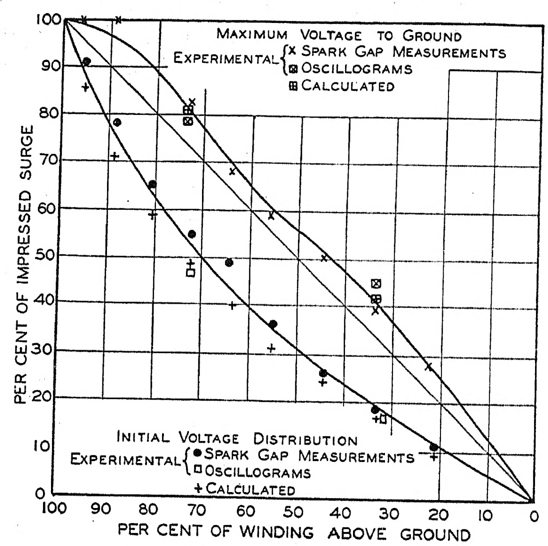

FIG. 7—SURGE-VOLTAGE DISTRIBUTION IN A 42,000-KVA., 220-KV. SHELL TYPE TRANSFORMER

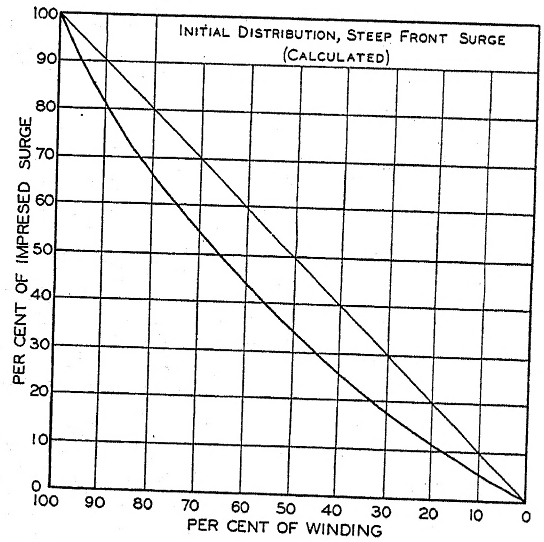

FIG. 8—CALCULATED INITIAL SURGE-VOLTAGE DISTRIBUTION IN A 20,000-KVA., 132-KV. SURGE-PROOF SHELL TYPE TRANSFORMER WITH FULL INSULATION FOR SINGLE-PHASE RAILROAD TRANSMISSION

Appendix II

IMPROVEMENTS IN TRANSFORMER INSULATION

It is well known that insulator strings, needle gaps and similar structures have a much higher dielectric strength when subjected to impulse voltages than when subjected to 60-cycle voltages. The ratio of the impulse strength to the 60-cycle strength is called the impulse ratio. For a particular structure, the impulse ratio is

constant for a given wave, but for very short waves the impulse ratio increases in value. Tests on insulation samples indicate similar characteristics. The report of the Transformer Subcommittee published in 1930 states that different types of transformers having the same low-frequency dielectric strength have about the same impulse strength, and therefore about the same impulse ratio.

A very simple experiment can be performed which

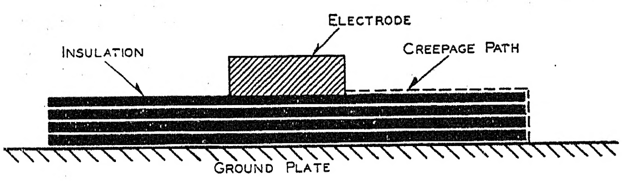

FIG. 9—EXPERIMENT TO DETERMINE CHARACTERISTICS OF CREEPAGE PATH

might cause one to question this conclusion. In Fig. 9 is shown an electrode separated from a ground plate by an insulation barrier. The whole is immersed in oil and tests made to determine the dielectric strength of the barrier as function of its extension beyond the electrode for both low-frequency voltages and impulse voltages. Fig. 10 shows the results. The low-frequency dielectric strength increases rapidly with the extension,

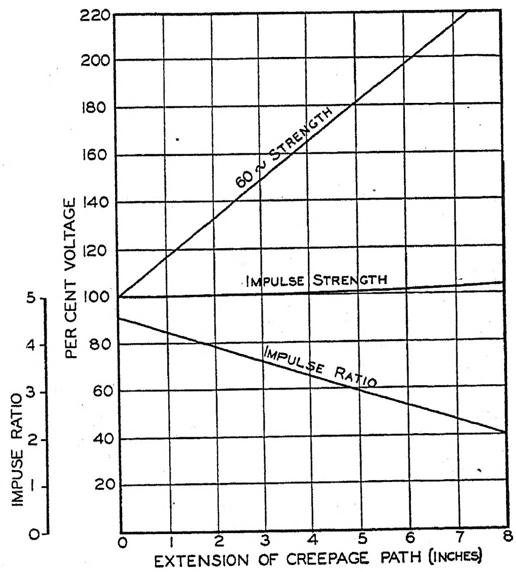

FIG. 10—DIELECTRIC STRENGTH OF A CREEPAGE SURFACE FOR 60-CYCLE AND IMPULSIVE VOLTAGES

but there is no measurable increase in the impulse strength as the extension is increased. If the impulse ratio is plotted as function of extension, it is seen that the greater the extension the lower the impulse ratio. Here at least is a humble example of a type of insulation structure whose impulse ratio may vary over a wide range.

This simple experiment seems to indicate that the creepage surface over the barrier gives practically no impulse strength. In Fig. 11 is shown the electrostatic field through the barrier of Fig. 9. The equipotential surfaces cut obliquely through the surface of the barrier. Similar creepage surfaces located in essentially the same manner with respect to the electrostatic field are found in both core or shell type transformers extending lengthwise of the coil groups or columns. See Fig. 2, Appendix I. It is also known from field experience and from tests on sample transformers that failures due to surge voltages very often take place along these surfaces.

Accordingly, an investigation was made to see what effect the elimination of these creepage surfaces from the transformer insulation might have on its impulse strength. Two sample transformers were built with the

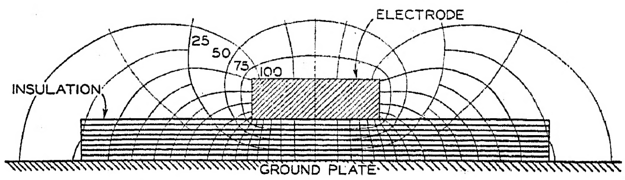

FIG. 11—ELECTROSTATIC FIELD DISTRIBUTION BETWEEN ELECTRODES OF TEST PIECE

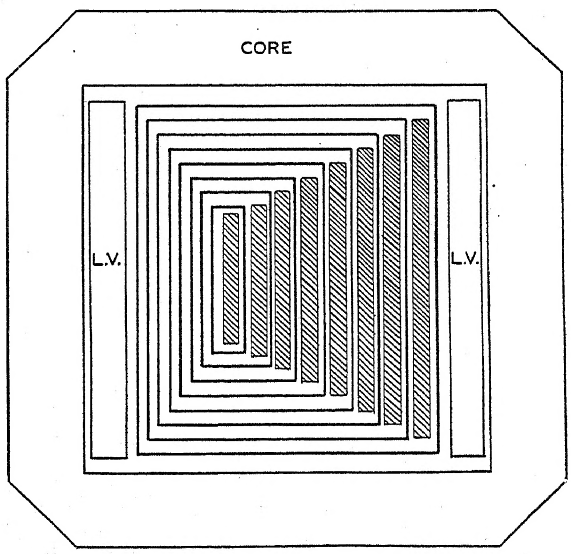

FIG. 12—ARRANGEMENT OF INSULATION BARRIERS IN THE HIGH-VOLTAGE WINDING OF THE SURGE-PROOF TRANSFORMER

same insulation clearances. One was insulated in the conventional manner; in the other, creepage surfaces were eliminated. Both were tested to destruction with impulse voltages. The one without creepage surfaces withstood nearly twice as much voltage as the other.

These and similar investigations led to the conclusion that the impulse ratio of a complete transformer depends on how much of the low-frequency dielectric strength is obtained from the use of creepage surfaces.

It follows that for the maximum impulse ratio, creepage surfaces should be eliminated entirely from the insulation structure. This means that the insulation barriers should be coincident with the equipotential surfaces of the electrostatic field. While this cannot be

accomplished exactly in a practical design, it has been very closely approximated. In fact, any failure in the insulation structure from one coil to another coil or to ground must result in the puncture of a number of barriers proportional to the normal potential difference between those points.

Fig. 12 shows how this is accomplished by the unique arrangement of the insulation barriers between coils and the core. By referring to Fig. 3, which shows the electrostatic field, it will be seen that the surfaces of the barriers lie essentially along the equipotential surfaces.

It will be noted that the same proportions and arrangements of coils which have made it possible to equalize the dielectric stress so well throughout the entire insulation structure have also made possible this simple arrangement of insulation barriers in such a manner as to eliminate creepage surfaces.

This is indeed a simple solution to the problem of building surge-proof transformers. Its effectiveness has been proved by the most drastic tests which it has been possible to devise. It resulted from a careful study of the basic fundamentals of the problem.

In conclusion, the writer desires to acknowledge the helpful assistance of P. L. Bellaschi for his invaluable theoretical investigations, of J. K. Hodnette for his suggestions and experimental work, and of C. L. Fortescue and F. J. Vogel for their advice and direction throughout this entire development.

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Discussion

R. E. Hellmund: While in Europe in the summer of 1925, I was very much surprised to find that it was quite general practise with most manufacturing concerns on the Continent to carry out both experimental and commercial surge tests on insulators and transformers. Apparently such practise had been initiated shortly after the war, and by 1925 had been quite generally adopted. Since this was in marked contrast to American practise at that time, I was naturally interested in learning why such a difference existed. Although it was of course impossible to obtain very definite reasons, answers to my inquiries gave me the impression that the practise of surge testing in Europe had been brought about by a greater number of failures of transformers in service, which in turn most likely should be considered as an aftermath of the war in so far as it had undoubtedly interfered with normal design progress. In other words, the European engineers evidently were forced to the expediency of surge testing at a time when, due to the lack of sufficient data on surges occurring in service and the lack of means for analyzing their laboratory and commercial test data, the entire procedure contained a great many elements of uncertainty. Nevertheless, this may have contributed to the improvement of their designs, which seems to be somewhat borne out by the fact that they have now obtained a great degree of reliability in service.

In contrast to this condition prevailing in Europe, the service record of high-voltage transformers in America always had been very satisfactory and it was therefore quite natural that the American engineers should have hesitated to introduce the practise of surge testing until sufficient data had been accumulated with regard to surges actually occurring in the various systems and until, by the use of the cathode-ray oscillograph and similar developments, it had become possible definitely to analyze and standardize factory tests as well as to study design features of the apparatus such as those described in Mr. Putman's paper. In the absence of definite operating troubles, such orderly procedure was undoubtedly justified until now, when after a great amount of work has been carried on and completed along this line, surge testing not only can be carried on but also can be interpreted as a measure of what might be expected of a piece of apparatus in actual service. The practise now arrived at in this country is, as a result of the more deliberate method of developing it, in marked contrast to the European methods previously mentioned. In nearly all cases, the latter consisted of the discharge of the stored energy in the transformer winding itself, usually at relatively low voltages, while, with the testing methods referred to in Mr. Putman's paper, a surge involving several times the rated voltage, considerable amounts of energy, and very steep wave fronts is impressed upon the transformer winding from a surge generator and recorded by means of the cathode-ray oscillograph. Although the method of testing with surge generators was used in Europe in 1925 in connection with lightning arresters, insulators, and switches, I did not find any instances of its use in connection with transformers.

F. W. Peek, Jr.: I hope that Mr. Putman will continue the research which he has begun on lightning-proof transformers. I will discuss only one phase of Mr. Putman's paper. Fig. 9 illustrates a test devised to show that creepage distance is of little value as an insulation. In reading the paper the impression might be gained that this corresponds to a usual transformer insulation arrangement. It does not. The poor results are obtained because of the very high stress between electrodes perpendicular to the creepage surface. This stress starts corona, for either impulse or 60 cycles, which causes leakage over the surface. The effect could be temporarily stopped by boxing in the electrodes. However, deterioration and failure would soon result. Such an arrangement is analogous to the transformer with very high stresses between coils. Such dangerous stresses are indicated by corona discharges during the induced voltage test if measures are taken to detect them. The skilled designer would avoid this condition because it would mean a transformer of short life.

C. L. Fortescue: Great progress has been made in recent years in the design of transformers and the manufacturers have made an important contribution to the electrical industry in bringing out these surge-proof non-oscillating transformers. The principles used in obtaining these results have been known for a good many years, perhaps the first germ of the idea was incorporated in a transformer constructed at Cornell in 1899 by Professor Harris J. Ryan with the aid of Mr. E. F. Scattergood using what he has called the "sectored guard surface." About 1908 Mr. Reynders conceived the idea of using alternate layers of insulation material and conductors in transformers but, while he had all the elements for obtaining the surge-proof transformer, he failed to coordinate the field distribution of the coils with that of the insulating structure. In March 1913 in a paper before the A.I.E.E., I showed that it was possible to group a system of conductors of different potentials so that they will assist in insulating one another. In the discussion there was

a great deal said about the principles of shielding insulating structures, illustrated by the theory of the condenser terminal and methods for obtaining uniform distribution of voltage over insulator assemblies, and so forth. The discussions illustrated rather forcibly the confusion of mind that existed at that time with regard to surges in general, surges being often referred to as high-frequency voltage and sometimes as high-frequency surges and this confusion of mind still exists at the present time as illustrated by the many requests by engineers as to the frequency of the lightning surge and references to artificial surges as having a certain frequency. In the discussion I pointed out that transformers designed on the electrostatic principle expounded in this paper could be made highly resistant to surges. In applying this principle to produce non-resonant transformers the so-called condenser type insulation could be used to insulate the core and case from the windings. In this way the winding space is isolated from the external space and through condenser action between the high-and low-voltage cylinders and plates produces a uniform electric field in the winding space which will therefore have the same potential distribution as the windings themselves as determined by electromagnetic induction. The use of condenser action to obtain desirable field distribution in isolated space is plainly stated in the paper and discussion, and in the text of my patent No. 1,129,463 and reissued patent No. 14,473. The essential requirement to obtain strength against impulses is that the dielectric circuit shall conform at every point with the electromagnetic circuit. This is the principle on which the present surge-proof transformers are based.

In 1915 Weed presented before the A.I.E.E. a paper on surges in transformers and in 1922 he was the author of a paper describing surge-resistant winding structures. More recently important contributions were made by Blume, Boyajian, Palueff, Hodnette, Bellaschi, and others which were directed towards simplifying the structures. Unfortunately there was not much incentive to designing transformers to withstand a lightning surge, the reason, I think, being that lightning was regarded as an act of God and it was assumed that nothing much could be done about it. It was realized that in most cases the lightning stroke was too far away to cause much damage and it was assumed that if it struck the line it was all up and nothing could be done about it. However, various protective devices were used and engineers were satisfied that nothing more could be done to insure the transformer windings against lightning.

The lightning failure at Wallenpaupack had the effect of rousing transformer designers from their false feeling of security and while I feel that danger to transformers has been somewhat exaggerated in some quarters, the jolt was just the stimulant required to start designers thinking about the importance of the dielectric design of transformers. I have always emphasized the importance of the dielectric circuit in transformers but exigencies of manufacture and the lack of knowledge of lightning surges acted as a deterrent in development progress. However, the work of recent years must be highly commended for the reason that the dielectric problem under surge conditions has been solved by recourse to relatively simple expedients which produce transformers presenting no serious difficulties in manufacture. In general, the simpler the insulating structure the better it is from a manufacturing and operating standpoint and in this respect the surge-proof shell type transformer must be acknowledged by everyone to be ideal.

A. B. Hendricks, Jr: The construction described appears to be logical and successful, and good results have been obtained under test.

Of "the three points of particular interest" mentioned, the third states; "transformers tested were of a new surge-proof construction involving fundamental improvements in the design of the insulation."

These "fundamental improvements" are not so very new. Fig. 1 is from my own drawing of a transformer which I designed and built and put in service in 1906, which exemplifies the fundamental principles involved. The winding is of the concentric form with circular coils, but the insulation is varied more or less in proportion with the voltages, and by the use of interleaved cylinders, flanged rings and formed insulating casing presents an absolute barrier between the high-voltage winding, low-voltage winding and core. I well remember my own statement at the time this transformer was built, that the insulation formed "an absolute and complete barrier and little or no dependence was placed on mere surface distance."

Since the middle point of the high-voltage winding is connected to the core, the voltmeter coil is located at the same point.

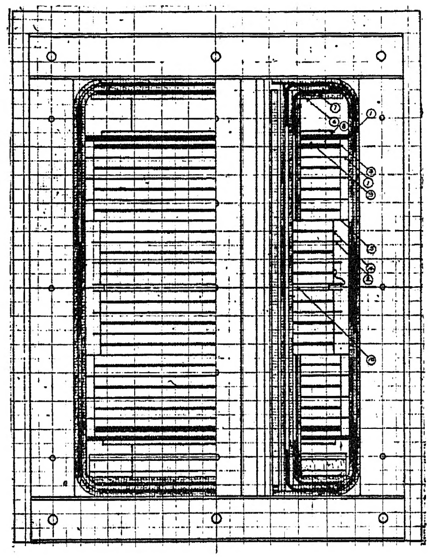

FIG. 1

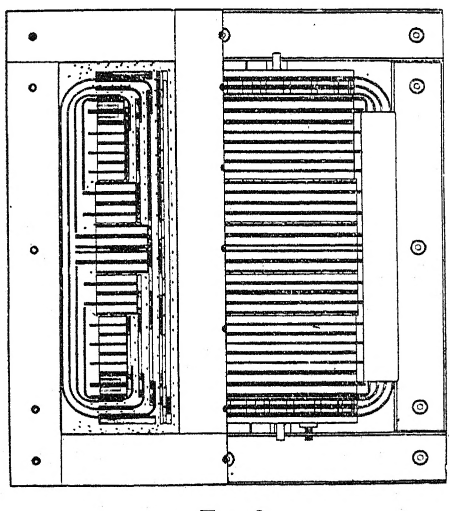

FIG. 2

The two coils at each end of the series are strip-wound, one turn per layer, the insulation between turns and coils being greatly reinforced. These were called "buffer coils," and I think this was the first design to which this term was applied, and it has persisted ever since.

Fig. 2 shows a 300,000-volt transformer of similar design built in 1913, the insulation in this case being in three steps.

Fig. 3 shows a 500-kilowatt, 578,000-volt transformer, being one unit of our original three-phase million volt outfit, as described in the A.I.E.E. JOURNAL October and November 1922. In this design, the insulation of the coil groups is varied in

many steps, each group being also heavily taped separately and insulated from the adjoining group by collars. A heavily insulated large radius static plate is placed at the top of the coil stack.

All of these three designs are testing transformers with concentric windings, in which the boxing covers three sides only of the coil group, the fourth side being insulated by the taping and collars.

The principle is identical with the scheme described in Mr. Putman's paper, in that the barriers are placed along the equipotential surfaces. Such an arrangement is common practise in transformer design. Naturally, our later designs show an improvement in arrangement of the insulation over the earlier ones.

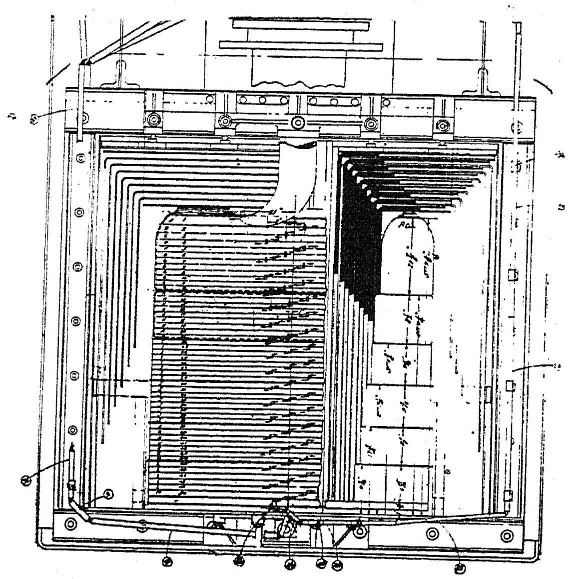

Fig. 3

While these designs described have a long and honorable history, I believe that they will be superseded by more modern and efficient designs of the non-resonating type.

J. F. Peters: Mr. Putman's paper covers an accomplishment in transformer construction that should be of great value in the economical transmission of power at high voltage. This type of construction contains flexibility that makes it possible to obtain practically any desired reactance in two- or three-winding transformers and at the same time obtain impulse strengths never obtained in transformers before. The tests of the 42,000-kva. units described in his paper show very definitely that proper proportioning of the windings gives almost perfect distribution of all impulse voltages.

This is not a surprise; it merely demonstrates theory. In a discussion of Mr. Palueff's paper on *Effect of Transient Voltages on Power Transformer Design*,* and of a paper by Mr. Brand and Mr. Palueff,* both presented in 1929. I pointed out that the proper physical proportions of the windings have a great deal to do with getting a good distribution of impulse voltage along the coil group and in keeping the amplitude of oscillations between the initial and the final distribution relatively small.

Mr. Putman's paper proves the correctness of those statements both as to voltage distribution and practical elimination of internal oscillations. In addition, he shows how to obtain impulse strengths of extraordinary value.

J. E. Clem: Near the end of the first page of this paper there is a statement to the effect that the "initial distribution of surges within individual shell type groups is practically uniform" and that this is an "inherent characteristic of shell type groups."

*A.I.E.E. Trans., 1929, Vol. 48, Part 3.

It is rather difficult for me to visualize the basis of these statements since my transformer experience and understanding of transients in transformers would lead me to draw an entirely different conclusion. In support of my belief that the initial distribution in a non-shielded shell type transformer is not uniform, I refer to Mr. Palueff's discussion of J. K. Hodnette's paper, A.I.E.E. Trans., Vol. 49, 1930, No. 1, p. 75, Figs. 4 and 5.

Reference to the coil sketch in Fig. 5 of the above discussion shows that the inside turn of the first coil was connected to the inside turn of the second coil and the outside turn of the second coil to the outside turn of the third coil (adjacent connection) and so on down the stack. It has been claimed that the difference between the voltages along the inside of the coil stock and those along the outside of the coil stack could be removed by changing the connections so that the inside of coil No. 1 was connected to the outside of coil No. 2 (cross connection) and so on down the stack. Although some shell type transformers may have been built in the past with this type of connection, I believe that the practise has been abandoned on account of the hazard. That a hazard will be introduced by this type of connection is substantiated by the following quotation from a discussion by Mr. J. F. Peters, which appears on page 701 of the 1929 Transactions, "but it is then necessary to carry the series connections through the oil ducts between coils where the space is necessarily limited and, therefore, produce somewhat of a hazard."

The initial distribution in a shell type transformer is not uniform even though the coils are cross connected. This is made clear by reference to Fig. 4, which shows the initial distribution over a group of shell type coils connected first one way and then the other. The solid curve shows initial distribution when the

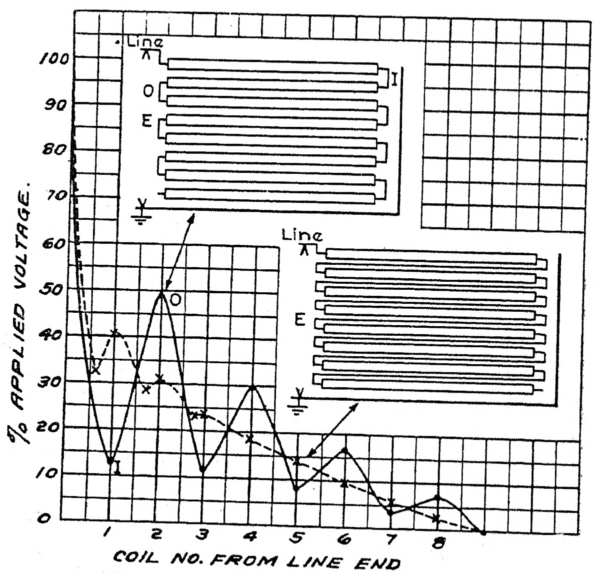

Fig. 4—Initial Voltage Distribution Non-Shielded Shell Type Coils (Clem)

coils are adjacent connected. The dotted curve shows the initial distribution when the coils are cross connected. The sawtooth appearance so pronounced when the coils are adjacent connected is not entirely eliminated by the use of the cross connection. The low points when the coils are cross connected occur at a point along the face of the coil nearer the inside than the outside end. Actually, the voltage between the first pair of coils along the outside is higher when the coils are cross connected than when they are connected in the other manner, the figures being 60 per cent and 51 per cent of the applied voltage respectively. It is very apparent from these curves that the initial distribution of a non-shielded shell type transformer can not

be made uniform by using the cross connection method of connecting the coils.

The author claims that a short stack of wide shell type coils inherently has a better initial distribution than a long stack of narrow (core type) coils. Comparison of Fig. 5 and the solid curve of Fig. 4 of this discussion indicates otherwise. The coil stack in Fig. 5 was obtained from the coil stack in Fig. 4 by making the coils narrower and the stack correspondingly longer,

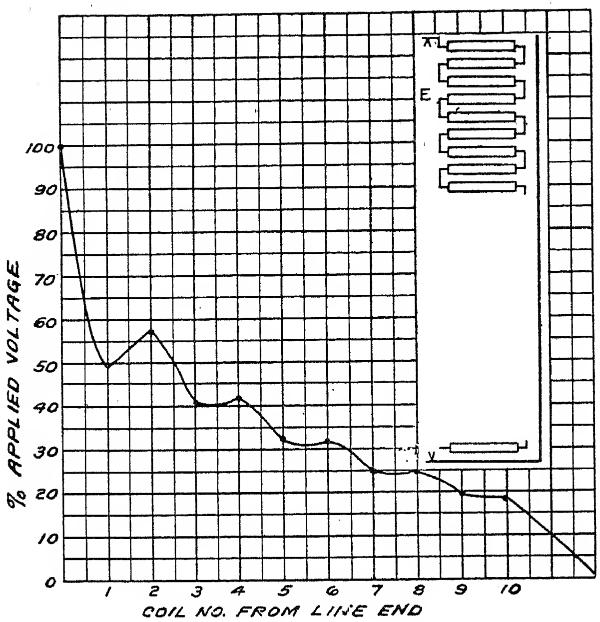

Fig. 5—Initial Voltage Distribution Non-Shielded Core Type Coils (Clem)

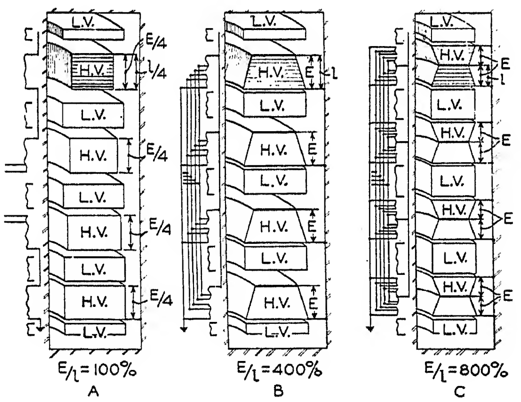

Fig. 6—Voltage Stresses Along Stack of High-Voltage Windings Under Assumption of Uniform Voltage Distribution

A. Conventional design of shell type transformer. (Total voltage is distributed between all groups)
B. New shell type design (full voltage is applied across each group)
C. New shell type design, twin groups. (Full voltage is applied across each half of each group. Maximum voltage per unit length of stack, maximum number of interconnections and leads)

the same values of winding being used, however, without changing the insulation. Non-modification of the insulation makes the comparison more favorable for the shell type group, since the wider coils would of necessity have a higher voltage between them and thus require more insulation. Comparison of the curve in Fig. 5 with the solid curve in Fig. 4 shows that the distribution for the shell type winding is actually less uniform than that for the equivalent core type winding. It follows, naturally, that the initial distribution in a non-shielded shell type transformer is not inherently uniform.

K. K. Palueff: The outstanding features of the core-type transformer as ordinarily built for grounded neutral service, are: first, the winding consists of two multiple paths from the middle of the stack to the ends (see Fig. 4 of Putman's paper); second, the winding is made non-resonating by means of an electrostatic shield connected to the line end and properly spaced from the high-voltage winding.

The first feature was adopted by us for high-voltage transformers about 1919. It eliminates the concentration of dielectric field at the end of the stack shown on Figs. 2 and 3 of the paper and therefore stresses more uniformly the insulation between high- and low-voltage windings.

The second feature was put into practise in 1926. It permits the most effective use of insulation throughout the winding since it allows accurate predetermination of voltage distribution throughout the entire structure. It also reduces the transient voltage stresses very materially by distributing them uniformly and preventing oscillation.

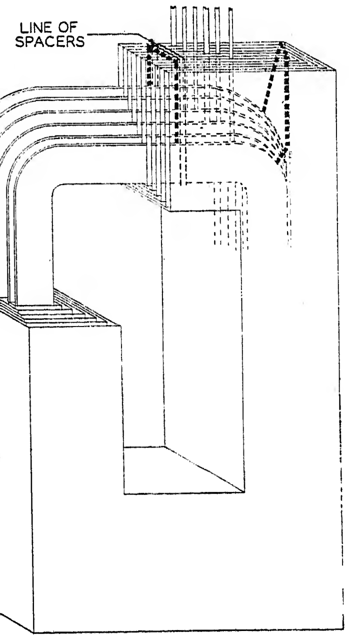

Fig. 7—Creepage Path Is Shown by Heavy Dotted Lines

It is most gratifying to find that these essential features of transformer design gradually are receiving more general recognition, as unqualified support given to them by Mr. Putman indicates.

The practical importance attached by Mr. Putman to these two features is evident from his willingness to adopt a distribution of windings radically different from that previously employed in shell type transformers in spite of the fact that this change resulted in as much as eightfold increase in voltage stress between coils and along the high-voltage stack, and also in an increase in the number of high-voltage leads (line and taps) in the same ratio as can be seen from comparison of sketch *A* and *C* of Fig. 6 of this discussion.

Mr. Putman believes that he succeeded in the introduction of a third major improvement in transformer design "through the elimination of creepage surface from the entire insulation structure."

It is obvious from Fig. 7 of this discussion that creepage path in the new design exists on the top and bottom of the insulation in essentially the same manner as it existed in the old design. It seems to me that such a creepage path is unavoidable because

if the windings were "boxed in" on the top and bottom in the manner shown in Fig. 12 of the paper, no oil could possibly circulate between the windings and tank which is an impossible condition for a power transformer.

The comparison of arrangement of windings and of the points of application of voltage in the old and new shell type design (Fig. 6 of this discussion) indicates that the full line-to-ground voltage is crowded in the new design into from 1/3 to 1/8 of the space used for it in the old design. This results in a correspondingly increased voltage between coils and density of dielectric field.

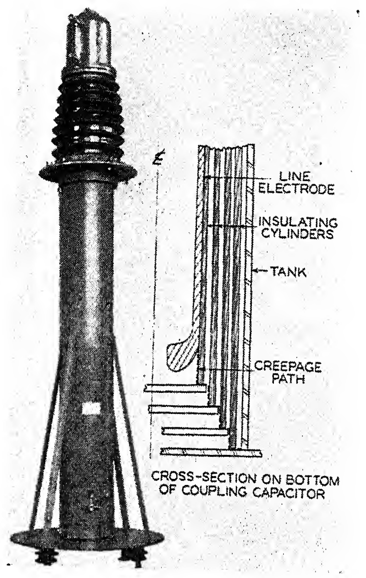

FIG. 8—RELIABILITY OF CREEPAGE PATH

Oil insulated (0.001 μf.) coupling capacitor for 110-kv. transmission system, tested in 1925 with 225 impulses of artificial lightning with amplitudes of applied waves just below or above bushing arcover

The introduction of barriers in those parts of the structure surrounded by the core, where proper length of creepage path can not be secured, therefore is absolutely imperative. Unfortunately, as Mr. Putman states, these barriers only double the strength of a given insulating space while, as shown here, the stress is increased as many times as there are groups (from two to eight times or so).

In view of all this, the stress along the creepage surfaces at top and bottom in the new design is necessarily appreciably greater than in the old. Since the vertical extension of the insulating barriers in the old design could be made just as long as in the new, it is not clear why the author expressed such a definite preference for the new arrangement of the windings.

Dielectric Strength of Creepage Path. The dielectric strength of a creepage path (whether for 60 cycles or impulse voltage) follows very definite laws, which are well known; where creepage path is used in accordance with these laws it is perfectly dependable.

One of the fundamental principles of design, where creepage path is used, is the proper proportioning of the thickness of barriers or other solid insulation to the length of the creepage path.

Another important characteristic of creepage path is illustrated in Fig. 9. It shows that the arcover voltage for a given creepage distance falls rapidly with decrease in distance between coils (or other electrodes).

This happens because the strength of a creepage path falls suddenly as soon as oil corona is formed on the edges of the coils. On the other hand, the voltage necessary to start corona is the lower the smaller the distance *d*.

Impulse Ratio. It is stated in the paper that the impulse ratio of creepage path is not a constant value, but depends on the length of the creepage path. I fail to see the practical significance of such a statement. The insulation allowances used between turns, between coils and between winding and ground, are selected by the designer without any reference to impulse ratio, but in accordance with the impulse strength determined by test for these various elements.

Non-Resonating Windings. To make a concentric type transformer non-resonating, it is only necessary to place an electrostatic shield on the outside of the high-voltage winding without any other change in construction.

Since coils of interleaved construction are generally many times wider than those of the concentric construction (they may reach some 25 to 30 in.) the high-voltage winding stack in interleaved transformers is necessarily much shorter than in concentric transformers. For this reason the stresses between coils and along the stack in concentric transformers are many times smaller than in interleaved transformers, particularly when the concentric transformer windings are located on two legs of the core, as is the general practise.

Another undesirable feature of the non-resonating transformer of interleaved type is the extremely high lightning voltage induced electrostatically in low-voltage windings from the high-voltage winding and shield as shown in Fig. 10. This happens because the low-voltage winding instead of being placed next to the core, as in concentric transformers, is placed in close proximity to the high-voltage coil and shield. From the approximate distribution of electrostatic field produced by a lightning wave, Fig. 10, it follows that the low-voltage winding rises above ground to some 60 or more per cent of the amplitude of the lightning wave impressed on the high-voltage terminal.

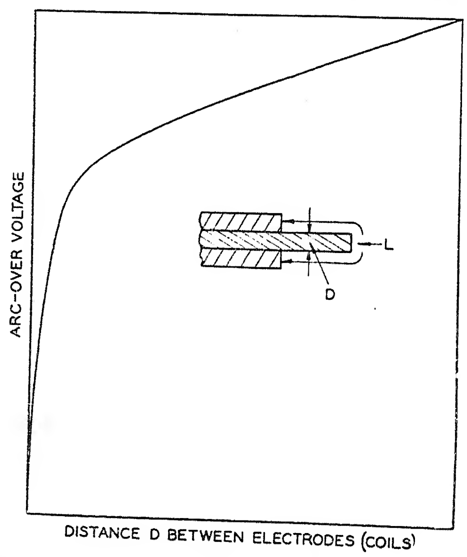

FIG. 9—REDUCTION OF ARCOVER VOLTAGE OF A CREEPAGE PATH

With reduction of distance *D* between electrodes when total length *L* of path is kept constant

Incidentally, it is interesting that a single shield placed on the top of the line coil as shown in Fig. 10 can not produce a uniform voltage distribution throughout the winding. This follows from the consideration of the electrostatic field shown, which results from the fact that only a few turns of the high-voltage winding, in the immediate vicinity of the grounded end of the winding, remain practically at the ground potential after the impact of lightning wave, while the rest of the ground coil, as well as all other coils, "float" in the electrostatic field as shown.

Short Stack of Wide Coils. Calculations and tests of electrostatic (or initial) distribution of voltage produced by steep wave

front are so simple that there should be no difference of opinion on the subject. In spite of it, the difference evidently exists.

For a number of years I have maintained that without a shield, initial distribution can not be made uniform or even made to approach uniformity by varying the proportions of the width of coils to the length of the winding stack. Furthermore, I know that in a short stack of wide coils without shielding, the initial distribution is extremely non-uniform. This knowledge is based on numerous tests and calculations, some of which were published by me before. Fig. 13 shows the initial distribution in an absurdly short stack of only four wide coils.

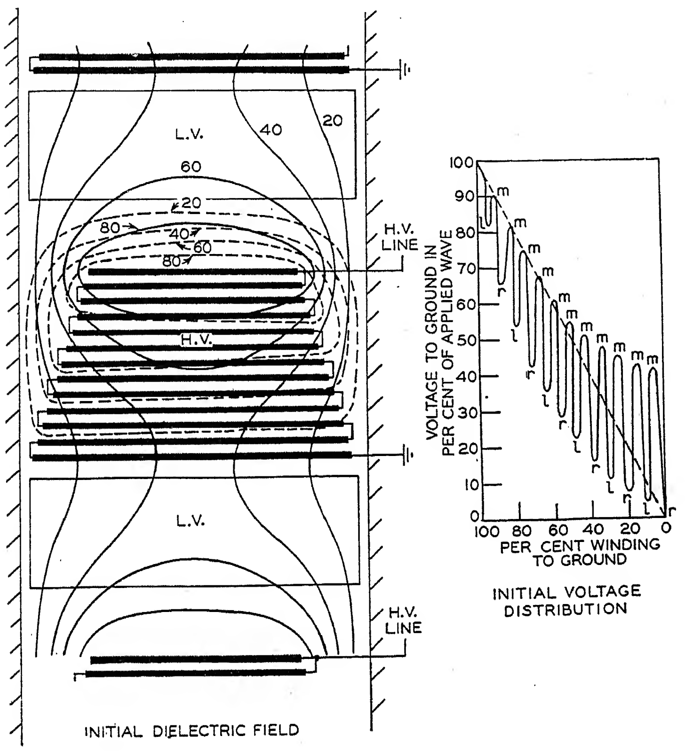

Fig. 10—Character of Initial Dielectric Field and Corresponding Initial Voltage Distribution Produced by Lightning Waves

Interleaved construction with high-voltage winding arranged similar to Fig. 3 of the paper
Solid lines—actual field
Dotted lines—Field necessary to make winding non-resonating and to prevent electrostatic induction of abnormally high potentials in low-voltage windings
l and *r* indicate the left and right edges respectively of the coils
m indicates the middle of the coils

I also previously explained that the increase in the width of the coils which gives, as stated in the paper, "a relatively high coil-to-coil capacity compared to the capacity to ground" can not have the beneficial effect, expected by some, on the initial distribution or on the subsequent oscillations.

It follows that an ordinary winding, not equipped with a shield, can not be made non-resonating by mere selection of proper width of coils and length of stack.

In spite of the simplicity of tests, certain precautions are necessary to secure correct results. In the case of wide coils, for example, it is imperative to measure voltages in different parts of each coil rather than limit these measurements only to turns located on the edges of the coils, as is frequently done. For instance, if on Fig. 10, a line were drawn over points marked *r* and *l*, which indicate the voltages on the outside and inside edges of coils, it would give quite a smooth curve which approaches the uniform distribution line. However, the actual distribution, shown by the solid wavy curve, is extremely non-uniform with the *middle* of each coil at much higher potential than its edges and with as much as 30 to 40 per cent of transformer terminal voltage appearing across half of a coil near the ground end of the stack.

In case measurements are made only on the outside edges, the difference between conditions indicated by such tests and the true conditions can be seen from a comparison of curves 1 and 1*a* of Fig. 13.

With the above in view, I ask Mr. Putman whether the curves shown in his Figs. 5, 6, 7 and 8 are the results of measurements made at the *middle* and at *both edges* of each coil.

Mr. Putman states that he uses a "static plate" but not a "shield." I should appreciate it if Mr. Putman were to give his definition of the word "shield" since, in the light of the above, it is obvious that, in case his tests indicate values measured on both edges and in the middle of each coil, such results could not be obtained without the use of some special means to neutralize, to a marked degree, the effect of ground capacitance.

Could Mr. Putman also tell us how essential is the presence of his "static plate" for securing distribution shown by the above curves? What would the distribution be without "static plates"?

Reduction in Major Insulation Stresses. The author of the paper points out that in the winding of Fig. 3 of his paper the dielectric field concentration at the edge of the line coil is not as high as in the old construction shown in Fig. 2. It is obvious that if the distances between the core and the line coil were made the same in the two constructions, the overall stress and the concentration of field on the winding of Fig. 2 (old design) would be less than that of Fig. 3, while the field concentration on the line coil will not change appreciably. The reduction shown in Fig. 3 is caused by a 30 per cent increase in distance between coil and core rather than by any improvement in construction. Since the voltage from line coil to core in both constructions is identical it is not clear why it was necessary to make such an increase.

Lightning Tests. It is stated in the paper that the lightning tests described therein were the most severe ever performed on assembled power transformers. I question the correctness of this

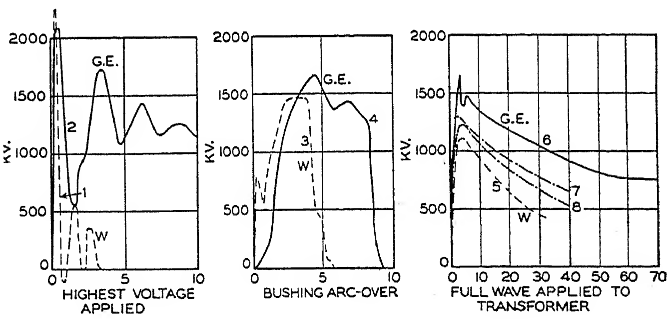

Fig. 11—Lightning Test on Core and Shell Type Transformers (230-Kv. Operating Voltage)

———— G. E. test August 1930 (core type)
— — — W test August 1931 (shell type) waves shown are applied to high-voltage terminal

statement. The tests we made about a year and a half ago (and reported at the time in the A.I.E.E. Transactions, January 1930) on one of the 13,000-kva., 230-kv. transformers built for the New England Power Co. consisted of over 200 impulses. Of these, over 40 were equal to or in excess of that necessary to arc over the 64-inch coordinating gap. Many were sufficiently high to arc over the transformer bushing. In Fig. 11 of this discussion are shown copies of the oscillograms of the most important waves used during these tests. Waves shown by dotted lines are taken from Dann's paper referred to by Mr. Putman

(*Electric Journal*, Dec. 1931). Wave No. 5 was taken from Mr. Putman's lecture given at Hazleton last fall. Our waves are shown by solid lines. No. 2 was not previously published. No. 4 was published in the *G. E. Review*. A wave practically identical to No. 6 was also published in the same periodical.

Mr. Putman considers that the wave No. 1 was the most severe because it rose at the rate of 11,000,000 volts per microsecond. It would be easier to agree with him if he gave some experimental data showing the effect on insulation of rate of voltage rise. I believe that this impulse was not particularly severe because of its extraordinary shortness. It lasted only two tenths of a microsecond.

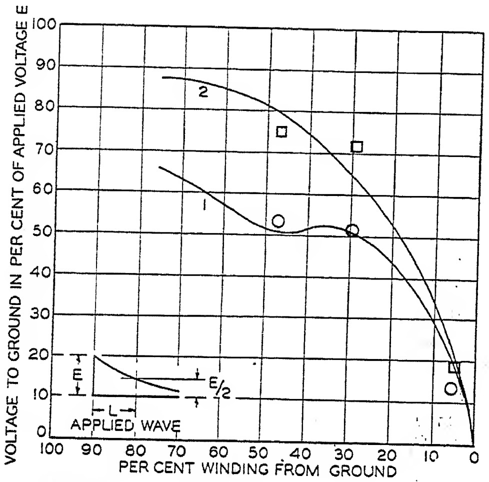

Fig. 12—Maximum Voltage to Ground Produced by Waves with Exponential Tails of Different Length (Test Results)

Length of wave in per cent of natural period of transformer 1—21 per cent (Palueff's test on core type transformer)
0 —22 per cent (Hodnette's test on shell type transformer)
2 —107 per cent (Palueff's test on core type transformer)
□ —113 per cent (Hodnette's test on shell type transformer)

Since it is recognized that the destructiveness of impulse waves increases not only with amplitude but also with the length of the wave, it is obvious that stresses produced by wave No. 2 were materially more severe than those produced by wave No. 1. Comparison of waves 3 and 4 also shows that our wave (No. 4) was more severe. The same is true in case of waves 5 and 6, where our wave is about 38 per cent higher in amplitude and twice as long.

Experience shows that real lightning as well as artificial may produce numerous "pin-hole" punctures or cause other localized damage to the insulation throughout the transformer without necessarily causing complete failure. At the present time there is no means available, without complete disassembly, to determine local damage of turn, coil or major insulation that may be produced by artificial lightning. I would like to know whether the 42,000-kva. transformers subjected to lightning tests by Mr. Putman were disassembled for minute examination of each coil and insulation parts.

For a number of years we have felt that the standard induced voltage test specified for grounded transformers was not an adequate test alone because the voltage stresses produced by such a test do not correspond either in magnitude or distribution to stresses produced by transient voltages in transformers of ordinary construction. This, in case of a grounded transformer for example, would permit building a transformer of given operating characteristics (and meeting all A.I.E.E. rules and tests) at a cost appreciably less than that of transformers designed to withstand transient voltage stresses. It is obvious that should lightning tests be made a part of A.I.E.E. acceptance test it would help to bring the standards of transformer design to a more uniform level.

For the above reason I heartily endorse Mr. Peek's suggestion made at the Convention, that the proper A.I.E.E. committee should consider the possibility of standardizing artificial lightning tests as a part of the acceptance tests on transformers, supplementary to the present induced voltage test of 3.46 times normal.

Effect of Length of Wave on Amplitude of Oscillations. Mr. Putman states "surges can not produce *severe* internal oscillations unless the duration of the surges is longer than the natural period of the transformer." A more accurate statement of this relation is as follows:

The voltage of any particular harmonic of transformer oscillation will not reach its *crest* value unless the length of the applied wave is at least half of the natural period of the harmonic.

W. M. Dann: In his paper Mr. Putman refers to shell type transformers with graded insulation, and this subject is discussed in the following.

About ten years ago Westinghouse engineers made an exhaustive series of tests to determine the distribution of surge voltage throughout the windings of shell type transformers of current design. In these tests, surges from a surge generator were impressed upon the high-voltage winding, and surge voltages to ground from various parts of the winding were measured with a sphere gap, as well as voltages between coils and between various parts of the windings. In those days we had no cathode ray oscillograph and it was not possible to reproduce a picture of the wave as it is done today. However, the shape of the wave was calculated from the constants of the circuit, and of course its magnitude could be measured directly with a sphere gap with a very fair degree of precision. It was on this basis that these distribution tests were made, first with a very steep wave having a long tail and then with a wave of less steepness decreasing to half value in approximately 20 microseconds. It

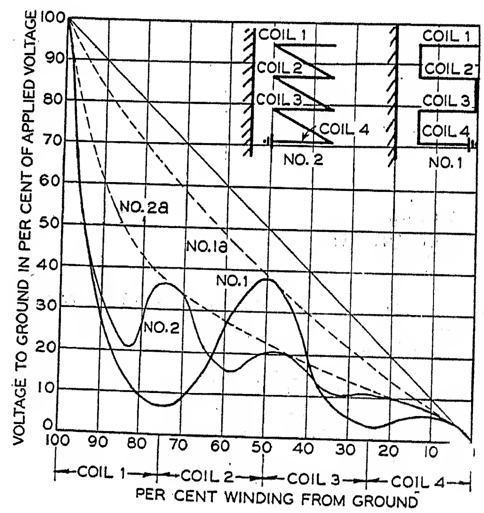

Fig. 13—Initial Distribution in a Stack of Four Very Wide Coils

1 and 2—Actual distribution (voltages measured throughout the coils)
1*a* and 2*a*—Apparent distribution (voltages are not measured throughout the coils)

took almost a year to complete them, and an idea of the thoroughness and care with which they were made may be gained from the fact that all results were checked by three of four different observers in repeated tests.

These tests showed very clearly that in the shell type transformer there were no dangerous voltages of the kind which one would expect if violent internal oscillations were developed by the entering surge. They furnished direct data on surge voltage stresses between winding and ground for the proper design of graded major insulation.

These actual tests supplemented in an important way the fundamental data on which Westinghouse design practises have been based. At the time they were made they were considered of prime importance; since then, they have become of great interest because of the repeated statements of Mr. Palueff and others to the effect that the practise of grading major insulation is hazardous.

That this practise is far from hazardous, I think, is clearly shown by the following facts:

1. There are more than three million kva. of 220-kv. Westinghouse transformers in successful operation with graded insulation, and the practise of grading the insulation for 220-kv. transformers is practically universal in this country.

2. There are approximately five million kva. of Westinghouse power transformers for 110-kv. and above in successful operation with graded insulation.

3. With all these transformers in service, to the best of our knowledge not a single failure of major insulation to ground due to lightning has been experienced in a Westinghouse shell type transformer having graded insulation; this record could hardly have been attained if dangerous voltages between winding and ground were actually set up by internal oscillations due to lightning surges.

References have been made at various meetings of the Institute by Mr. Palueff to the failure of a 220-kv. transformer at the Leaside station of the Hydro-Electric Commission at Toronto during a lightning storm. This transformer happens to be a Westinghouse shell type unit having graded insulation. These references appear on page 79 of the January 1930 TRANSACTIONS, and again on page 1195 of the July 1930 TRANSACTIONS, and once more on page 69 of the March 1931 TRANSACTIONS. In these references, this failure has been explained as a failure of the graded insulation to ground produced by violent internal oscillations, and it has erroneously been pointed to as an example of the hazard of grading the major insulation.

As a matter of fact, there was no failure of insulation to ground in this transformer. It was purely a failure across the high-voltage group of coils due to the presence of a tap lead, required for temporary 154-kv. operation, and this lead proved to have insufficient surge strength. It was not an inherent weakness due to the principle of grading the insulation, because the grading had nothing whatever to do with the failure.

The new surge-proof transformer, which Mr. Putman describes, represents a distinct forward step in methods of insulating, and in the practise of grading major insulation. Heretofore the insulation of a transformer having graded insulation has been graded by groups; that is, the high-voltage group nearest the line has been given the full amount of major insulation, and the group nearest the grounded end has been given a reduced amount of major insulation to correspond to the voltage stresses developed in the group, and intermediate groups, if any, have had an intermediate amount of insulation. I think Mr. Putman has made it plain in Figs. 3 and 12 and in the text of his paper, that in the new surge-proof transformer the coils themselves have been graded in width and the insulation around the individual coils has been graded in amount. The result is that concentrations of stress have been eliminated and the amount of solid puncture insulation around each coil has been proportioned to match the stresses developed in that coil.

It seems to me that the remarkable surge strength which has been demonstrated by the actual application of surges to these transformers shows the soundness of the principles on which the new design is based.

V. M. Montsinger: I discuss Mr. Putman's paper from the standpoint of 60-cycle and impulse arcover of barriers as affected by creepage. My experience does not check the data given in Fig. 10 showing no increase in the impulse arcover voltage for long collar extensions.

In the first place if the impulse strength shown as a practically horizontal line in Fig. 10 is correct, this shows that creepage strength across the edge of the barrier must be approximately the same as the puncture strength. This follows because the creepage certainly can be made great enough to cause puncture. Since the impulse strength curve is practically horizontal the two strengths; *i. e.*, creepage over edge and puncture, must be approximately equal. This does not seem reasonable.

Numerous tests have shown that for a practical barrier as used in design work puncture occurs at approximately twice the voltage at which arcover occurs when the electrodes are even with the edge. This is true for both 60-cycle and ½/5 microsecond impulse waves. It follows, therefore, that under these limiting conditions the impulse ratio is approximately the same.

Fig. 14 shows the results of tests made on a barrier with various creepage distances.

My experience in breaking down complete transformers shows that approximately the same impulse ratio is found whether failure is entirely by creepage or by a combination of puncture of solid insulation and creepage when using the ½/5 microsecond wave. This refers to cases where oscillations are not involved.

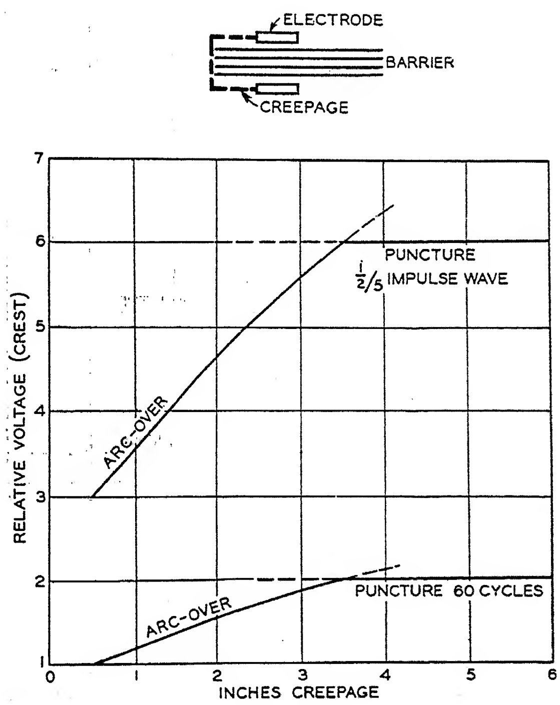

FIG. 14

I feel, therefore, that it would not be advisable to make any radical change in design practise as a result of the data given in Fig. 10 of Mr. Putman's paper.

So far my remarks have been confined entirely to breakdown, no reference being made to the appearance of corona in oil.

Fig. 15 illustrates the very wide range in 60-cycle voltage between the beginning of corona and the breakdown of two barriers of different thicknesses. These data show the importance of properly proportioning the barrier extension.

It is evident that the observable corona depends only on the thickness of the barrier and shape of the terminals; also that it is not of any use so far as suppressing corona is concerned to make the extension very much in excess of twice the thickness of the barrier.

If the barrier extension is many times its thickness and if the voltage is raised above the corona point, heavy local arcing may result in serious damage to the barriers long before arcover or puncture takes place. This is true of either 60-cycle or impulse voltage. For obtaining the data shown in Figs. 14 and 15 the

electrodes were symmetrically located on both sides of the barrier.

To see if I could check the constant impulse voltage curve, shown in Fig. 10 of Mr. Putman's paper, tests were made on a barrier in oil as near like this one as possible. Fig. 16 shows that with a fast-chopped wave the impulse voltage increased quite definitely with an increase in extension. For the ½/5 full wave the voltage increased more than with the chopped wave. For a still slower wave the increase in impulse voltage was more marked. Mr. Putman did not define the wave he used. I would not say that the constant voltage *versus* creepage can not be obtained though I feel that a condition that gives this result is not a practical one.

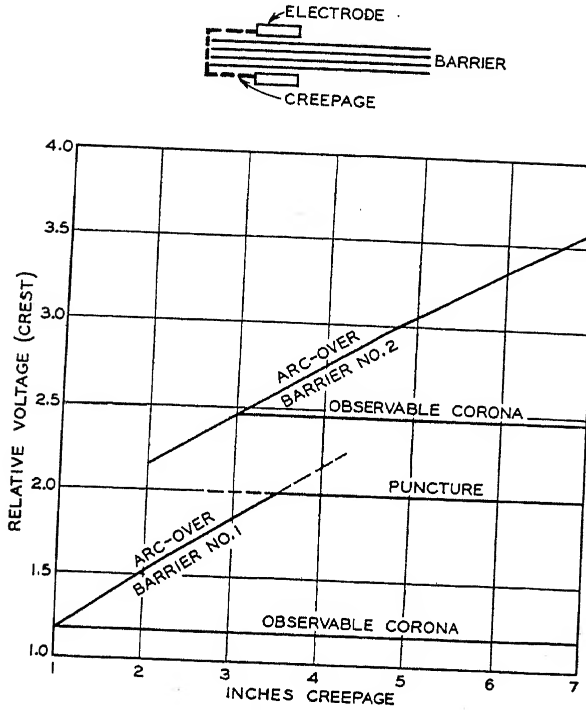

FIG. 15

On account of the possibilities of damage from corona it has been a long established practise with the General Electric Company to proportion the insulation barriers and to insulate the (electrodes or) ends of coil stacks by the use of taped buffer coils so that no corona occurs during any dielectric test. Dangerous corona in the oil usually results in the surface of the oil being disturbed.

In conclusion I want to emphasize two points—(1) that creepage distance if properly used can be depended upon for a certain insulation strength even with impulse voltage. (2) That no matter what tests on barriers show, impulse breakdown tests are finally made on complete transformer windings by testing them to destruction.

Transformers covering the complete voltage range have been tested to destruction in our laboratory by impulse voltage.

J. R. Gaston: Mr. Putman has obtained apparent success in building a surge-proof "pan-cake coil" shell type transformer. Unquestionably the design is successful between limits that will be shown. But to assume that the impulse tests signify, for this type of power transformer, the results in the degree that the paper seems to imply, fails to be impressive because of the omission of the following important data, namely:

1. Polarity of Impulses. An eleven-million-volt negative impulse is not necessarily as effective as a five-million-volt positive impulse. Both the paper and bibliography fail to give the polarity of the impulses.

2. Temperature of Transformer. The dielectric strength of both oil and fibrous material decreases with a rise in temperature, while their conductivity may increase several thousand fold or more in the operating temperature range of the average transformer. The paper does not give these data, and therefore from engineering point of view we must assume that the surge tests were made cold.

In commenting on Appendix II, it appears that explanations for Figs. 9 to 11 are in error, as follows, namely:

1. Breakdown creepage distance varies logarithmetically with the voltage, and not directly.

2. Fig. 11 would be correct if K for oil and the fibrous insulating materials were the same; the deduction being that unless the corners are heavily reinforced breakdown would result, and after all, that is the purpose of "angle rings" or "flanged collars" as used on core type transformers.

I agree with Mr. Palueff that under the present system of impulse testing the purchaser's property is more apt to be unnecessarily exposed to incipient breakdown, if tested hot, and I do not consider a negative cold surge test of much significance, with the present data available.

A. A. Emlen: In connection with the suggestion that the machinery committee investigate the possibility of incorporating in the Institute Rules a "surge test for transformers," I would offer the following comments:

It is not clear from the data given on surge tests as actually applied to transformers at what temperature these tests were

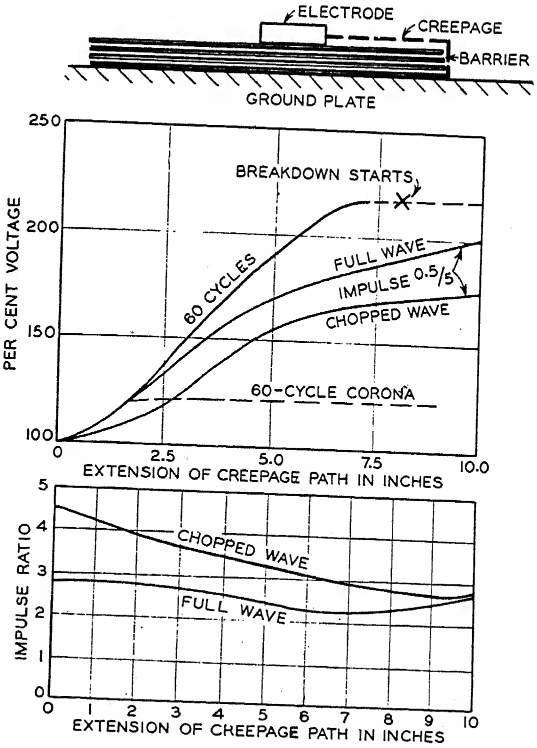

FIG. 16

made. Since the temperature of the dielectric, whether oil or solid, has a decided bearing on the voltage a transformer will stand (either 60 cycle or impulse) it would be interesting to learn whether the results obtained were at low temperatures or at 75 deg. cent.

If studies are to be made regarding surge tests for commercial transformers it seems that the temperature at which these tests are made is a very important point to consider.

Considerable progress seems to have been made in the development of apparatus to determine the existence of incipient

corona or breakdown. However, till these detectors are practically foolproof it would not seem advisable to apply surge tests of any magnitude to commercial transformers.

J. Murray Weed: It is most gratifying to me to see the progress which has been made and which still is being made in the development of surge-proof transformers. When the pioneer paper on this subject was discussed by the Institute in 1922,[1] the consensus of opinion was that, although it possessed great theoretical interest, its application was either unnecessary or impracticable. In closing that discussion, I put the best face I could on the situation by saying, "I have gone as far as I have been able to go in pointing the way and I would not be surprised to see the application of the general method here illustrated carried much further than seems to be expected by most of those who have discussed the paper."

The designers of the transformer described in the present paper must be given credit for having made an approach to a uniform gradient for surge voltages in shell type transformers. It is interesting to observe that, however much the arrangement here shown appears to differ from any of those shown in my paper referred to or from the non-resonating transformers which have been built by others, the fundamental principle involved is the

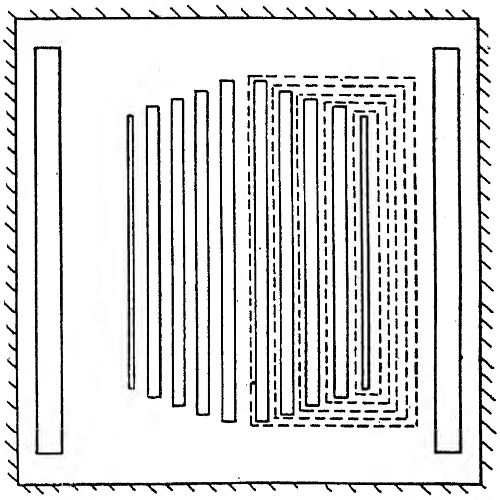

Fig. 17—Single-Group Shell Type Transformer Designed for Elimination of Creepage Surfaces

same in all cases. As stated in my paper, the method consists in so embedding the winding within the dielectric of a condenser, whose plates are connected to the winding terminals, that the potential imparted to each turn through the dielectric (by condenser action) corresponds to a uniform distribution of voltage from turn to turn. If, in this case, the line static plate and the tongue iron be recognized as the chief plate elements of the condenser and the plot of electrostatic field shown in Figs. 3 and 4 be completed to cover the region occupied by the winding, this will give a true picture of the initial surge voltage distribution. The potential of each turn would be indicated by its position with respect to the equipotential lines.

If the measurements of the surge voltages shown in Figs. 5 to 8 were made on the outside turns of the respective coils, the part of the field external to the coils would need some correction, of course, to correspond with the actual voltage distribution. If the whole field were correctly plotted, I fear that the location of the equipotential lines would reveal voltages on some of the interior turns of the coils which would fall a considerable distance from the curves in Figs. 5 to 8. It is readily seen, however, that the initial surge voltage distribution near line end of the winding has been improved by the introduction of the plate.

1. A.I.E.E. Trans., Vol. XLI, p. 151.

F. J. Vogel: The primary importance of Mr. Putman's paper rests in the principle that to obtain the highest impulse voltage strength, it is necessary to eliminate creepage surfaces. This is not a new principle, since in the insulation between the high- and low-voltage windings of all transformers, barriers of insulating material are common practise. What is new is the further application of this principle between coils of a single winding to increase the insulation strength of the winding to the greatest extent.

One of the most interesting features of this development is the broad extent of its application. It is not limited to designs using

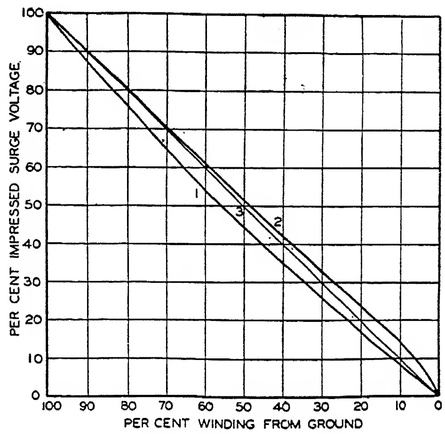

Fig. 18—Surge Voltage Distribution

In single-group shell type transformer surge entering one end—1½—40 wave
Curve 1—Initial distribution
Curve 2—Maximum oscillation

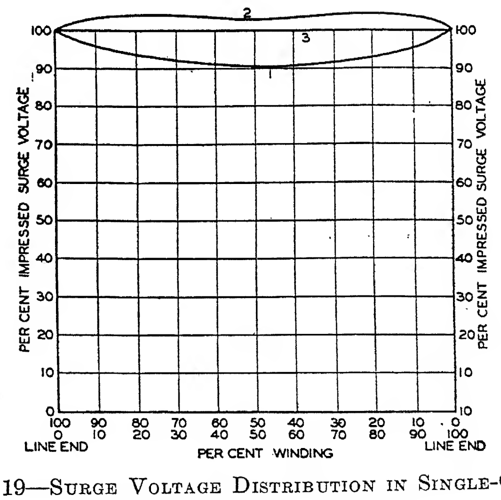

Fig. 19—Surge Voltage Distribution in Single-Group Shell Type Transformer

Equal surges entering both ends—1½—40 wave
Curve 1—Initial distribution
Curve 2—Maximum oscillation

a single high-voltage group, nor even to designs with grounded neutral. It can be applied to transformers grounded through reactance at the neutral or fully insulated star- or delta-connected transformers. To demonstrate this, the possibility of elimination of creepage between coils of a shell type transformer winding capable of withstanding a high dielectric test to ground is indicated in Fig. 17.

The extent of the application of such a group as shown in Fig. 17 is very important. For example, the distribution of surge voltages is just the same, whether the surge comes in from either

end of the winding due to the symmetry of its arrangement. Fig. 18 shows the initial distribution of voltage and maximum voltage to ground when the surge enters from one end, and Fig. 19 shows both the initial distribution and the maximum voltage to ground when equal surges enter both ends of the winding. Therefore, it is easily possible to apply this type of design to delta-connected transformers and obtain surge proof characteristics.

It is but a step from the fully insulated single-group design, to the fully insulated two-group design. As previously brought

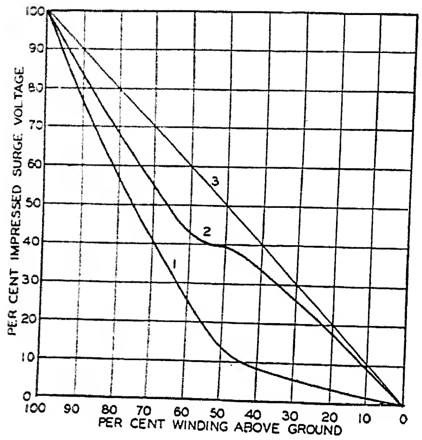

FIG. 20—SURGE VOLTAGE DISTRIBUTION IN TWO-GROUP SHELL TYPE TRANSFORMER

Surge entering one end—1½—40 wave
Curve 1—Initial distribution
Curve 2—Maximum oscillation

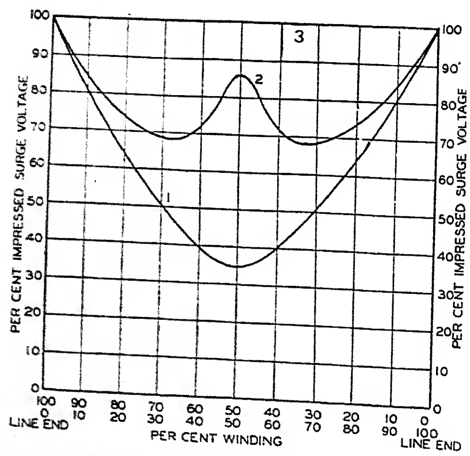

FIG. 21—SURGE VOLTAGE DISTRIBUTION IN TWO-GROUP SHELL TYPE TRANSFORMER

Equal surges entering both ends—1½—40 wave
Curve 1—Initial distribution
Curve 2—Maximum oscillation

out in a paper by Mr. J. K. Hodnette, (TRANS. A.I.E.E., Jan. 1930, p. 68) if the surge enters from one line, nearly all the stress from the initial distribution will be concentrated in the one group, and will be nearly uniform in this one group. This is shown in Fig. 20. This one group can be made as resistant to insulation failure during the period of initial voltage distribution as the one group design previously described. The voltages to ground resulting from a long wave are also shown in Fig. 20. This shows that the stresses to ground at the middle of the winding will not be excessive, particularly since a wave as long as this will be much lower in magnitude than the very short ones. The initial distribution and maximum voltage to grou for simultaneous equal surges on both ends of the winding, shown in Fig. 21 taken from test data. Thus, the possible ap cation of the design to delta connected transformers is clea known.

One further step is the application of this principle to grounded neutral design, but with insulation graded in prop tion to the induced stress at the interleaved low-voltage c and in the group connected to ground. Such an arrangement shown in Fig. 22. The distribution within the winding for bo short and long waves would be similar to that indicated in F 20. Taking into consideration the relative magnitudes of su

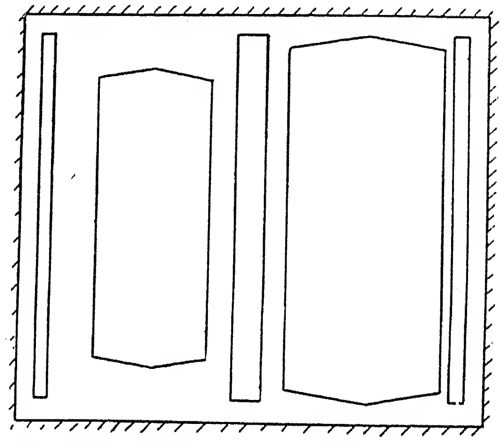

FIG. 22—OUTLINE OF GRADED INSULATION IN A TWO-GROU SHELL TYPE TRANSFORMER ELIMINATING CREEPAGE

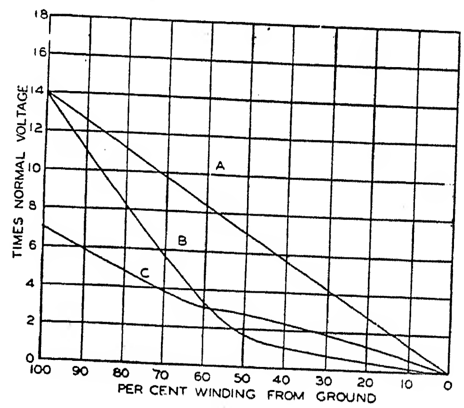

FIG. 23—SURGE VOLTAGE DISTRIBUTION

Two-group shell type transformer with graded insulation
Curve B—Voltage in transformer due to highest line surges
Curve C—Voltage in transformer due to longest line surges (just low enough not to flash over line insulation)

waves (approximately 14 to 7 times normal), it can be readily seen that the insulation at the interleaved part is not unduly stressed. (Fig. 23.)

The natural period of interleaved shell type transformer windings, although not in itself of great importance, is of interest due to its effect in reducing the magnitude of oscillation of such windings. A comparison of the oscillations within a single group, and also of an interleaved winding is shown in Fig. 20. The natural advantages of the shell form of construction are shown quite clearly by the fact that with a single group, good distribution is obtained, and hence the oscillation, although short in period and relatively completely developed is not important (Fig. 19). When interleaved, good distribution is obtained in the

line group and the high ground capacity at the interleaving prevents high oscillation and also increases the natural period so that unless the surge is extremely long, the maximum oscillation can not be developed. For example the maximum potential by oscillation at the mid point of the winding as shown by Fig. 25 and by Fig. 20, curve 2, is 38 per cent of the impressed surge.

The curves of voltage distribution and oscillograms shown in this discussion were taken during the development of some 4,500 and 20,000-kva. transformers for the Pennsylvania Railroad. These transformers are single-phase and insulated for a 309,000-volt test from the high-voltage to low-voltage windings and ground. They are also designed to be co-ordinated with a 41-inch gap for impulse strength. To demonstrate this, the first

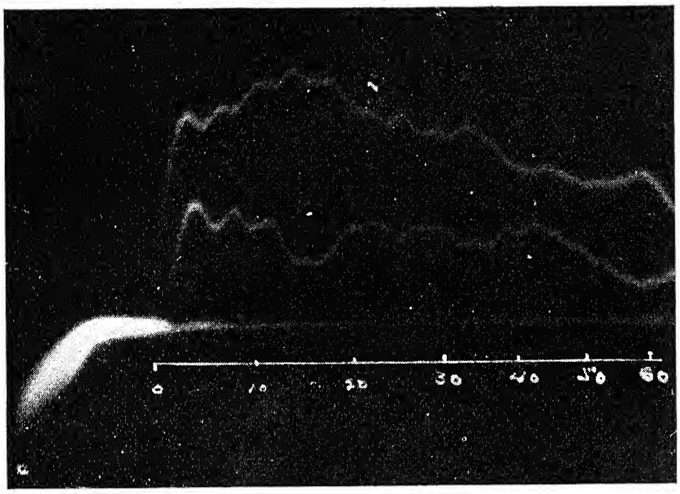

FIG. 24—OSCILLOGRAM SHOWING VOLTAGES TO GROUND IN A SINGLE SHELL TYPE GROUP AT POINTS 1/3 AND 2/3 FROM THE LINE END

No violent oscillations are indicated

transformers completed have been subjected to impulse voltage tests, with waves of both short and long duration. No indications of weakness either during the test or by complete disassembly and inspection of the units were found.

Thus the application of the principle has been made to fully insulated transformers which are operated single-phase, and which could equally well be operated in delta.

H. V. Putman: Several of the discussors appear to be much concerned about the core-type transformer. They have rushed to its defense, but I have made no attack on it. My company builds core-type transformers—many more core types than shell. We recognize the merits of core type construction, but also its limitations. We recognize the merits and limitations of shell type construction also. Our engineering policy is to use the type best suited for any given application. Our development work for many years has been carried out on both types in parallel. We have done quite as much surge voltage investigational work on core types as on shell. We have millions of kva. of each type in service.

In discussing differences between core and shell, some of my critics are at a disadvantage, since their experience has been largely with core types. For instance, Mr. Clem condemns all shell type construction on the basis of experiments on ten round coils ten inches wide—a mere toy. Our knowledge of both core and shell types represents results on real transformers—some of them among the world's largest.

This background of parallel experience on both types places my associates, Bellaschi, Vogel, Fortescue and Hodnette, in a favorable position to speak authoritatively on the *differences in characteristics* of the two types.

Creepage Surfaces

Mr. Palueff's discussion is characterized by sketches representing his idea of Westinghouse transformers. These sketches are incorrect. No transformers have ever been built employing the winding arrangements shown in his Fig. 6, *B* and *C*, nor employing the insulation structure shown in his Fig. 7. He does not see why I prefer the new insulation arrangement, because, according to him, "creepage paths still exist at the top and bottom of the insulation structure (his Fig. 7), and the stress across these creepage paths is now three to eight times as great as in the old design." I prefer the new arrangement, because, based on actual tests, it has set a new standard of performance in the matter of resistance to surge voltages.

The insulation barriers employed in the new surge-proof transformers do not extend straight up above the coils (as in Mr. Palueff's Fig. 7), but they are turned over at the top substantially coincident with the equipotential surfaces. The creepage paths shown by Mr. Palueff and referred to by Mr. Peek therefore do not actually exist in the transformer. I fear neither of these gentlemen really understands what I mean by eliminating creepage surfaces. I mean simply the location of the insulation barriers so that their surfaces are substantially coincident with the equipotential surfaces of the electrostatic field. I do not claim to have enclosed each coil in a sealed box of insulation. It is, of course, possible to trace a devious path from one coil to any other coil. But with my arrangement, the direction of voltage stress is approximately normal to the insulation surfaces at all points, and this is the condition that gives the highest impulse strength.

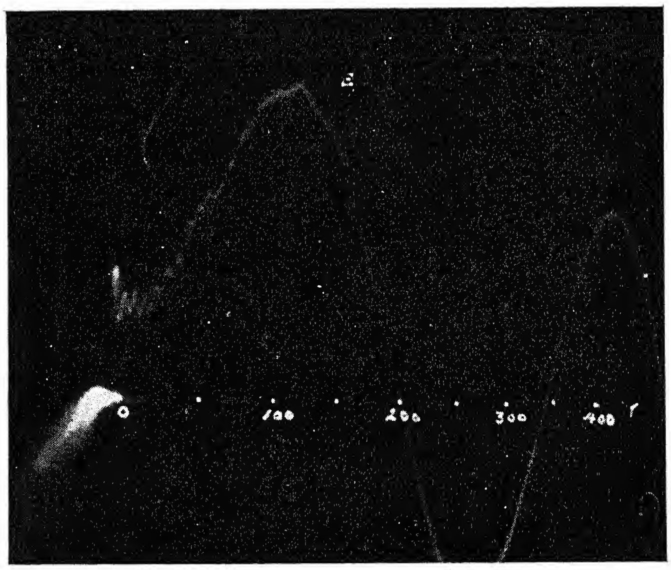

FIG. 25—OSCILLOGRAM SHOWING VOLTAGE TO GROUND AT THE MIDPOINT OF A TWO GROUP SHELL TYPE WINDING

Note long natural period of over 300 in. due to interleaved windings

Mr. Palueff states that in my new design I have crowded the full line-to-ground voltage into 1/3 to 1/8 the space used for it in the old. This statement is based on the winding arrangements imagined by Mr. Palueff in his Fig. 6, B and C, which, of course, never have been and never would be used. He appears to refer to the voltage lengthwise of the high-voltage stack, and presumably to the surge voltage, since there would be no question about the normal voltage. His statement would not be true even if the winding arrangements in his Fig. 6 were used, since in any case practically the whole surge voltage would appear across the group connected to the high-voltage line. This is due to the large ground capacity to the low-voltage coil group at the end of the first high-voltage group.

In any transformer there are only a few inches of space between the high-voltage winding and the low, and across these few inches the entire surge voltage exists. Here, indeed, is a high gradient. Mr. Palueff is greatly concerned about gradients of much lower value lengthwise of the high-voltage stack; why isn't he concerned about this high gradient in the few inches between the high and low? Simply because in these few inches lie puncture barriers separated by oil ducts—the best kind of insulation known. It is this kind of insulation—puncture barriers separated by oil ducts—that I have used throughout my entire insulation structure *lengthwise of the stack*, as well as between high- and low-voltage windings. The gradients through the barriers lengthwise of the stack are, of course, appreciably less than through the barriers in the space between high and low.

Both Mr. Montsinger and Mr. Palueff insist that creepage surfaces are entirely dependable if used intelligently within their limitations. I fully agree (we use them in our core type transformers), but is that any reason for not discarding them in favor of something superior wherever practical?

It is only when creepage surfaces are involved, that designers begin to compare gradients in different types of transformers and worry about volts per inch. These matters do not concern our new surge-proof transformers; they belong to the past.

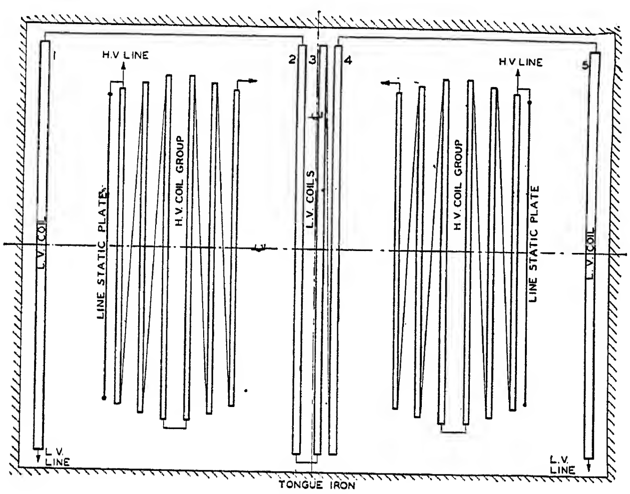

Fig. 26

Surges Induced in Low-Voltage Windings. I desire to correct a statement made by Mr. Palueff regarding the surge voltages induced in the low-voltage windings of shell type transformers. He states, "Another undesirable feature of the non-oscillating transformer of the interleaved type is the *extremely* high lightning voltage induced electrostatically in the low-voltage windings, from the high-voltage winding and static plate shown in Fig. 10. This happens because the low-voltage winding, instead of being placed next to the core, as in core type transformers, is placed in close proximity to the high-voltage line coil and static plate. From the approximate distribution of the electrostatic field produced by a lightning wave, Fig. 10, it follows that the low-voltage winding rises above ground to some 60 or more per cent of the amplitude of the lightning wave impressed on the high-voltage terminal."

It can be shown that this statement is incorrect. Of all the surge-proof transformers we have built, the 4,500-kva., 138-kv. fully insulated, single-phase transformers recently built for the Pennsylvania Railroad, had relatively the highest surge voltage induced in the low-voltage winding. These transformers have interleaved windings, as shown in Fig. 26.

Surely one could not conceive of a winding arrangement in which the low-voltage and high-voltage windings are more effectively coupled electrostatically. Having chosen the worst possible winding arrangement, let us also choose for investigation the worst possible case which occurs when a surge enters both high-voltage terminals simultaneously. Note that the high-voltage winding is not connected to ground at any point. Both ends are raised to the potential of the incoming surge and the remainder of the winding is free to float; closely coupled to it is the low. Even the terminals of the low are not solidly grounded but are connected to ground through 450 ohms to simulate the surge impedance of the low-voltage circuit. Under these conditions, let us examine the voltage induced in the low.

During the design of these transformers, Mr. Bellaschi made such an examination. His plot of the electrostatic field is shown in Fig. 27. The induced voltages shown by this plot are given in the figure. In Fig. 28 are also shown the test results for the potentials at all points in both the high and low voltage windings. It will be noted by reference to the table below that the maximum induced potential found in the low is 18 per cent of the lightning surge voltage as compared wtih 20 per cent calculated by Mr. Bellaschi.

CALCULATED AND MEASURED VALUES FOR VOLTAGES AT DIFFERENT POINTS IN THE L.V. WINDING

	Measured	Calculated
Coil No. 1—Line end	6%	
Coil No. 1—Finish Coil No. 2—Finish	15%	17%
Coil No. 2—Start Coil No. 3—Start	16%	
Coil No. 3—Finish Coil No. 4—Start	18%	20%
Coil No. 4—Finish Coil No. 5—Finish	16%	17%
Coil No. 5—Line end	6%	

It may be remarked that the measured values shown above were obtained by spark gap measurements between the various points in the winding and ground, and therefore represent maximum values, while the calculated values are for the initial voltages. However, it can be easily shown that under these conditions the maximum potentials in the low occur as a result of the initial distribution and not as a result of any subsequent oscillation.

Both the calculated and measured values reported above show that the surge voltages induced in the low of an interleaved shell type winding are not even of the same order of magnitude as Mr. Palueff believes.

In making his analysis, Mr. Palueff made the mistake of assuming that the potential of the points in the space occupied by the windings are unaffected by the presence of the windings themselves. He takes no account of the fact that the windings are made up of heavy copper conductors along which the charges may move—indeed, do move very appreciably in even a small fraction of a microsecond. When the presence of the windings themselves is taken into account, the shape of the field is materially changed, as can be seen from Bellaschi's sketch, Fig. 27, which is made correctly. In reaching conclusions in matters of this kind, practical experience with real transformers is desirable, since a superficial analysis can so easily be wrong.

In comparing the induced surge voltages in the low in core and shell type transformers, care should be taken to compare transformers of like specifications. For instance, Palueff says that in core types the low-voltage winding is placed next to the core, but in a core type transformer, to meet the low 4 per cent reactance required in the 4,500-kva. transformers, it is necessary to place a section of the low-voltage winding outside the high-voltage stack as well as next to the core. The low-voltage winding thus consists of two concentric cylinders with the high between the two. With this winding arrangement, which is common in

core types, designed for low reactance, the electrostatic coupling between high- and low-voltage windings is quite as good as in an interleaved shell type. In our experience we have found that core and shell type transformers are on a par so far as surge voltages induced in low-voltage windings are concerned.

Distribution of Surges in Individual Coils. From the incorrect field plot in his Fig. 10, Mr. Palueff draws another surprising conclusion relative to the distribution of surge voltages within the individual high-voltage coils. He says, "However, the actual distribution shown by the solid wavy curve is extremely non-uniform with the middle of each coil at much higher potential than its edges, and with as much as 30 to 40 per cent of the transformer terminal voltage appearing across half of a coil near the ground end of the stack." This conclusion is, of course, absurd. Let Palueff refer to his own Fig. 13. According to the

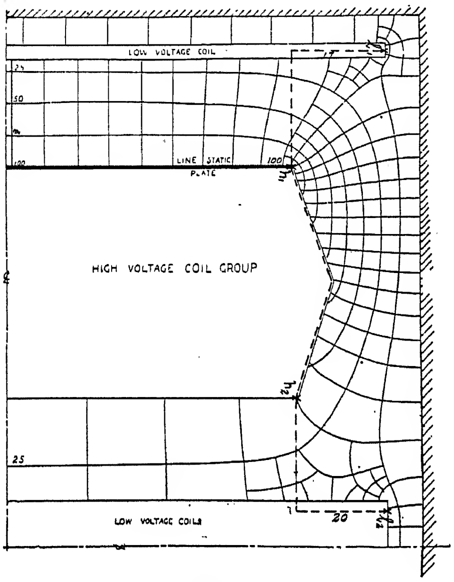

FIG. 27

title, these curves represent actual measurements of the surge voltage distribution in a stack of four very wide coils. The voltage distribution within the individual coils is not uniform, and the distribution curves are characterized by what we call "saw teeth." The magnitude of these saw teeth is greater the fewer the number of coils in the group, so that in a group of only four, these saw teeth would be very pronounced.

An examination of these curves shows that the potential of the mid point of the coil at the ground end is about 5½ per cent for connection No. 1 and about 7 per cent for No. 2, instead of the 30 to 40 per cent mentioned earlier by Palueff. A further examination of these curves shows that in every case except coil No. 4, connection No. 1, the potentials at the mid points of the coils lie either in between those of the edges of the coils or else are lower; whereas Palueff concluded from his analysis, the potentials of the mid points would be *very much* higher than the potentials of the edges, as illustrated in his Fig. 10.

Effect of Coil Connections on Surge Voltage Distribution. Mr. Clem suggests that cross connections are hazardous and have been abandoned in shell type transformers. Mr. Peters' statement in this connection, quoted by Mr. Clem referred to transformers (core type, for instance), where the coils are spaced close together and sufficient room for properly insulated cross connections is difficult to obtain. In shell type transformers it is usually possible to obtain sufficient clearance for start-finish (or cross) connections and, in fact, this often results in the more economical winding. Our insulation tables cover both start-finish and start-start connections, and the designer employs the one best adapted to the particular design at hand.

In Mr. Palueff's discussion of Hodnette's paper referred to by Mr. Clem, Palueff claims that our results of surge voltage distribution were incorrect because we failed to measure the surge voltages on the *inside* edges of the coils and predicted that when we did so we would find the extremely non-uniform distribution claimed by him. Mr. Palueff's assumption in this case was unwarranted, because the measurements were made on a winding having start-finish connections, in which the inside edges of each coil are connected directly to the outside edges of each adjacent coil. Measurements were therefore made on both inside edges and outside edges simultaneously. Much of our early work was done with this type of winding because it did make it possible to measure the potentials on the inside edges. And it was the inside edge potentials in which we were vitally interested, because they established the potential distribtion along the creepage surface lengthwise of the group, along which failures took place when tests to destruction were made.

Uniform Distribution in Shell Type Stacks. Both Mr. Clem and Mr. Palueff discuss at great length the relative merits of core and shell type groups from the standpoint of initial surge voltage distribution. They present again the same old arguments and data which Mr. Palueff has repeated time and again in the A.I.E.E. discussions. These gentlemen apparently do not yet understand what we mean by saying that in a shell type group the surge voltage distribution is inherently substantially uniform. I am going to explain this statement very carefully and clearly. But before doing so, let us orient ourselves in this discussion to see what it is all about, and what practical conclusion it is desired to reach.

An ordinary transformer of either core or shell type construction, when tested to destruction with surge voltages, will usually fail lengthwise of the high-voltage stack, the failure starting at

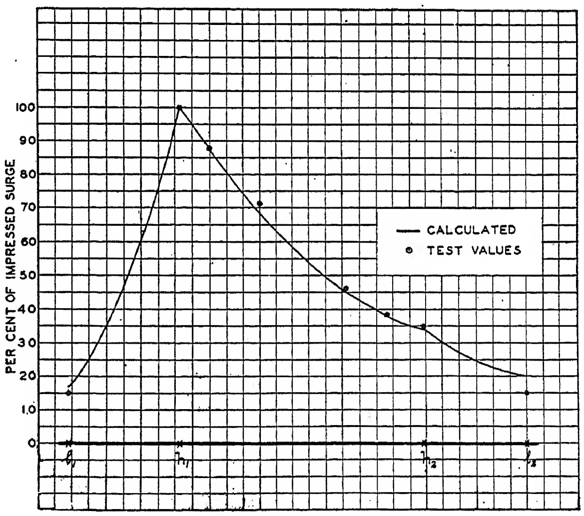

FIG. 28

the inside edge of the line coil and extending along the creepage path on the insulation barrier adjacent to the coils. If the surge voltage stress is uniformly distributed along this creepage path, it is obvious that the maximum gradient will be lower and its resistance to breakdown will be greater than in a similar transformer, where the distribution is non-uniform. So the point under discussion is whether the distribution *along the creepage surface* lengthwise of the high-voltage stack is or is not substantially uniform in a shell type group. It is well known that the initial distribution along this creepage surface in a core type is extremely non-uniform, there being a very high gradient at the line end. Indeed, the use of shielding in a core type has as one of its principal objects, the elimination of this non-uniform stress distribution.

But in an ordinary shell type, there appears still to be a difference of opinion about the uniformity of the gradient along this creepage surface. I shall therefore at this point present a careful explanation of this situation in a shell type group.

In Fig. 29 I illustrate a group of nine coils. In Fig. 4, Mr. Clem shows the initial distribution in such a group of nine coils for both start-finish and finish-finish connections. In an actual transformer of conventional construction, the line coil would have comparatively few turns heavily reinforced, so that the percentage of the total surge voltage appearing across it would not be as large as shown by Mr. Clem's curves, but the principle is the same and his curves will serve our purpose. In an actual transformer also, the duct space between the first two coils would be made larger than between the others, and additional insulation barriers employed in this duct.

The creepage surface in such a winding exists on the insulation barrier inside of the coils at the crown above the iron. It extends between points 2 and 4 as illustrated by the dotted line in Fig. 29. We have tested a number of conventional shell type transformers to destruction with surge voltages and they invariably fail along the path shown by the dotted line. We have never experienced a failure either across the face of the first coil nor

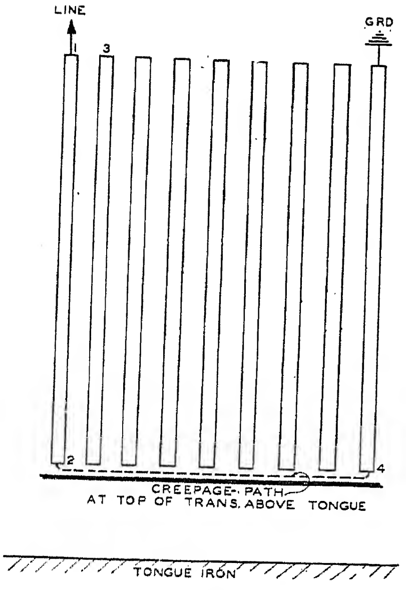

FIG. 29

between the first two coils at the outside, that is, between points 1 and 3, Fig. 29. The reason for this is that the great width of the shell type coil, which might be 20 or 30 inches in a large transformer, effectively protects the line coil against flashover. This width may often be greater than the length of the creepage path along the stack, so that there is not even a remote possibility of flashover across the face of the line coil. Furthermore, failures have not been experienced between points 1 and 3, Fig. 29, because there is no creepage surface here, solid insulation barriers being used, just as in the space between the high- and low-voltage windings. It is clear, therefore, that no difficulty has been experienced in insulating the first coil and duct for the surge voltages appearing across them.

The real problem in shell type transformers has been the gradient along the creepage surface just as in the core type.

In Fig. 30 I have plotted the voltage along this creepage surface, using the values from Mr. Clem's curves, Fig. 4, to illustrate the point that this gradient *is* substantially uniform. In plotting these curves, I have used the potentials on the inside edges of the coils which are adjacent to the creepage path and which establish the gradient along it, reading the data from Mr. Clem's curves.

These curves clearly show the uniformity of the gradient along the creepage path, which is characteristic of shell type groups. The difference between the two types is that in the core type this gradient is far from uniform, while in the shell type it is substantially uniform. The same result can be obtained from any of the other distribution curves for shell type groups presented by Palueff and Clem. With this point clearly established, let us examine its practical significance. It is this; shielding, however desirable in high-voltage core type transformers, would be totally ineffective in shell type so far as improving the dielectric strength of the creepage path lengthwise of the stack is concerned, since the gradient along this path is already uniform. And if any improvement is to be made in a shell type transformer, it must be made here, since this creepage surface is its weakest point. That improvement has been made a major improvement through elimination of the creepage surface itself. Shielding would obviously be worse than useless.

Mr. Palueff has sought to show that the distribution in a shell type is just as bad as or worse than in a core type; that a shell type, therefore, needs shielding just as bad as a core type; but that it is practically impossible to shield it. He has beclouded the entire issue by drawing curves of the distribution within the winding, that is, in and out among the turns, rather than along the creepage path, where failures actually take place. I believe this discussion has clarified this matter. It has shown that shielding could fulfill no useful purpose in shell type transformers. It has also shown clearly the logic of eliminating creepage surfaces in this type of transformer.

Static Plate. Mr. Palueff asks me to define a shield. Surely he is not going to call my static plate a shield. The static plate was patented many years ago by Moody. I define it just as he did and use it for the same purpose. I employ no shield.

Mr. Palueff also asks if I can tell him how essential the static plate is to the securing of uniform distribution along the stack. It is not essential at all. Clem's curves, Fig. 4 show this. Its slight effect is negligible. It eliminates the voltage drop across the face of the line coil resulting from the penetration of the surge into the winding, thereby making possible the use of ducts all of the same width and a uniform conductor insulation throughout. The use of a static plate with shell type transformers of conventional construction would result in general in lower impulse strength, because it would place the entire surge voltage across the creepage path lengthwise of the group, whereas without the plate this voltage across the creepage path *is* reduced by the voltage drop across the line coil. The line coil is much more able to stand this voltage than the creepage path.

Distribution in Core Type Stacks. Mr. Clem presents certain data in his Fig. 4 which show that the distribution along a stack of core type coils only two inches wide is substantially uniform. In fact, Fig. 4 shows a perfectly uniform distribution along the inside edges of the coils, and hence along the creepage surfaces. These data are, to say the least, most misleading. Surely he would not have any one ascribe any practical significance to such results. If he is correct, then transformer designers have been grossly misled these many years in reinforcing the insulation between coils at the line end of core type transformer windings and there is no purpose in Palueff's shielding. Such results can be obtained in a stack of coils without ground capacity, but not in a real transformer.

Surge Voltage Tests. I would like to say a few words about the relative severity of our lightning tests. My claim to first place was based on the G.E. published data relative to its New England power test. However, comparing data as Palueff has done in Fig. 11 is rather fruitless until it can be established why a Pittsfield volt is not as large as a Sharon volt. Our laboratories will no doubt eventually be able to measure the same voltage for the flashover of the same gap. At present, however, when dealing with short surges of the order of 5 microseconds, Pittsfield measures voltage from 1/4 to 1/3 higher than Sharon and

Trafford. Thus a given voltage measured by us as 1,500 kv. would be measured by Pittsfield as approximately 2,000 kv. If my test on the 42,000-kva. Baltimore Consolidated transformer were calibrated in Pittsfield volts, it would register close to 3 million volts, while our measurements indicated 2¼ million. This question might well have been deferred until the data published by our laboratories show better agreement.

Messrs. Palueff, Montsinger and Peek have all referred to the pinhole punctures between turns resulting from repeated surge voltage tests. This arouses my curiosity, because we have not found these in our experience. If such failures have actually been experienced, it must indicate some weakness in conductor insulation. We have built and tested experimentally many full size power transformers. Some of these were tested to destruction, others not; in all cases they were torn down and examined. Many were rebuilt, retested, and re-examined the second time. In some cases our coils were examined under a magnifying glass in an effort to find some of these pinholes, but without success.

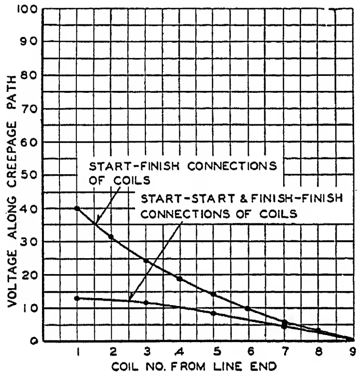

Fig. 30

I might say also that in all of our experience in testing shell type transformers to destruction with surge voltages, we have never seen a coil fail across its face. There need be no concern about the unequal distribution of surges within individual coils.

Mr. Palueff asks if we have torn down and examined commercial transformers after impulse tests. The Baltimore transformer, on which the measurements of distribution within the winding were made, was torn down sufficiently to attach the necessary connections between coils. We were able to examine those parts of the insulation structure which our experience had indicated would be the first to show any marking. Every piece of insulation was clean.

Some of the 4,500-kva. fully insulated transformers mentioned previously were disassembled and minutely examined. One of these transformers was given a total of 106 surges, all above the flashover of the protective gap. On one occasion the coils and insulation were laid out on the assembly floor, where they were examined by a party of 40 or 50 visiting engineers.

If was after making a number of successful surge voltage tests on commercial transformers that we recommended early last fall to the N.E.L.A. the use of surge tests to demonstrate guarantees of impulse strength. I am glad to see that others are now also coming to recognize that low-frequency dielectric tests do not constitute any demonstration of impulse strength and to agree that surge tests are practicable. I feel sure that we have already reached a satisfactory solution to the problem of failure detection mentioned by Mr. Peek.

Messrs. Gaston and Emlen ask about the temperature at which impulse tests have been made. All of our commercial transformers have been tested hot. Tests on our experimental transformers have been made both hot and cold.

Corona. Messrs. Montsinger and Peek mention the undesirability of corona during the low-frequency test. Since high-voltage transformers came into use, we have been building trnasformers which in all probability have some slight corona during the high-voltage test. Our experience is that in some types of insulation the corona point is very close to the breakdown point. Here the presence of corona during test would indicate overstressed insulation, and we are careful to avoid it. On other types of insulation structures, the corona point occurs at a much lower voltage than breakdown, and here slight corona during test need give no alarm. Copious corona discharges will usually produce visible marking on the insulation, and should, of course, be avoided. Experienced witnesses, who have seen insulation tests on many transformers of different makes, have often complimented us on the quietness of our transformers during the insulation test and on the lack of disturbance in the oil.

Barrier Test. I am much interested in Mr. Montsinger's comments on the barrier test. I went through the same process of reasoning myself at first and concluded that the impulse strength must increase with barrier extension. But repeated tests with very short surges (we used 1 microsecond or less) showed only very slight increase. Fig. 31 shows a complete set of time lag curves for barriers 1 inch thick, and with different extensions. These curves converge together for the very short waves, giving the result indicated in Fig. 10 of my paper. Mr. Montsinger's curve of impulse ratio against extension shows that the impulse ratio varies from 4.5 for zero extension to 2.7 for 8 inches extention for a ½-5 wave. With a much shorter surge, my Fig. 10 shows the impulse ratio to vary from 4.5 to 2. With surges of the same length, our data would not be far apart.

Internal Oscillations. Mr. Dann's discussion answers this question of internal oscillations in shell type transformers effectively, and I hope finally. One of the important differences between core type transformers and shell type transformers having interleaved windings is the long, natural period of the latter, which effectively prevents internal oscillations of high magnitude. As an example, the natural period of the 4,500-kva. transformers previously mentioned was found from test to be 300 microseconds.

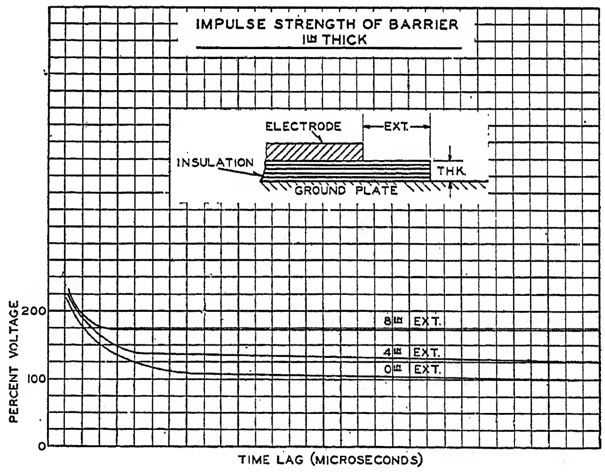

Fig. 31

Fully Insulated Transformers. One of the fine things about the improvements in shell type transformers which I have described is that they are in no way limited to grounded neutral transformers. Mr. Vogel in his discussion shows their application to fully insulated transformers and to transformers of the interleaved type.

Conclusions. The following paragraphs present a few important facts on which there is no difference of opinion. Core type transformers of conventional construction are characterized by an extremely non-uniform distribution of surge voltage

lengthwise of the coil stack with high concentration at the line, and by relatively severe internal oscillations which produce correspondingly severe voltage stresses in the major insulation between high- and low-voltage windings, even with fairly short surges. These characteristics are recognized by all manufacturers of core type transformers, and the design of the insulation is made to take account of them.

There are two schools of thought regarding the best method of handling these core type insulation problems. One is represented by Palueff and his associates, who advocate graded insulation and the use of shielding to secure uniform distribution and prevent internal oscillations. The other is represented by European engineers, who prefer to utilize the space required by the shielding structure for full major insulation between high and low, and properly to reinforce the end turns rather than insulate a shielding structure at line potential from the entire high-voltage winding. This reinforcement or padding of end turns and coils insulates for the non-uniform initial distribution. The use of major insulation provides adequate dielectric strength against internal oscillations. The European engineer attempts to know the surge voltage stress at every point in his winding and to insulate for it directly. This is sound engineering and its simplicity commends it.

Both schools of thought are one in recognizing these insulation problems in core type transformers. They differ only in the method of solution advocated. For low-voltage transformers, the simpler European method of solution is used universally. It is in the higher voltage classes that these insulation problems become paramount and require careful consideration.

In these higher voltage classes we prefer to avoid entirely these difficult insulation problems by employing the simple construction disclosed in my paper. Reinforcement of insulation on end turns and coils is not required because the distribution of surge voltage stress throughout the entire insulation structure is substantially uniform and there is no creepage surface to protect. Where grounded neutral operation is required, graded insulation can be utilized with perfect assurance because severe internal oscillations cannot occur.

The simplicity of this new construction is apparent. Our claims of surge strength are backed by sound theoretical analysis and by tests on complete transformers. Operating engineers are not likely to be impressed by purely theoretical discussions. Only by actual demonstrations with surges representative of those produced by natural lightning will they be convinced of the surge-proof qualities of transformers.

Effect of Transient Voltages on Power Transformer Design—IV

Transition of Lightning Waves from One Circuit to Another Through Transformers

PART I—BY K. K. PALUEFF*
Member, A.I.E.E.

PART II—BY J. H. HAGENGUTH*
Associate, A.I.E.E.

Summary of this and preceding papers.—*Three preceding papers of the same general title have dealt with transient voltage stresses developed within high-voltage transformer windings. The most important conclusions arrived at by the author and presented in these papers and the discussions are:*

1. Transformers of all conventional constructions undergo oscillations when subjected to lightning or switching surges.

2. The amplitudes of these oscillations may be dangerously high, depending on the amplitude and the shape of the applied voltage. Their frequencies range from a few thousand to a few hundred thousand cycles per second.[1,2]

3. A lightning wave of a given shape produces very different stresses in different transformers. [1,2]

4. In practical design, neither amplitude nor frequency of these oscillations can be controlled by arrangement or proportions of windings.[1,2]

5. A lightning wave chopped by flashover of line insulation can produce stresses in transformer windings equal to or even in excess of those produced by a long wave of the same amplitude and front. [1,2]

6. Unless means are taken to obtain uniform distribution under all lightning conditions it is entirely possible to design transformers that will pass A.I.E.E. test which are inadequate for service conditions. This follows because test voltages in neither magnitude nor distribution of stresses correspond to that produced by transient voltages in transformers of ordinary construction. This difference is particularly great in transformers with solidly grounded neutrals, where the potential test allows the insulation from high-voltage winding to low-voltage winding and ground to be reduced as the neutral is approached.[1,2]

7. If a transformer winding is equipped with a properly designed electrostatic shield, voltages of all surges will distribute uniformly, and oscillation or resonance will not occur. Over 2,000,000 kva. of such non-resonating transformers are in successful operation.[1,2,4]

To demonstrate the lightning strength of such a transformer, one of fourteen 13,000-kva., 230-kv. transformers was subjected to artificial lightning in August 1930.[4,10] *Over 200 high-voltage surges were applied. Forty were in excess of the flashover voltage of the 64-inch coordinating gap. The majority of these were full waves just below the flashover of a string of 14 standard suspension insulators. A few were full waves just below flashover of the transformer bushings. Also a few waves high enough to flash over the bushing were applied. To be certain that no damage was done by these tests the transformer was completely disassembled for examination, since there is no other positive practical method available at present for determining a partial or local damage to the insulation inside of the windings that may be caused by lightning tests. These tests were preceded and followed by complete A.I.E.E. Acceptance Tests, including a 460-kv. induced potential test.*

8. A neutral impedor has been developed, extending the application of non-resonating design to transformers with partially or entirely isolated neutral.[3]

9. A non-resonating design has been adapted to auto-transformers.[5]

10. It has been shown theoretically and later experimentally, that a choke coil is worthless as a protective device, and may actually increase stresses.[1,2]

In this paper, the phenomena of the transmission of voltage surges through transformers to the connected secondary circuits are critically examined, a quantitative theory for their calculation is developed, and general conclusions on their magnitude are presented.

Part I

Introduction

THE impact of a surge on the high-voltage terminals of a transformer produces transient voltages not only within that winding, but also within the low-voltage winding, as well as at its terminals. The latter voltage is impressed on the network and apparatus connected to this winding and is the subject of this paper.

Some occasional failures, during lightning storms, of rotating apparatus connected to lines through transformers, indicate the possibility of transmission of high-transient voltages through transformers.

Dry insulation used in rotating machinery has much lower impulse strength than transformer insulation for the same operating voltage. On account of this it is more difficult to protect generators from transient voltages than transformers.

The first part of the paper deals with the subject from a practical standpoint. It establishes certain approximate relations between circuit constants, that are generally available, and transient voltages.

The second part is the mathematical treatment of some of the problems dealt with in the first. It shows first: that the approximations used in the first part are justified; second that in some cases these approximations are unavoidable because in practise most of the constants of the complete equations ((1a) for example) can not be determined.

Four Components of Transient on the Secondary Circuit

It is convenient to consider that the surge voltage produced at the low-voltage terminals of a transformer by a rectangular traveling wave is a result of superposition of four components. Fig. 1.

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*Both of Power Transformer Dept., General Electric Co., Pittsfield, Mass.

1, 2. See Bibliography.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

32-49

1. At the moment of impact of a lightning wave, an *electrostatic* voltage is produced at the terminals of the low-voltage winding which depends upon the relation of the distributed capacitances of the low-voltage winding to high-voltage winding and ground, and is independent of turn ratio.

2. A *free oscillation in the high-voltage winding,* consisting of a number of space harmonics,[1,2] induces a corresponding oscillation in the low-voltage winding. The induction is accomplished by means of the electrostatic and electromagnetic fields of these harmonics, and is dependent upon the distributed constants and turn ratio of the windings.

3. A *free oscillation in the low-voltage winding* follows immediately after the impact of the wave. The resulting voltages depend upon the distributed constants of the low-voltage winding.

4. Through a *purely electromagnetic induction* between the two windings, a unidirectional surge is produced on the secondary circuit. Its voltage rises from zero to a certain maximum, then decays to zero. The voltage is directly proportional to the turn ratio and is independent of the distributed capacitance. It is a simple function of the short-circuit inductance of the transformer and of the surge impedances of external circuits.

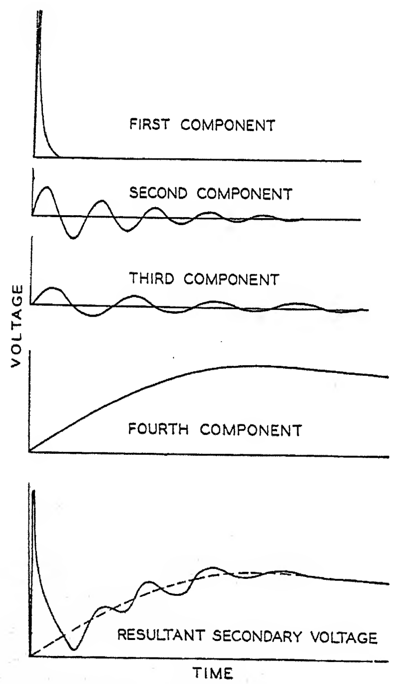

Fig. 1—Secondary Terminal Voltage and Its Four Components

All four components depend, to a different degree, upon surge impedances and lengths of external circuits connected to the transformer.

Relative Importance of Four Components

Let $2E$ be the voltage produced across the high-voltage ("primary") winding of a transformer by a rectangular wave of E volts and of infinite length, E_2' the crest voltage of the wave transmitted to the low-voltage ("secondary") circuit, Z_1 and Z_2' the surge impedances of external circuits connected respectively to the primary and secondary of the transformer, t_2' the *total* length in microseconds of the transmitted secondary wave, and t_1 the length of primary wave which would contain the same amount of energy as the secondary wave. It is assumed that the secondary circuit is at least $t_2'/2$ microseconds long.*

It can be shown that for an exponential secondary wave with a sheer front and a tail that falls from E_2' to $E_2'/2$ in t_2 microseconds, the maximum secondary voltage is

$$E_{2e}' = 0.59 \times 2E \sqrt{\frac{Z_2'\, t_1}{Z_1\, t_2}}$$

Similarly, for a secondary wave, half of a cycle of a sine wave in shape with a length t_2 during which the amplitude is above $E_2'/2$, it can be shown that

$$E_{2s}' = 0.58 \times 2E \sqrt{\frac{Z_2'\, t_1}{Z_1\, t_2}} \cong E_{2e}'$$

While for oscillatory secondary voltage which falls to zero in say five cycles, the crest voltage of the first half of a cycle is

$$E_{2o}' = 0.41 \times 2E \sqrt{\frac{Z_2'\, t_1}{Z_1\, t_2}} = .71\, E_{2e}'$$

Voltages produced by the first and the fourth components are exponential and those produced by the other two are essentially sinusoidal and oscillatory.

Simple calculations show that for each of the first three components, t_1 is less than *two-* or *three-tenths* of a microsecond. It follows that t_2 for a given component must be exceedingly short to permit that component to produce a secondary voltage that is comparable to $2E$. (Fig. 2.)

Fig. 2 permits estimate of maximum possible secondary voltage where order of magnitude of t_2 is known. It also gives the maximum possible length t_2 of the secondary wave where its crest is known.

Under the assumption that the primary circuit is an overhead transmission line, curve 4.0 corresponds to a case where the secondary circuit is a winding of a small rotating machine, curve 1.0 where it is an overhead transmission line, and curve 0.1 where it is an underground cable. The range of large rotating machines is between the last two curves.

Time t_2 is materially shorter for the first component than for the second or third, due to the difference in the nature of these transients, but generally speaking the energy (proportional to t_1) involved in the first component is somewhat less. The value t_2 for the first com-

*By the "length" of a given circuit is meant the length of *time* T_3 required for a traveling wave to traverse the circuit from end to end. In one microsecond, a wave travels 1,000 ft. on an overhead transmission line and about 500 ft. on an underground cable. Propagation along a generator winding depends on many design factors, and is of the order of 100 ft. per microsecond.

ponent ranges between a fraction of a microsecond and say 5 microseconds. Furthermore, since the voltage of this component is induced purely electrostatically it obviously can not exceed the voltage applied to the primary ($2E$) but is a fraction (K_c) of this voltage. K_c depends only on the electrostatic relation of windings. The voltage produced by the first component becomes more dangerous as the rated secondary voltage, and correspondingly the insulation of secondary apparatus are reduced, because the voltage of the transmitted wave is independent of the turn ratio.

The time t_2 for the second and third components is relatively long, being in the order of a few microseconds to several hundred microseconds, particularly in a case where energy of these components can not be transmitted to the secondary circuit in one-half cycle of their respective frequencies as generally is the case. Keeping in mind the fact that t_1 in each case is a small fraction of

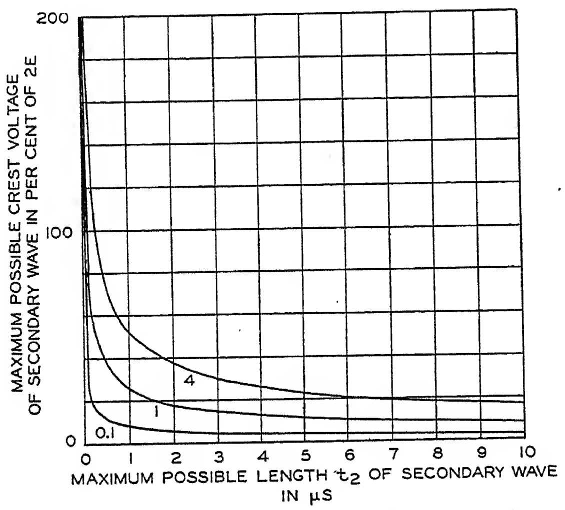

FIG. 2—RELATION BETWEEN THE MAXIMUM SECONDARY VOLTAGES AND THE MAXIMUM LENGTH OF THE SECONDARY WAVES CONTAINING THE SAME AMOUNT OF ENERGY

Curves based on: t_i = 0.2 microseconds. Numbers on curves indicate the corresponding value of Z_2'/Z_1

a microsecond (t_1 of the second component is much greater than t_1 of the third), it follows from the equations above that E_2' for these two components must be a very small fraction of $2E$, Fig. 2.

Thus it appears that under average service conditions it is possible but not probable that secondary waves of high amplitude may be produced by the second and the third components of the secondary transient. However, high voltages may be produced if Z_2' is abnormally high or if the secondary winding is not connected to its network.

The amount of energy that can be transmitted to the secondary network by the fourth component may reach the order of magnitude of that contained in a *few hundred* microseconds of the primary wave. If t_2 were small compared to t_1, it would follow that E_2' could be many times E. However, the mechanism of creation of the secondary wave of this component, is such that t_2 is necessarily greater than t_1, to the extent that E_2' is never more than $2E/r$ in case the secondary circuit is sufficiently long and $4E/r$ in case the secondary circuit is very short (r is the ratio of turns of the primary winding to the secondary winding). It is seen, therefore, that very powerful secondary surges of dangerously high voltage can be produced by purely electromagnetic transformation of a lightning wave.

Where the secondary circuit is short in comparison with the length of secondary wave voltages in excess of those of Fig. 2 can be produced.

ELECTROMAGNETIC TRANSFORMATION OF TRAVELING WAVES

A. Single-Phase Transformer

1. Infinite Rectangular Traveling Wave

If the secondary winding were perfectly interlaced with the primary, then when a rectangular wave of infinite length is applied, the secondary voltage would rise instantly to a value equal to the primary voltage divided by the turn ratio.* The voltage of both windings would fall gradually to zero as the current in the primary circuit increases and approaches a certain constant value.

In an actual transformer, however, there is considerable leakage or "short-circuit" inductance between the two windings. This causes the shape, and to a small extent the amplitude of the secondary wave to be different from that stated above. The rise of voltage is not instantaneous, but requires from a fraction of one microsecond to a few hundred microseconds.

For the analysis of the electromagnetic transient it is convenient to think of it as consisting of two parts. The first is the comparatively rapid rise of secondary voltage. This is dependent almost entirely upon the *short-circuit inductance* L_s of the transformer, and will be referred to as the L_s transient. The second is the comparatively gradual decay of secondary voltage to zero. This is dependent almost entirely upon the *self inductance* L_1 of the primary winding, and will be referred to as the L_1 transient.

As is customary for mathematical analysis and laboratory tests, surge impedances of transmission lines, cables, generator windings, etc., may be replaced with resistances of equal ohmic values, and the voltage E of the traveling wave replaced by a d-c. voltage of $2E$. This permits the circuit consisting of the transformer (with both windings grounded) and its primary and secondary surge impedances, (Fig. 3a) to be replaced by that in Fig. 3b. The application of this method in the laboratory has known limitations which, however, do not apply in this case.

*The effect of mutual inductance on transient phenomena is well known. The mathematical derivations and resulting equations however are quite complex and difficult to visualize. Here, by taking advantage of relative values of transformer constants affecting the phenomena, the solution is arrived at in a most elementary manner. The results obtained agree very closely with the exact solution given in Part II.

To take care of the effect of turn ratio between primary and secondary windings, all of the constants of the secondary circuit may be replaced by equivalent constants which would produce the same effects in the primary circuit if the turn ratio were unity. This requires that the actual secondary surge impedance Z_2' be replaced by an equivalent value Z_2 which is equal to $Z_2'r^2$. Also this permits the circuit to be redrawn as in Fig. 3c, in which L_1, L_2 and M are respectively the self inductance of the primary and secondary (equivalent) windings and the mutual inductance between them. This may be simplified to the approximate equivalent circuit of Fig. 3d.

After a solution is obtained for an equivalent circuit, all secondary current and voltage values (I_2 and E_2) *must be corrected by the turn ratio to obtain actual values* (I_2' *and* E_2') *for the secondary circuit.*

To make the analysis more concrete, it will be followed in parallel with a numerical example. E is as-

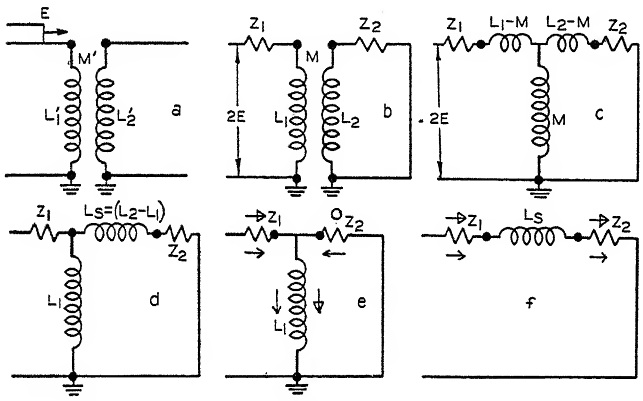

FIG. 3—DEVELOPMENT OF EQUIVALENT CIRCUIT FOR SINGLE-PHASE TRANSFORMER

◄ Final current ← Transient current

sumed to be 500,000 volts, Z_1 is 500 ohms. Single-phase transformer, 60 cycles, 22,500 kva., 66,000 volts (115,000 volts Y) to 13,200 volts, 6 per cent leakage reactance, $L_s = 0.031$ henrys, $L_1 = 0.31$ henrys, $r = 5$, $Z_2' = 600$ ohms, $Z_2 = 25 \times 600 = 15{,}000$ ohms.

L_1 Transient. Neglecting the effect of leakage inductance, it becomes possible to investigate the L_1 transient alone (See Fig. 3e).

At the moment of impact of a rectangular wave, L_1 acts as an open circuit because of the steepness of the front of the wave. A current equal to $2E/Z_1 + Z_2)$ $= (2 \times 500{,}000)/(500 + 15{,}000) = 64.5$ amperes will flow through Z_1 and Z_2, and no current will flow through L_1. The voltage across Z_1 will be $2E\ Z_1/(Z_1 + Z_2)$ $= 64.5 \times 500 = 32{,}250$ volts. The voltage across Z_2 and L_1 will be $E_2 = 2E\ Z_2/(Z_1 + Z_2) = 64.5 \times 15{,}000$ $= 967{,}750$ volts. These are called initial values.

With $2E$ maintained sufficiently long across the circuit, a constant current will finally be established through it. To such a current the reactance of L_1 will be zero. Therefore, all the current will flow through L_1 and none through Z_2. The full voltage $2E$ or 1,000,000 volts will appear across Z_1 and no voltage will appear across L_1 or Z_2. The current through Z_1 and L_1 will be $2E/Z_1 = 1{,}000{,}000/500 = 2{,}000$ amperes, and that through Z_2 will be zero. These are called the *final* values.

Since the amplitude of the transient currents and voltages is the *difference* between the final and initial values, it follows that the amplitudes of transient currents are:

$$\text{in } Z_1,\ \frac{2E}{Z_1 + Z_2}\left(\frac{Z_2}{Z_1}\right) = \frac{2 \times 500{,}000}{500 + 15{,}000}\ \frac{(15{,}000)}{500}$$

$$= 1935.5 \text{ amperes}$$

$$\text{in } Z_2,\ \frac{2E}{Z_1 + Z_2} = \frac{2 \times 500{,}000}{500 + 15{,}000} = 64.5 \text{ amperes}$$

$$\text{in } L_1,\ \frac{2E}{Z_1} = \frac{2 \times 500{,}000}{500} = 2{,}000 \text{ amperes.}$$

Taking into account the direction of these currents (See Fig. 3e) it is seen that the sum of the Z_1 and Z_2 currents is equal to that of L_1. Furthermore, the values of the Z_1 and Z_2 currents are inversely proportional to Z_1 and Z_2. Thus the *transient* is equivalent to charging inductance L_1 from a zero current to $2E/Z_1 = 2{,}000$ amperes through Z_1 and Z_2 in parallel.*

The effective surge impedance for such a charge is

$$\frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{500 \times 15{,}000}{500 + 15{,}000} = 484 \text{ ohms}$$

and the voltage across L_1 and Z_2 is

$$E_2\, e - \frac{Z_1 Z_2}{Z_1 + Z_2}\, t/L_1$$

Thus during the transient the voltage across L_1 and Z_2 falls exponentially from $E_2 = 2E\ Z_2/(Z_1 + Z_2) = 967{,}750$ volts to zero. (Curve 1, Fig. 4.) From the laws of exponential functions, the voltage falls 95 per cent of this value in

$$T_1 = \frac{3\,L_1}{\dfrac{Z_1 Z_2}{Z_1 + Z_2}} \text{ seconds} = \frac{3 \times 0.31}{484} \text{ seconds}$$

$$= 1{,}860 \text{ microseconds.}$$

L_s Transient. As was previously stated, the leakage inductance L_s (See Fig. 3d) prevents the voltage across Z_2 from rising instantly to its maximum value. During the time required for this voltage to rise only an exceedingly small current has time to build up in L_1, due to the fact that in transformers L_1 is generally much larger than L_s. For the analysis of the L_s transient this small current can be neglected and the equivalent circuit becomes that of Fig. 3f.

*This is in accordance with one of the useful rules of transients which permits an imaginary short-circuiting of all points between which voltage of a given transient is zero during the entire period. In this case voltage between terminals 1 and 2 is maintained constant (2E) and therefore has no transient component. This rule alone permits writing the expressions for E_2 and T_1 directly from inspection of the circuit of Fig. 3a.

Since the front of the incident wave is infinitely steep, at the first instant the entire applied voltage appears across L_s, and none across Z_1 and Z_2. No current will flow. This is the *initial* condition.

With $2E$ maintained sufficiently long across the circuit, the current will increase and approach a constant value. To this constant current L_s will offer zero reactance. The full voltage $2E$ will appear across Z_1 and Z_2 in series, and the current through them will be $2E/(Z_1 + Z_2) = 2 \times 500{,}000/(500 + 15{,}000) = 64.5$ amperes. The voltage across Z_2 will be $E_2 = 2E\, Z_2/(Z_1 + Z_2) = 2 \times 500{,}000 \times 15{,}000/(500 + 15{,}000) = 967{,}750$ volts. This is the *final* condition.

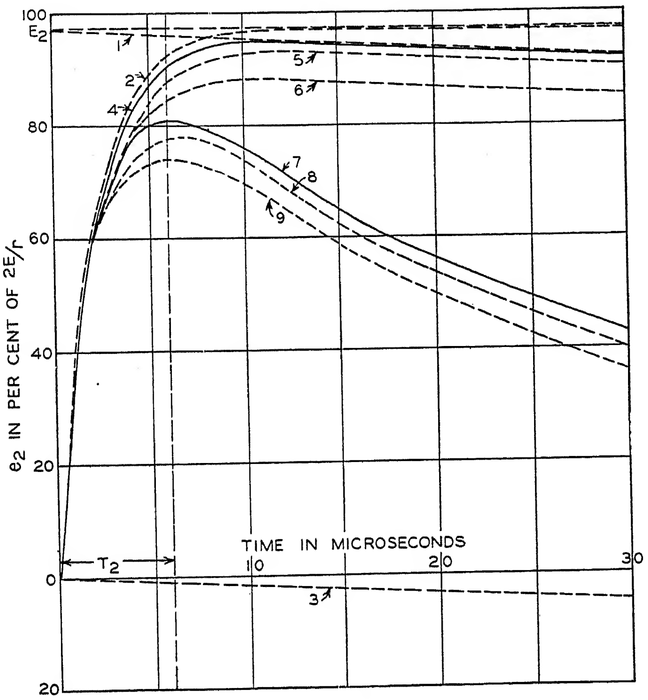

Fig. 4—Secondary Terminal Voltage as Obtained from Equivalent Circuits of Fig. 3 for the Numerical Example

Infinite rectangular primary wave
1. Voltage across Z_2 due to L_1 transient
2. Voltage across Z_2 due to L_2 transient
3. Difference between curve 1 and E_2
4. Resultant secondary voltage obtained by superposition of curves 2 and 3
5 and 6. Resultant secondary voltage (using limits of $L_1 = M$ and $L_2 = M$ respectively) obtained by rigorous solution of circuit Fig. 3c (equation (2), Part II)

Twenty-five microsecond exponential primary wave
7. Resultant secondary voltage in circuit of Fig. 3f
8 and 9. Resultant secondary voltage for circuits Fig. 3c (8) $L_1 = M$ (9) $L_2 = M$ (equation (9a), Part II)

Since the transient is caused by the *difference* between the final and initial conditions, the amplitude of transient current is $2E/(Z_1 + Z_2)$ and the amplitude of transient voltage across Z_2 is $2E\, Z_2/(Z_1 + Z_2)$. The transient consists therefore of charging the inductance L_s from a zero current to $2E/(Z_1 + Z_2)$ through Z_1 and Z_2 in series. It follows that the transient voltage across Z_2 rises exponentially from zero to $2E\, Z_2/(Z_1 + Z_2) = 967{,}750$ volts. It will reach 95 per cent of its ultimate value in $T_2 = 3L_s/(Z_1 + Z_2) = 3 \times 0.031/(500 + 15{,}000)$ seconds or in 6 microseconds. Thus,

$$e_2 = 2E\frac{Z_2}{Z_1 + Z_2} - 2E\frac{Z_2}{Z_1 + Z_2}\, e^{-\frac{Z_1+Z_2}{L_s}t}$$

$$= E_2\left(1 - e^{-\frac{Z_1+Z_2}{L_s}t}\right) \qquad \text{Curve 2, Fig. 4}$$

Resultant Z_2 Voltage. The voltage across Z_2 would be as shown by curve 2, Fig. 4 if the ultimate voltage across Z_2 were E_2. However, on account of the presence of L_1 the voltage across Z_2 gradually falls below E_2 and finally reaches zero (curve 1, Fig. 4). Therefore, voltage across Z_2 at any moment will be lower than that shown by curve 2 by the amount very nearly equal to the difference between E_2 and curve 1, *i. e.*, by:

$$-\left(E_2 - E_2\, e^{-\frac{Z_1 Z_2}{Z_1+Z_2}t/L_1}\right) = -E_2\left(1 - e^{-\frac{Z_1 Z_2}{Z_1+Z_2}t/L_1}\right)$$

(curve 3, Fig. 4).

The resultant voltage across Z_2 therefore (curve 4, Fig. 4) is obtained by superposition of curves 2 and 3 of Fig. 4.

$$i.\,e.,\ e_2 = E_2\left(e^{-\frac{Z_1 Z_2}{Z_1+Z_2}t/L_1} - e^{-(Z_1+Z_2)\,t/L_s}\right)$$

Compare this with exact solution equation (1b) and (2) of Part II.

It can be stated that the secondary voltage is essentially a copy of the voltage across the primary terminals reduced by the turn ratio (curve 1 of Fig. 4) with the front slanted to T_2 microseconds due to the effect of leakage inductance.

To arrive at the above solution the approximate equivalent circuits of Figs. 3e and 3f were used in analyzing the L_1 and L_s transients, and the effect of the transients on each other was neglected. To illustrate the degree of accuracy obtained by this simple method, curves 5 and 6 of Fig. 4 are shown, which were obtained by rigorous solution of the exact equivalent circuit of Fig. 3c for two extreme cases (one $L_1 = M$ and the other $L_2 = M$). (See equation (2) in Part II.)

In T_2 microseconds the voltage of the L_s transient will rise to 95 per cent of its maximum value (E_2) while the voltage of the L_1 transient will hardly begin to fall from the same numerical value. Therefore, the first exponential term of the above equation is approximately equal to unity for at least T_2 microseconds and

$$e_2 = E_2\left(1 - e^{-\frac{Z_1+Z_2}{L_s}t}\right)$$

This means that at $t = T_2$ $e_2 = 0.95$ $E_2 = 919{,}350$

$$\left(\text{divide it by turn ratio to get actual secondary voltage } \frac{919350}{5} = 183{,}830 \text{ volts}\right).$$

For the sake of simplicity this value will be considered

as the practical crest of secondary voltage, and T_2 as the time of its occurrence. Fig. 5.

It is important to realize the relative duration of the L_1 and L_s transients as expressed by the ratio

$$\frac{T_1}{T_2} = \frac{L_1}{L_s} \frac{(Z_1 + Z_2)^2}{Z_1 Z_2} = \frac{1860}{6} = 310$$

In practise L_1/L_s for lightning voltages is possibly never less than five and generally is above ten. Irrespective of values of Z_1 and Z_2, $(Z_1 + Z_2)^2/Z_1 Z_2$ cannot be less than four and generally is more than ten. It follows that the L_1 transient is at least twenty times slower than the L_s transient. This justifies the above approximation of the maximum secondary voltage, and the neglecting of the effect of the above transients on each other which made possible the independent use of the simplified equivalent circuits of Figs. 3e and 3f.

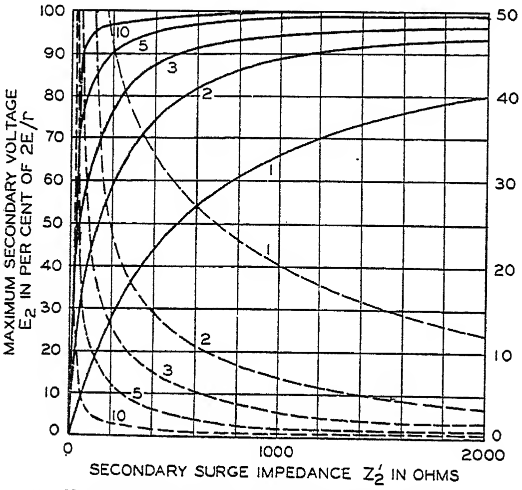

Fig. 5—Effect of Secondary Surge Impedance Z_2' and Turn Ratio on Maximum Voltage and Front of Secondary Wave

Primary surge impedance: Z_1 = 500 ohms
Short-circuit inductance: L_s = 0.01 henrys*
Numbers on curves indicate the corresponding value of turn ratio
Solid lines = maximum voltage
Dotted lines = wave front
*(For any other value of L_s the length of the front has to be multiplied by L_s/0.01)

Secondary Winding Not Grounded. In the above, one terminal of the transformer secondary winding was connected to ground, and the other to a surge impedance Z_2. If both terminals were connected to identical circuits of surge impedance Z_2, then in effect the surge impedance of the secondary circuit would be doubled. Then

$$E_2 = 2E \frac{2Z_2}{Z_1 + 2Z_2} \text{ and } T_2 = \frac{3L_s}{Z_1 + 2Z_2}.$$

A voltage to ground of $\frac{E_2}{2}$ would appear on each of the two surge impedances.

Transmission of Surges Through Auto-Transformers. The relations derived above for single-phase transformers apply equally well for auto-transformers. The transmission of surges through auto-transformers was described in a previous publication.[5]

2. Finite Rectangular Primary Wave

If a rectangular primary wave has a finite length of t microseconds, the secondary wave produced is the resultant of superposition of two secondary waves, one

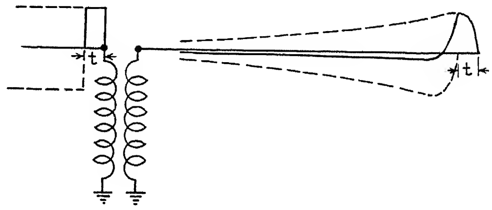

Fig. 6—Secondary Voltage Produced by Rectangular Wave, t Microseconds Long

produced by an infinite rectangular primary wave of the same polarity as the finite primary wave, and the other by an identical primary wave of opposite polarity lagging t microseconds behind the first (See Fig. 6). Since the L_1 transient is extremely slow in comparison with the length of the lightning waves found in service, it is apparent that during the short interval of t microseconds the current through L_1 will be negligible. Therefore, for

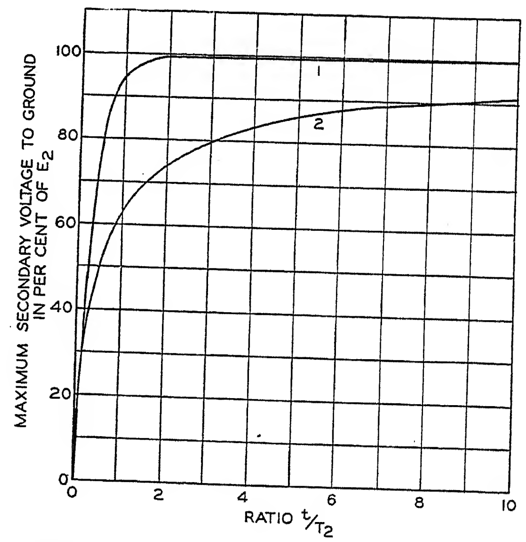

Fig. 7—Effect of the Length t of Primary (Applied) Wave on the Amplitude of the Secondary Wave

1. Finite rectangular wave applied
2. Wave with exponential tail applied
E_2 is secondary voltage produced by infinitely long rectangular wave
T_2 is secondary front produced by infinitely long rectangular wave

the analysis of conditions found in practise, *the effect of L_1 can be completely neglected*, and the equivalent circuit of Fig. 3f is sufficiently accurate, as shown by comparison of curve 7 with 8 and 9 of Fig. 4.

From curve 1, Fig. 7, it is apparent that the secondary voltage will not reach 0.95 E_2 unless the length t of the rectangular primary wave is at least equal to the length $T_2 \doteq 3L_s/(Z_1 + Z_2)$ of the front of the secondary wave.

The fraction of this value that will be reached depends upon the ratio t/T_2 as shown in curve 1 of Fig. 7.

3. Effect of Shape of Primary Wave

The maximum secondary voltage produced by the primary wave having vertical front and exponential tail is shown on curve 2, Fig. 7. In this case the length t of the primary wave is measured at the level $E/2$ as is commonly done.

The steepest front T_2 of the secondary wave is produced by a rectangular primary wave. The length of secondary wave front increases as the length of the primary wave front is increased, and the two become approximately equal when the latter is more than $3T_2$.

The steepness of the secondary wave is important from the standpoint of stresses produced in rotating machines or other apparatus in the secondary circuit. The stresses, particularly between adjacent turns, are much greater for steep wave fronts. As has been shown in previous publications, these stresses depend on the ratio of T_2 to the natural period of voltage oscillations of a given piece of apparatus, or to the length T_3 of its circuit.

The voltages to ground at different points of a secondary circuit of given length depend upon the ratio T_2/T_3 as shown by Fig. 8. These curves are based on the assumption that the far end of the secondary circuit is kept at ground potential, so that secondary waves suffer negative reflection from that point. It follows from the curves that the secondary voltage cannot reach the maximum value at any point whose "distance" from the far end is less than $T_2/2$. As a result of this phenomenon, that part of Z_2 nearest the line end is subjected to high voltages more often than the rest.

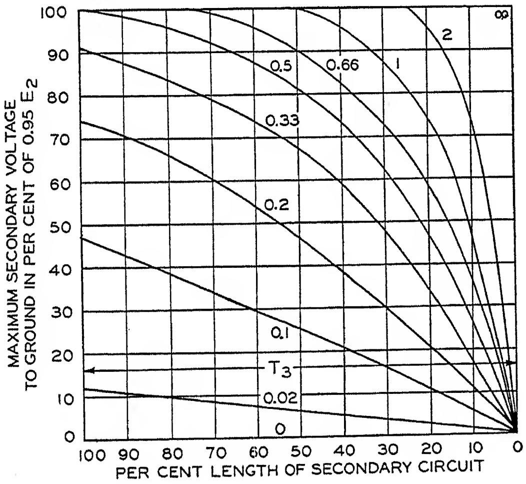

FIG. 8—EFFECT OF LENGTH OF FRONT T_2 OF SECONDARY WAVE ON MAXIMUM VOLTAGE ALONG EXTERNAL SECONDARY CIRCUIT

Total length of secondary circuit is T_3 microseconds. Far end grounded
Numbers on curves indicate corresponding values of T_3/T_2

B. Effect of Transformer Bank Connection and the Number of Phases Struck by Lightning

Relations between the circuit constants and the crest E_2 and front T_2 of the secondary wave were established above for a single-phase transformer. Since installations of power transformers are seldom single-phase, it is important to determine the secondary voltage obtained in three-phase banks. It is apparent that the results obtained for the single-phase transformer apply directly in case both windings and generator are connected in Y with all neutrals grounded, because in such a case each phase is independent of the others. Table I shows the effect on the secondary voltage of other bank connections. For convenience of comparison the front (T_2) and crest (E_2) obtained in a single-phase transformer is taken as a reference. Figures without brackets give relative voltages and wave fronts for banks which have the same ratio of *line* voltages, while figures in brackets give the corresponding values for banks which have the same turn ratio or ratio of *phase* voltages.

TABLE I

BANK CONNECTIONS			*RELATIVE MAXIMUM SECONDARY VOLTAGE TO GROUND		AMPLITUDE FACTOR AND TIME CONSTANT (EQUATION 13-b)	
	PRIM.	SEC.	AMPLITUDE	FRONT LENGTH	A	B
1	Y⏚	Y⏚	100	100	$\frac{1}{r}\frac{q}{1+q}$	$\frac{Z_1(1+q)}{L_s}$
2	Y⏚	Y	67 (67)	100 (100)	$\frac{2}{3}\frac{1}{r}\frac{q}{1+q}$	$\frac{Z_1(1+q)}{L_s}$
3	Y	Y⏚	67 (67)	100 (100)	$\frac{2}{3}\frac{1}{r}\frac{q}{1+q}$	$\frac{Z_1(1+q)}{L_s}$
4	Y	Y	67 (67)	100 (100)	$\frac{2}{3}\frac{1}{r}\frac{q}{1+q}$	$\frac{Z_1(1+q)}{L_s}$
5	Δ	Δ	67 (67)	33 (33)	$\frac{2}{3}\frac{1}{r}\frac{q}{1+q}$	$3\frac{Z_1(1+q)}{L_s}$
6	Δ	Y⏚	57 (100)	300 (100)	$\frac{1}{r}\frac{q/3}{1+q/3}$	$3\frac{Z_1(1+\frac{q}{3})}{L_s}$
7	Δ	Y	57 (100)	300 (100)	$\frac{1}{r}\frac{q/3}{1+q/3}$	$3\frac{Z_1(1+\frac{q}{3})}{L_s}$
8	Y	Δ	57 (33)	11 (33)	$\frac{1}{r}\frac{q}{1+3q}$	$\frac{Z_1(1+3q)}{L_s}$
9	Y⏚	Δ	57 (33)	11 (33)	$\frac{1}{r}\frac{q}{1+3q}$	$\frac{Z_1(1+3q)}{L_s}$

*Amplitudes and lengths of fronts of the bank connections (2 to 9) are expressed in per cent of corresponding values of bank connection (1) as the base

Figures without brackets are based on equal ratio of operating line-to-line voltage

Figures in brackets are based on equal transformer turn ratio

To illustrate the simplicity of the method used in obtaining this table, the solution for secondary voltage will be analyzed for single-phase excitation of a Y-delta bank with neutral isolated and generator connected in Y.*

*There is considerable similarity between the method used here and that used in analysis of single-phase excitation of a three-phase system at operating frequency.

Construction of the equivalent circuit of the bank is accomplished in three steps in Fig. 9. From the relations of currents of the secondary and primary lines it is evident that in spite of the fact that each individual transformer was reduced to an equivalent 1:1 transformer the combined effect of interconnection of the three units resulted in a 1.5:1 ratio of transformation from the primary terminals (a to b and c) to the secondary terminals d to e). Therefore, in the final equiva-

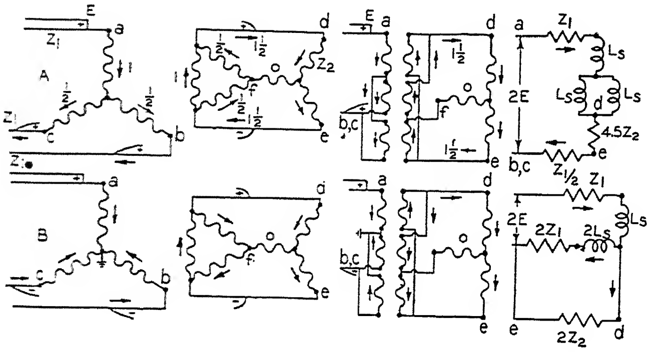

FIG. 9—DEVELOPMENT OF EQUIVALENT CIRCUITS FOR Y-DELTA TRANSFORMER BANKS CONNECTED TO A GENERATOR (BASED ON ONE-TO-ONE TRANSFORMER RATIO)

A—Primary neutral isolated
B—Primary neutral grounded

lent circuit, the surge impedance Z_2' is multiplied not by r^2, but by $(1.5\,r)^2$ or $2.25\,r^2$. Furthermore, since none of the secondary lines is grounded, the voltage to ground at d is $+\dfrac{e_2}{2}$ and at e is $-\dfrac{e_2}{2}$. The current in each corresponding generator phase is

$$\frac{e_2}{2 \times 2.25\,Z_2};$$

which means that in the equivalent circuit $2 \times 2.25\,Z_2$ must be substituted for the secondary surge impedance.

Since the primary current flows first through one transformer and then through the other two in multiple, the total inductance of the circuit is equal to $L_s + \dfrac{L_s}{2}$. For the same reason the combined surge impedance of phases b and c is equal to $\dfrac{Z_1}{2}$. These considerations lead to the final equivalent circuit of Fig. 9. This circuit is very similar to that of Fig. 3f and therefore we can write the following expressions:

$$E_2 = \frac{2E}{1.5\,Z_1 + 4.5\,Z_2}\,4.5\,Z_2 \times \frac{1}{1.5}$$

$$= \frac{2E}{Z_1 + 3Z_2} \times 2Z_2 \text{ volts}$$

$$T_2 = \frac{3L_s}{Z_1 + 3Z_2} \text{ seconds.}$$

The voltage from one phase (f) of the secondary circuit to ground is zero, and from the other two (d and e) is $\pm E_2/2$.

Where $Z_1/3Z_2$ is very small, $E_2 = 2E/1.5$ and $T_2 = L_s/Z_2$. Thus the maximum possible line-to-line voltage in this case is 67 per cent and the line-to-ground voltage is 33 per cent of that for a single-phase transformer, and the front of the wave is three times as steep.

To obtain the actual secondary voltage, E_2 must be divided by the turn ratio r.

Since the two secondary waves are equal but opposite in polarity, the resultant voltage to ground will be zero at the point of the secondary circuit where the waves meet as at the neutral of a generator.

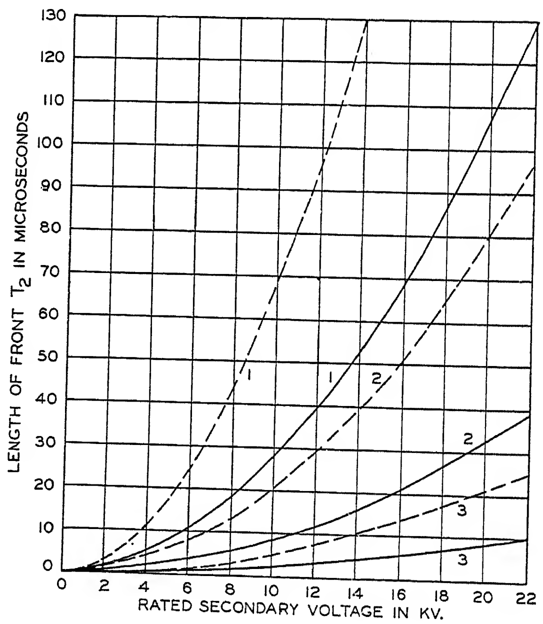

FIG. 10—LENGTH OF FRONT OF SECONDARY WAVE AS A FUNCTION OF RATED SECONDARY LINE-TO-LINE VOLTAGE

Solid curves $Z_2' = 500$ ohms. Dotted curves, $Z_2' = 200$ ohms

1. Transformer capacity 1,000 kva. per phase. Short-circuit reactance 5 per cent
2. Transformer capacity 5,000 kva. per phase. Short-circuit reactance 7.5 per cent
3. Transformer capacity 25,000 kva. per phase. Short-circuit reactance 10 per cent

Frequency 60 cycles per second. Z_2 is very large compared to Z_1
These curves apply to Y-Y connection with both neutrals grounded
For other bank connections see Table I

If identical lightning waves arrive simultaneously on two phases (b and c) with no wave on the third phase (a), the voltage from b and c to a will be the same as above (Fig. 9) and the secondary voltages therefore will be the same.

When identical lightning waves arrive simultaneously on all three phases in banks having one side connected in delta, no voltage is produced on the secondary side because the voltages in the three phases of the delta are equal and in phase.

C. *Finite Secondary Circuit*

All voltage relations established in the paper were arrived at under assumption that the time length T_3 of the secondary circuit is at least half of the total length t_2' of the secondary wave. In case the secondary circuit is shorter than that, the crest and the front of the secondary wave may be essentially different from those given above. This is caused by the phenomena of repeated reflections of the secondary wave from both ends of the circuit.

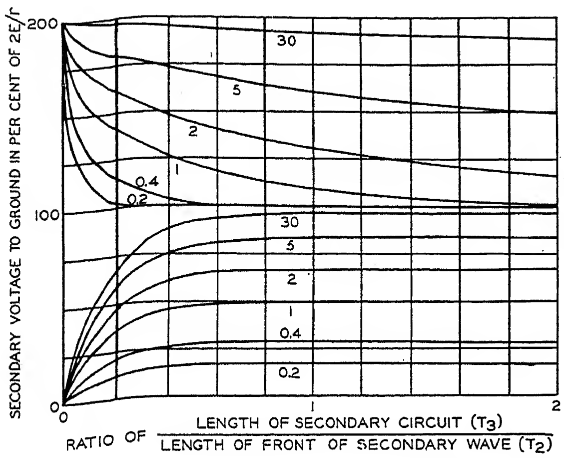

FIG. 11—SECONDARY TERMINAL VOLTAGE DUE TO REPEATED REFLECTIONS IN FINITE GROUNDED AND ISOLATED SECONDARY CIRCUITS. NUMBERS ON CURVES INDICATE CORRESPONDING VALUES OF Z_2/Z_1

In case the far end of the secondary circuit is grounded directly or through a relatively (to Z_2') low surge impedance the resultant voltage will be lower. This is the most common condition in practise because, as was explained in the paper (Fig. 9), in case the secondary external circuit is connected in delta or Y the point in that circuit, (like the neutral for example), where two secondary waves (caused by a single primary wave) meet, acts as if it were grounded (except bank connection in No. 1, Table I with primary wave on more than one phase). Fig. 11. Zone grounded.

In case the far end of the secondary circuit is open or grounded through a relatively (to Z_2') high surge impedance the voltage will be higher and its front steeper than was given previously in this paper. Such a condition is found, for example, where low voltage (secondary) buses are left connected to transformers but are disconnected from generators. Fig. 11. Zone isolated.

In case a generator is connected to transformer by means of a cable, the repeated reflections in the cable may produce voltages higher than those produced in case the generator is connected to the transformer directly. However, this voltage will be lower than that produced in the cable with generator disconnected. Fig. 12. The shape and the amplitude of these voltages can be calculated with sufficient accuracy on the assumption that the cable of surge impedance Z_2 and of a given length is equivalent to a condenser C_2 whose capacitance is equal to that of the cable. (Fig. 12.)

CONCLUSIONS

The voltage on the secondary terminals of a transformer produced by an impact of a lightning wave on the primary side consists of four components. The magnitude of the voltage due to the fourth component generally is much higher than that produced by the second and third components and may be exceeded only by that produced by the first, under certain special conditions.

When the surge impedance Z_2' of the external secondary circuit is very high or the circuit is short, amplitudes of each of the four components may be dangerously high.

The voltages due to the first three components depend on the relative position and distribution of windings and of the core, and may be essentially different in transformers of identical operating characteristics, like short-circuit reactance, kva. rating, etc., but of different types of design.

The fourth component, the electromagnetic, depends only on the short-circuit reactance of a transformer and is the same for transformers having the same reactance (in henrys) and the same turn ratio no matter how different may be their design features.

For the above reasons, in a general discussion of characteristics of lightning voltages transmitted to the secondary network, little can be said about the first three components that would be applicable to all transformers, particularly because the essential design features of a given transformer are generally known only to its manufacturer. On the other hand, the value of short-circuit reactance and of turn ratio of transformers is generally known and therefore very definite and specific conclusions can be drawn in regard to the voltage produced by the fourth component.

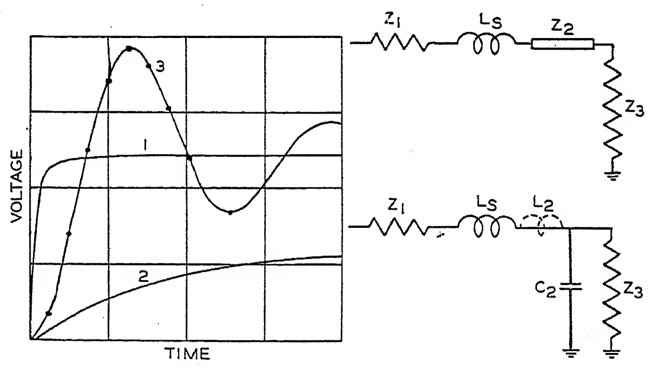

FIG. 12—EFFECT OF CABLE AND CAPACITOR ON SECONDARY VOLTAGE TO GROUND (CALCULATED)

1. Generator (Z_3) connected directly. $Z_2 = 0$
2. Generator (Z_3) connected through cable (Z_2)
   Repeated reflections are neglected
3. Generator (Z_3) connected through cable (Z_2)
   Repeated reflections are taken into account. Dots indicate points calculated by method of repeated reflections. Curve is calculated with total capacitance (C_2) and total inductance (L_2) of cable substituted for its surge-impedance (Z_2)

The following conclusions may be drawn concerning voltages produced on secondary circuits by the *electromagnetic components*.

1. The effect of a transformer on lightning voltages, transformed electromagnetically to the secondary network, is practically the same as that of an inductance equal to the short-circuit inductance of the transformer, connected between the primary and secondary lines (in the equivalent circuit reduced to 1:1 ratio). Figs. 3 and 4.

2. The ratio of crests of primary and secondary voltage is practically equal to the ratio of turns of the respective windings, where surge impedance of primary circuit is negligible in comparison with the effective surge impedance ($Z_2 = Z_2' r^2$) of the secondary circuit, as often is the case. (Fig. 5.)

3. On overhead transmission lines the amplitude of a relatively long (20 microseconds or more) traveling wave may be six times the system operating voltage (crest). It follows that such a wave may produce a secondary voltage of 3.4 to 6 times the secondary operating voltage, depending upon the transformer bank connection used. (Table I)

4. Although the front of an incident traveling wave may be vertical, the front (T_2) of the secondary wave is always slanted due to the effect of short-circuit inductance of the transformer. The front (T_2) is shortest where transformers of large kva. capacity and of relatively low voltage and reactance are connected to circuits of high-surge impedance, and is longest where the opposite is the case. In general the length of the front ranges from a few to a hundred microseconds (Fig. 10), but in exceptional cases may reach several thousand microseconds.

5. For the secondary voltage to reach 75 per cent of its possible maximum, the length of the primary wave must be at least $T_2/2$ if it has a vertical front and flat top or at least $2T_2$ if it has vertical front and exponential tail. (Fig. 7.)

6. The slanting of the front tends to reduce stresses in windings of apparatus connected to the secondary, particularly stresses between adjacent turns.

7. A secondary voltage to ground is the same whether one or two phases of the primary are struck by lightning. If either primary or secondary, or both, are connected in delta, then, when all three primary phases are struck by equal traveling waves, no secondary voltages will be produced.

8. With the exception of the Y-Y bank with both neutrals grounded, all three-phase bank connections produce zero lightning voltage at the isolated neutral of generators connected in Y.

9. A few hundred feet of metal-clad cable, connected between a transformer and generator, may materially slope the front of a secondary voltage wave, but substantially increase the amplitude if the primary wave is sufficiently long. The same is true of relatively small capacitances connected from the generator terminals to ground. (Fig. 12) (See Part II)

10. A neutral grounding resistance on the primary side tends to reduce the amplitude and shorten the front of the secondary wave. A reactance tends to increase the length of the front and thereby reduce the maximum amplitude produced by relatively short waves. In practise, the effect of the resistance will be small, while that of the reactor may be appreciable.

11. Protective gaps and lightning arresters are more effective in reducing secondary voltage when connected on the primary side. When connected on the secondary side, the sudden arcing of the gap or of the arrester may cause considerable stresses between turns of connected apparatus. This is not true in case the arrester has the proper negative volt-ampere characteristics.

12. Where capacitances or metal-clad cables are used on the secondary side, an arcover of the primary line may cause very serious oscillation between the capacitance of the secondary network and the transformer short-circuit inductance. If the applied voltage is a train of waves such as a switching surge, the secondary voltage may be built up to higher values than that produced by a single wave, due to the phenomenon of resonance. In such cases it is particularly desirable to use lightning arresters on the secondary side in addition to those on the primary.

13. If the operating voltage of the generator is the same as that of the transmission line, a device described by the author in an A.I.E.E. discussion in June 1930 can materially reduce the transient voltage. In many cases this device is more effective and economical than a 1:1 delta Y transformer bank that reduces voltages to 57 per cent (or less) of the value of the incoming wave.[6]

14. Where the "time length" (T_3) of a secondary circuit is short, in comparison with the length of the secondary wave, voltages in excess of that mentioned in conclusion 2 may result (Fig. 11). For this reason, particularly high secondary voltages may be expected on low-voltage buses disconnected from the apparatus and the network.

15. The above analysis was made for a transformer. It is obvious, however, that the same formulas apply also to the analysis of a rotating machine and of an induction regulator due to the presence of "transformer action" between windings.

Part II

When a lightning wave strikes the terminal of the primary winding of a transformer, four distinct phenomena contribute to the transient in the secondary winding. These are discussed in the first part of the paper. The electromagnetic transfer of the lightning wave has predominating influence on the secondary transient as far as secondary terminal voltages are concerned, provided lines, cables or generators of relatively low surge impedance are connected to the secondary winding.

A. Single-Phase Transformer with Grounded Neutral

Calculations for single-phase connection are chosen to derive simple equations for the electromagnetic transformation of lightning waves in inductive windings. They can be applied directly for Y-Y connected trans-

former banks with both neutrals grounded, and are the basis for solution of other transformer connections.

Fig. 3a shows the diagrammatic sketch of a two-winding transformer, with primary and secondary transmission lines. This can be simplified to Fig. 3b, where the surge impedances are reduced to resistances (provided the lines are long) and the applied voltage has twice the amplitude of the traveling wave of Fig. 3a. The internal resistances of the windings are in general too small to affect the result and therefore can be neglected. If in any case it should appear necessary to take these resistances into account, they may be inserted in series with the corresponding surge impedances of the lines. The fundamental equations for voltages and currents in operational form are:

$$2E = (Z_1' + pL_1')\, i_1 \pm pMi_2'$$
$$0 = (Z_2' + pL_2')\, i_2 \pm pMi_1'$$

where $p = \dfrac{d}{dt}$ is Heaviside's operator. Solving these two simultaneous equations for i_2 gives

$$i_2 = \mp \frac{2E\, M'}{L_1'L_2' - M'_2} \frac{p}{p^2 + 2\alpha' p + \omega_o'^2};$$

$$e_2 = i_2 Z_2' = \mp \frac{2E\, M' Z_2'}{L_1'L_2' - M'^2} \frac{p}{p^2 + 2\alpha' p + \omega_o'^2}$$

The solution[8] of this equation is

$$e_2 = \mp \frac{2E\, M'Z'_2}{L_1'L_2' - M'^2} \frac{\epsilon^{-m't} - \epsilon^{-n't}}{n' - m'}$$

$$= \mp \frac{2E\, M'Z_2'}{\sqrt{(L_1'Z_2' - L_2'Z_1')^2 + 4Z_1'Z_2'M'^2}} (\epsilon^{-m't} - \epsilon^{-n't}) \quad \text{(1a)}$$

where

$$n' = \frac{L_2'Z_1' + L_1'Z_2' + \sqrt{(L_2'Z_1' - L_1'Z_2')^2 + 4Z_1'Z_2'M'^2}}{2\,(L_1'L_2' - M'^2)};$$

$$m' = \frac{L_2'Z_1' + L_1'Z_2' - \sqrt{(L_2'Z_1' - L_1'Z_2')^2 + 4Z_1'Z_2'M'^2}}{2\,(L_1'L_2' - M'^2)} \quad \text{(1b)}$$

The same solution has been published by various authors. In a practical case the sign should be determined by the polarity of the transformer. It will be assumed positive in the rest of the paper. The constants used in the equation are all physical constants, which should be calculated from the physical configuration of the coils and transmission lines. The equation can be rewritten using constants which are converted to the primary side (or secondary) as for the equivalent transformer circuit (Fig. 3c).

$Z_1 = Z_1'$; $L_1 = L_1'$; $M = rM'$; $L_2 = r^2L_2'$; $Z_2 = r^2Z_2'$

r = ratio of primary turns to secondary turns.

Then

$$e_2 = \frac{2E}{r} \frac{Z_2M}{\sqrt{(L_1Z_2 - L_2Z_1)^2 + 4Z_1Z_2M^2}} (\epsilon^{-mt} - \epsilon^{-nt}) \quad (2)$$

$$e_2\, max = 2E\, A\, C \quad (3)$$

where amplitude factor

$$A = \frac{1}{r} \frac{Z_2M}{\sqrt{(L_1Z_2 - L_2Z_1)^2 + 4Z_1Z_2M^2}} \quad (4)$$

and crest factor C is the maximum value of $(\epsilon^{-mt} - \epsilon^{-nt})$ as t is varied. The expressions for n and m are identical to those for n' and m' using constants of the equivalent circuit in place of the physical constants. Values of Z_1 and Z_2, representing surge impedances of transmission lines, cables and generators can be obtained quite readily, while the evaluation of L_1, L_2 and M requires involved calculations or tests. In any given transformer the values of L_1, L_2 and M are not constant due to saturation and skin effect in the core. Fortunately results within engineering accuracy can be obtained although the absolute values of inductances are not known. The following transformation will show that a wide variation in absolute values of inductance produces only a small variation in the amplitude of the secondary wave and therefore a simplified equation can be used.

The amplitude factor (4) can be rewritten as

$$A = \frac{1}{r} \frac{1}{\sqrt{\left(\frac{L_1}{M} - \frac{Z_1}{Z_2}\frac{L_2}{M}\right)^2 + 4\frac{Z_1}{Z_2}}}$$

and substituting:

$c = \dfrac{M}{L_1}$; $d = \dfrac{M}{L_2}$; $q = \dfrac{Z_2}{Z_1}$ = effective ratio of secondary to primary surge impedance

$$q = r^2 s;\ s = \frac{Z_2'}{Z_1'}$$

and

$L_s = \dfrac{L_1L_2 - M^2}{L_2} \cong L_1 + L_2 - 2M$ = Short-circuit inductance in henrys.

$$A = \frac{1}{r} \frac{q}{\sqrt{\left(\frac{q}{c} - \frac{1}{d}\right)^2 + 4q}} \quad (5)$$

The crest factor is a function of n/m

$$n = \frac{Z_1}{2L_s}\left(1 + \frac{dq}{c} + \sqrt{\left(1 + \frac{dq}{c}\right)^2 - 4\frac{L_s}{M}dq}\right)$$

$$m = \frac{Z_1}{2L_s}\left(1 + \frac{dq}{c} - \sqrt{\left(1 + \frac{dq}{c}\right)^2 - 4\frac{L_s}{M}dq}\right) \quad (6)$$

Approximate Solution. In concentric construction either L_1 or L_2 is approximately equal to M, depending on the position of the coils with respect to each other and the core, and therefore either c or d will be approximately equal to unity. In interleaved construction the value of L_1 is approximately equal to L_2 and therefore c is approximately equal to d. In the following tables the variation of the maximum secondary voltage is given for two conditions, *i.e.*, $M/L_1 = 1$ and $M/L_2 = 1$. The short-circuit inductance L_s remains constant at 0.2 henrys, while it is arbitrarily assumed that the primary or secondary inductances change from 1 henry (air-core) to 100 henrys (iron-core). Amplitude factors, crest factors and maximum secondary voltages obtained with air-core inductance are expressed in per cent of the corresponding values obtained with iron-core inductance

Constants	L_1 henrys	L_2 henrys	M henrys	$C = \frac{M}{L_1}$	$d = \frac{M}{L_2}$	$\frac{L_s}{M}$
Iron-core (i)	100	M	99.8	0.998	1	0.002
Air-core (a)	1	M	0.8	0.8	1	0.25
Iron-core (i)	M	100	99.8	1	0.998	0.002
Air-core (a)	M	1	0.8	1	0.8	0.25

$q = \frac{Z_2}{Z_1}$	$\frac{A(a)}{A(i)} \times 100$	$\frac{C(a)}{C(i)}$	$\frac{e_2 \max (a)}{e_2 \max (i)} \times 100$	
1	99.4	78.8	78.4	$L_2 = M$
100	80.3	99.5	80.0	
1	99.2	75.5	74.8	$L_1 = M$
100	98.3	98.0	97.3	

This table shows that the maximum secondary voltages for air-core are 75 to 97 per cent of those for iron-core or in other words, the secondary voltage varies not over 25 per cent even for the extreme conditions assumed here. In nearly all practical cases L_s/M will be smaller than 0.1, which will decrease the variation of maximum voltage to 10 per cent or less. Considering the narrow range between air-core and iron-core values of maximum secondary voltages, it is justifiable to substitute $c = 1$ in equations (5) and (6). The equation for secondary voltage then is reduced to a very simple form (exponent m becomes so small compared to n that ϵ^{-mt} equals 1 for all practical purposes).

$$e_2 = 2E \frac{1}{r} \frac{q}{q+1} \left(1 - \epsilon^{-\frac{Z_1}{L_s}(1+q)t}\right) \tag{7}$$

This equation gives a good approximation for the front and the maximum amplitude of the secondary wave and e_2 is reduced to a function of only the effective ratio of secondary to primary surge impedances, the primary surge impedance and the short-circuit inductance of the transformer. In the case of infinitely long wave this equation would obviously give incorrect results for the tail of the secondary waves. However, for waves of the length found in practise, good approximate results are obtained when the approximate equations (**8b**) and (**9b**) are used (Fig. 4). The circuit representing equation (7) is shown in Fig. 3f and consists of two transmission lines joined by the short-circuit inductance of the transformer.

Finite Wave. Equations developed so far represent the secondary wave when the lightning wave has rectangular front and infinitely long tail. By means of Duhamel's theorem or Heaviside's shifting theorem[8] equations can be obtained for the secondary voltage, when the applied wave is not rectangular. Equations (**8a**) and (**9a**), give voltages for the circuits of Fig. 3c and (**8b**) and (**9b**) for circuit Fig. 3f.

If the applied wave at the terminals of the equivalent circuits

$e = 2E\,(\epsilon^{-at} - \epsilon^{-bt})$*

then

$$e_2 = 2E\,A \left\{ \left(\frac{a}{m-a} - \frac{b}{m-b}\right)\epsilon^{-mt} - \left(\frac{a}{n-a} - \frac{b}{n-b}\right)\epsilon^{-nt} + \left(\frac{a}{n-a} - \frac{a}{m-a}\right)\epsilon^{-at} - \left(\frac{b}{n-b} - \frac{b}{m-b}\right)\epsilon^{-bt} \right\} \tag{8a}$$

$$e_2 = 2E\,A \left\{ \left(\frac{n}{n-a}\right)\epsilon^{-at} - \frac{n}{n-b}\,\epsilon^{-bt} - \left(\frac{a}{n-a} - \frac{b}{n-b}\right)\epsilon^{-nt} \right\} \tag{8b}$$

or if the applied wave is

$e = 2E\,(\epsilon^{-at})$*

$$e_2 = 2E\,A \left\{ \left(\frac{a}{n-a} - \frac{a}{m-a}\right)\epsilon^{-at} + \frac{m}{m-a}\,\epsilon^{-mt} - \frac{n}{n-a}\,\epsilon^{-nt} \right\} \tag{9a}$$

or

$$e_2 = 2E\,A_1\,(\epsilon^{-at} - \epsilon^{-nt}) \tag{9b}$$

where

$$A_1 = \frac{n}{n-a} A = \frac{n}{n-a}\,\frac{1}{r}\,\frac{q}{q+1}$$

B. Three-Phase Banks

In the previous discussion it is shown that the transformer circuit of Fig. 3a can be simplified to that of Fig. 3f without appreciable error. The same principle can be applied to three-phase connections of transformers. Fig. 9 shows the equivalent circuits for Y-delta connected banks. Only the solution for the condition of grounded primary neutral is discussed here.

The general solution for the voltage e_b across surge impedance Z_b is

$$e_b = 2E \frac{1}{r} \left\{ \frac{Z_b Z_c}{Z_a Z_b + Z_a Z_c + Z_b Z_c} \left(1 - \frac{n\epsilon^{-mt} - m\epsilon^{-nt}}{2\omega}\right) + \frac{Z_b}{Z_a}\,\frac{\epsilon^{-mt} - \epsilon^{-nt}}{2\omega} \right\} \tag{10}$$

*These applied waves are equivalent to traveling waves of $e = E\,(\epsilon^{-at} - \epsilon^{-bt})$ and $e = E\,\epsilon^{-at}$ respectively.

and the general solution for voltage e_c is

$$e_c = 2E\frac{1}{r}\;\frac{Z_bZ_c}{Z_b+Z_aZ_c+Z_bZ_c}\left\{1-\frac{1}{2\omega}(n\epsilon^{-mt}-m\epsilon^{-nt})\right\} \qquad (11)$$

where $m = \alpha - \omega$; $n = \alpha + \omega$

$$\alpha = \frac{1}{2}\,\frac{L_a(Z_b + Z_c) + L_b(Z_a + Z_b)}{L_aL_b};$$

$$\omega_o^2 = \frac{Z_aZ_b + Z_aZ_c + Z_bZ_c}{L_aL_b};\ \omega = \sqrt{\alpha^2 - \omega_o^2}$$

By inspection the following relations will be found (all constants are transferred to the primary phase on which the lightning wave travels).

$Z_a = Z_1$; $Z_b = 2r^2Z_2' = 2r^2sZ_1 = 2qZ_1$; $Z_c = 2Z_1$

$L_a = L_s$ = short-circuit inductance of one transformer.

$L_c = 2\,L_s$

Substitution of these values in equation (**10**) gives

$$e_b = 2E\,\frac{1}{r}\,\frac{2q}{1 + 3q}\,(1 - \epsilon^{-nt}) \qquad (12)$$

The value of n is given in Table I.[9] Keeping in mind that e_b is the voltage across the secondary winding and that Z_b represents the surge impedance of two secondary lines in series it will be found that the waves traveling on two secondary lines (the voltage at the third secondary line is zero) have an amplitude of one-half the value of e_b and are of opposite polarity.

$$e_2 = \pm\,2E\,\frac{1}{r}\,\frac{q}{1 + 3q}\,(1 - \epsilon^{-nt}) \qquad (13a)$$

or $\quad e_2 = \pm\,2E\,A\,(1 - \epsilon^{-nt}) \qquad$ (**13b**)

Due to this fact the voltage at the generator neutral, where the two waves meet will be zero. The same reasoning can be applied to other bank connections, with the exception of the *Y-Y* bank with both neutrals grounded. Similarly the super-position of positive and negative waves in the phases of a delta-connected generator will cause a great reduction of voltage, where the secondary waves meet.

Equation (**13b**) is the solution for secondary voltages when identical lightning waves come in on one line only or on two lines simultaneously. If identical lightning waves come in on all three primary lines simultaneously no voltages will appear on the secondary lines. The same is true for all other bank connections of Table I with the exception of *Y-Y* bank with both neutrals grounded. Equation (**13b**) applies equally well for all bank connections, when tabulated values of amplitude factors (A) and time constants 1/11 are substituted.

Equation (**13b**) has the same form as (**7**) for single-phase connection (or *Y-Y* bank with both neutrals grounded) and equations (**8**) and (**9**) for waves with finite front and tail apply equally well requiring only the substitution of the correct amplitude factors and time constants.

The voltage on either of the two unexcited high-voltage lines becomes:

$$e_c = 2E\left(\frac{q}{1 + 3q} - \frac{1}{3}\,\epsilon^{-mt} + \frac{1}{3}\,\frac{1}{1 + 3q}\,\epsilon^{-nt}\right);$$

$$m = \frac{Z_1}{L_s} \qquad n = \frac{Z_1}{L_s}\,(1 + 3q) \qquad (14)$$

For an infinite rectangular wave the final value of e_c is r times as great as that of the secondary wave. The fronts of the waves on the high-voltage lines are, however, in general much slower than the fronts of the secondary waves. The maximum value of the wave is at the most 1/3 that of the lightning wave.

C. Repeated Reflection

In drawing the simplified circuit of Fig. 3b it was assumed that primary and secondary lines are very long. This assumption made possible the change from surge impedance to pure resistance. Although this assumption may be justified for the primary lines, it would give wrong results when secondary lines consist of short cables or overhead lines between the secondary of the transformer and the generator terminal or the finite length of generator windings. In such cases the replacement of the surge impedance is valid only until the wave returns to the transformer terminal after being reflected from the far end of the secondary line. From that instant on the returned wave and its reflection on the transformer terminals should be added to the initial secondary wave (see Fig. 12). If the reflection operator on the far end of the line is R_g and the reflection operator on the transformer terminal is R_T, then the resulting secondary voltage for single-phase transformation due to reflection can be written as:

$$e_{2res} = e_2 + (R_ge_2 + R_TR_ge_2)_{(t-2T)} + (R_g^2R_Te_2 + R_g^2R_T^2e_2)_{(t-4T)} + (R_g^3R_T^2e_2 + R_g^3R_T^3e_2)_{(t-6T)} + \ldots \qquad (15)$$

where $2T$ indicates the time interval during which the original wave travels twice the length of the secondary line, taking into account the velocity of propagation, which is different for transmission lines (1,000 ft. per microsecond) cables (500 ft. per microsecond) and generators (100 ft. per microsecond). The subscript $(t - 2T)$ indicates that the two waves in the brackets have to be added to the previous wave at time $t = 2T$. Suppose the secondary line or cable with surge impedance Z_2 is starting at the far end to a generator with a surge impedance of Z_3 ohms then

$$R_g = \frac{Z_3 - Z_2}{Z_3 + Z_2}$$

In *Y*-connected generators, Z_3 is the surge impedance of one phase, in delta-connected generators Z_3 = one-half the surge impedance of one phase.

For the simplified circuit of Fig. 3f the reflection factor R_T at the transformer terminals becomes

$$R_{\mathrm{T}} = \frac{Z(p) - Z_2}{Z(p) + Z_2} = \frac{pL_s + Z_1 - Z_2}{pL_s + Z_1 + Z_2}$$

$$= \frac{p + \dfrac{Z_1 - Z_2}{L_s}}{p + \dfrac{Z_1 + Z_2}{L_s}} = \frac{p + \beta}{p + \alpha}$$

The solution for R_{T} for an infinite long wave with rectangular front is

$$R_{\mathrm{T}} = \frac{\beta}{\alpha} + \epsilon^{-\alpha t}\left(1 - \frac{\beta}{\alpha}\right)$$

The applied wave in this case is, however, the original secondary wave (equation (7)) reflected at the generator end and equals

$$R_g e_2 = 2E\,\frac{Z_3 - Z_2}{Z_3 + Z_2}\,A\,(1 - \epsilon^{-\alpha t})$$

Using the Heaviside shifting theorem or Duhamel's theorem the reflected wave resulting when $R_g e_2$ is applied becomes

$$R_g R_{\mathrm{T}} e_2 = 2E\,\frac{Z_3 - Z_2}{Z_3 + Z_2}\,A\left\{\frac{\beta}{\alpha} - \epsilon^{-\alpha t}\left[\alpha t\left(\frac{\beta}{\alpha} - 1\right) + \frac{\beta}{\alpha}\right]\right\}$$

This wave travels out on the line, is reflected at the generator, and a wave $R_g^2 R_{\mathrm{T}} e_2$ returns. $R_g^2 R_{\mathrm{T}} e_2$ is the wave to be used as applied wave in this case.

$$R_g^2 R_{\mathrm{T}} e_2 = 2E\left(\frac{Z_3 - Z_2}{Z_3 + Z_2}\right)^2 A\left\{\left(\frac{\beta}{\alpha}\right) - \epsilon^{-\alpha t}\left[(\alpha t)\left(\frac{\beta}{\alpha} - 1\right) + \frac{\beta}{\alpha}\right]\right\}$$

and the reflected voltage at the transformer becomes

$$R_g^2 R_{\mathrm{T}}^2 e_2 = 2E\left(\frac{Z_3 - Z_2}{Z_3 + Z_2}\right)^2 A\left\{\left(\frac{\beta}{\alpha}\right)^2 - \frac{\epsilon^{-\alpha t}}{2}\left[(\alpha t)^2\left(\frac{\beta}{\alpha} - 1\right)^2 + 2\alpha t\left\{\left(\frac{\beta}{\alpha}\right)^2 - 1\right\} + 2\left(\frac{\beta}{\alpha}\right)^2\right]\right\}$$

or in general the nth reflection at the transformer terminal becomes

$$R_g^n R_{\mathrm{T}}^n e_2 = 2E\left(\frac{Z_3 - Z_2}{Z_3 + Z_2}\right)^n A\left\{\frac{\beta^n}{\alpha^n} - \epsilon^{-\alpha t}\left[\frac{(\beta t)^n}{\underline{|n}} + \sum_0^n{}_k\left\{\left(\frac{\beta}{\alpha}\right)^k \frac{(\beta t)^{n-k}}{\underline{|n-k}} - \frac{\underline{|n}}{\underline{|n-k}}\,\frac{\beta^{n-k}}{\underline{|k}}\right.\right.\right.$$

$$\left.\left.\left.\sum_0^{k-1}{}_\gamma\,\frac{\alpha^{k-\gamma}}{\underline{|n-\gamma}}\,\frac{t^{n-\gamma}}{\underline{|k-1-\gamma}}\,\frac{\underline{|k-1}}{\underline{|\gamma}}(-1)^{k-1-\gamma}\right)\right\}\right]\right\} \quad (16)$$

or if the applied wave is of the form $e = 2E\,\epsilon^{-at}$

$$R_g^n R_{\mathrm{T}}^n e_2 = 2E\left(\frac{Z_3 - Z_2}{Z_2 + Z_3}\right)^n A_1\left\{\left(\frac{\beta - a}{\alpha - a}\right)^n \epsilon^{-at}\right.$$

$$- \epsilon^{-\alpha t}\left[\frac{\beta - a}{\alpha - a}\,\frac{((\beta - a)t)^{n-1}}{\underline{|n-1}} - \frac{(-(\alpha - a)\,t)^{n-1}}{\underline{|n-1}}\right.$$

$$+ \frac{(\beta t)^n + (-\alpha t)^n}{\underline{|n}} + 1$$

$$+ \sum_1^{n-1}{}_k\left(\frac{\beta - a}{\alpha - a}\,\frac{((\beta - a)t)^{n-1}}{\underline{|n-1-k}}\,\frac{1}{((\alpha - a)t)^k}\right.$$

$$- \frac{(-(\alpha - a)t)^{n-1-k}}{\underline{|n-1-k}\ \underline{|k}}\,\frac{\underline{|n-1}}{\underline{|n-1-k}}$$

$$+ \frac{\underline{|n}}{\underline{|n-k}}\,\frac{(-\alpha t)^{n-k}}{\underline{|n-k}\ \underline{|k}} - \frac{\underline{|n}}{\underline{|n-k}}\,\frac{(\beta - a)^{n-k}}{\underline{|k}}$$

$$\sum_0^{k-1}{}_\gamma\left\{\frac{(-(\alpha - a)^{k-1-\gamma}}{\underline{|n-1-\gamma}}\,\frac{t^{n-1-\gamma}}{\underline{|k-1-\gamma}}\,\frac{\underline{|k-1}}{\underline{|\gamma}}\right\}$$

$$+ \frac{\underline{|n}}{\underline{|n-k}}\,\beta^{n-k}\sum$$

$$\left.\left.\left.\sum_0^k{}_\gamma\left\{\frac{(-\alpha)^{k-\gamma}}{\underline{|n-\gamma}}\,\frac{t^{n-\gamma}}{\underline{|k-\gamma}}\,\frac{\;}{\underline{|\gamma}}\right\}\right)\right]\right\} \quad (17)$$

$$A_1 = A\,\frac{\alpha}{\alpha - a};\ \beta = \frac{Z_1 - Z_2}{L_s} = \frac{Z_1}{L_s}(1 - q);$$

$$\alpha = \frac{Z_1 + Z_2}{L_s} = (1 + q)\,\frac{Z_1}{L_s}$$

a = exponent of the applied wave.

Equations (16) and (17) can be used for single-phase connections and also for those bank connections, which can be represented by equivalent circuits similar to Fig. 9A.

The equation for equivalent circuit of Fig. 9B for rectangular primary wave when a short cable or line (Z_2) is placed between the generator (Z_3) and the secondary winding of the transformer is

$$R_g^n R_{\mathrm{T}}^n e_2 = 2E\left(\frac{Z_3 - Z_2}{Z_3 + Z_2}\right)^n A\left[D^n(1 - \epsilon^{-\alpha t})\right.$$

$$+ \alpha D^n \epsilon^{-\alpha t}\sum_1^n{}_k\left(\frac{\underline{|n}}{\underline{|n-k}}\left(\frac{B}{D}\right)^k \frac{1}{\underline{|k}}\sum_0^{k-1}{}_\gamma\right\{$$

$$\left.\sum_0^{k-1}{}_\gamma\left\{\frac{(-\alpha)^{k-1-\gamma}}{\underline{|\gamma}}\,\frac{t^{k-\gamma}}{\underline{|k-\gamma}}\,\frac{\underline{|k-1}}{\underline{|k-1-\gamma}}\right\}\right)\right] \quad (18)$$

$$D = \frac{1-3q}{1+3q};\ B = \frac{6q}{1+3q};\ \alpha = \frac{Z_1}{L_s}(1+3q);\ q = \frac{Z_2}{Z_1}.$$

Simplified Solution. A simpler method of solution for secondary voltages due to repeated reflections consists in the replacement of the surge impedance of the secondary line or cable by their capacitance to ground (Fig. 12). In case the inductance of the line or cable is comparable to the short-circuit inductance of the transformer it should be added to L_s. The solution for single-phase transformation and all bank-connections, which can be represented by an equivalent circuit similar to Fig. 3a, is for

$e = E$ (traveling wave)

$$e_2 = 2E\,A\left\{1 - \frac{\omega_o}{\omega}\epsilon^{-\alpha t}\sin\left(\omega t + \tan^{-1}\frac{\omega}{\alpha}\right)\right\}$$

$$\text{if } \omega_o^2 > \alpha^2 \qquad \textbf{(19a)}$$

$$e_2\,max = 2E\,A\left(1 + \epsilon^{-\frac{\alpha}{\omega}\pi}\right)$$

since $t_1 = \dfrac{\pi}{\omega}$ is the time at which the maximum occurs.

$$e_2 = 2E\,A\left\{1 - \frac{1}{n-m}(n\epsilon^{-mt} - m\epsilon^{-nt})\right\} \text{ for } \alpha^2 > \omega_o^2 \qquad \textbf{(19b)}$$

for $e = E\,\epsilon^{-at}$ (traveling wave)

$$e_2 = 2E\,A\left\{\frac{\omega_o^2}{(\alpha-a)^2+\omega^2}\left[\epsilon^{-at} - \frac{\sqrt{(\alpha-a)^2+\omega^2}}{\omega}\epsilon^{-\alpha t}\sin\left(\omega t + \tan^{-1}\frac{\omega}{\alpha - a}\right)\right]\right\}$$

$$\text{if } \omega_o^2 > \alpha^2 \qquad \textbf{(20a)}$$

$$e_2 = 2E\,A\frac{\omega_o^2}{(\alpha-a)^2+\omega_1^2}\left\{\epsilon^{-at} - \frac{(a-m)\,\epsilon^{-nt} - (a-n)\,\epsilon^{-mt}}{n-m}\right\}$$

$$\text{if } \alpha^2 > \omega_o^2 \qquad \textbf{(20b)}$$

where

$$\alpha = \frac{1}{2}\left(\frac{Z_1}{L_s} + \frac{1}{C_2 Z_3}\right);\ \omega_o^2 = \frac{Z_1+Z_3}{Z_3}\ \frac{1}{L_s C_2};$$

$$\omega = \sqrt{\omega_o^2 - \alpha^2}$$

$$n = \alpha + \omega_1;\ m = \alpha - \omega_1;\ \omega_1 = \sqrt{\alpha^2 - \omega_o^2}$$

It should be noted that in this case $q = r^2\dfrac{Z_3'}{Z_1'}$.

Solution by means of equations (**19**) and (**20**) shows a good agreement, with the solution obtained by method of repeated reflections when the front of the secondary wave is longer than twice the length of the line or cable. If the front of the secondary wave is shorter than double the length of the line or cable only approximate results are obtained since in such a case the repeated application of steep fronts is neglected. Equations (**19**) and (**20**) can also be used, to calculate the voltage rise at the generator terminals, when a protective capacitor is used.

Conclusions

1. For the purpose of calculation of the electromagnetic transformation of lightning waves, the complicated transformer circuit can be replaced by an inductance equivalent to the short-circuit reactance of the transformer. The results obtained are in nearly all practical cases within 10 per cent of the real values.

2. Similarly bank connections can be reduced to simple equivalent circuits, which allow ready calculation of secondary voltages in networks.

3. Repeated reflections on short secondary lines or cables which are connected to generators or transformers on the far end may materially increase the amplitude of the secondary voltage over that obtained if the generator had been connected directly although the rate of rise of secondary voltage may be lower. The same is true, if small protective capacitors are used.

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8. "Heaviside's Operational Calculus," E. J. Bush, *G. E. Review*, Vol. 31, No. 9, Sept. 1928, pp. 506-509.

9. *Traveling Waves on Transmission Systems*, L. V. Bewley, A.I.E.E. Trans., Vol. 50, June 1931, pp. 532-550.

10. "Lightning Test on Power Transformers," F. W. Peek, Jr., *G. E. Review*, Oct. 1930.

"Lightning Test on Commercial Transformers," *Elec. World*, August 1930.

Full Lightning Test on Commercial Transformers, A.I.E.E. Journal, Sept. 1930.

Discussion

F. W. Peek, Jr.: The paper gives a practical method of determining the voltages that pass through the low side when the high side of a transformer is subjected to lightning. By means of this method the stresses on generators and other low side apparatus can be predetermined and adequate protective measures taken. This paper is one of a pioneer series by Mr. Palueff and his associates (Messrs. Weed, Blume, Boyajian, Brand and others) that has placed the subject of transients in transformer windings upon a scientific and engineering basis. The results of this study have been more than a theoretical treatment. From it came the now well-known non-resonating transformer in which the voltage stresses in any part of the winding and through the insulation structure are always uniform and relatively the same whether the applied voltage is at operating frequency, lightning impulses or high frequency. This ideal transformer thus has the same relative strength under all types of voltages. The designer knows just how much insulation to place at any particular part. He also knows that, whatever the type of test—60 cycles or lightning—the stresses are the same. Thus for this transformer the usual A.I.E.E. acceptance tests are equivalent to lightning tests.

I bring up the matter of testing because I feel that in view of the new knowledge resulting from this series of papers the commercial testing of transformers at operating frequencies should be reviewed by the Machinery Committee of the Institute and recommendations made for revision, if it seems advisable. A method of determining the corona point in oil in the 60-cycle test should be considered for reasons that I will bring out later. *I further recommend that this same committee set up methods, waves, etc., for making a lightning test on transformers* if such a test seems desirable. I suggest an A.I.E.E. committee for this work because of the highly technical problems involved. High-voltage testing is rather expensive and, since much depends upon it, should be made as effective as possible. I do not hesitate to recommend the consideration of commercial lightning testing after years of experience with such testing on complete transformers for the purpose of design. This testing was necessary because we have been building transformers lightning proof for a number of years. Someone has referred to impulse tests made in Europe. These tests are of such a mild value as to be useless.

One of the most important problems in establishing lightning tests is to devise means for detecting incipient turn-to-turn, coil-to-coil, or other failures. This is so because the operator must feel assured beyond any reasonable doubt that the transformer is as good after the test as it was before the test. Follow-up 60-cycle tests will not necessarily detect such injuries but they will be developed later by lightning strokes incidental to operation. Detection is difficult because of the short duration of the stroke. Long experience has shown us that in obtaining design data it is of the utmost importance to know when damage starts. This follows because the effect of lightning on partly damaged highly stressed insulation is cumulative.

A casual or superficial consideration of the subject would probably lead to the conclusion that the present A.I.E.E. tests should be abolished if lightning tests are established. Experience shows that such conclusions are erroneous. The tests at commercial frequencies are applied long enough to establish the strength of the major insulation, to detect manufacturing mistakes and, in case of over-stress, to indicate it by copious corona discharges. It is true that tests at operating frequency may also cause deterioration in poor designs, but such designs are easily detected by copious corona formation during such tests. Such discharges not only indicate high stresses but also a weak transformer for lightning. For example, corona between coils would indicate a design giving a high gradient between coils and short life under lightning. For this reason it has been our practise to design so that corona does not occur over the test range and to apply methods of detection of corona during tests.

F. W. Gay: The time is probably not far distant when all transformers to be used on high-voltage lines will receive a surge test and some manufacturers are even now waiting for the standardization of tests before purchasing one or more surge generators.

Transformers used on lines having arcover distances greater than 30 inches will probably be equipped with some means for assisting the equalization of stresses among winding turns and layers.

The electrical engineer is supplied by nature with three fundamental tools which he may use if he will in solving all his problems, *i. e.*, resistance, inductance, and capacitance. It is however, a characteristic of the electrical engineer to try and complete an undertaking while using only one of his tools.

It is entirely possible to provide a transformer with a resistance path in parallel with a winding to be protected and electrostatically couple the resistance path with the winding and with the transformer terminals, so that relatively very great currents will pass to charge the winding for high-frequency surges but relatively insignificant current will pass through the resistance at normal frequency.

It is also entirely possible to wind in juxtaposition with the many turn low-frequency high-voltage winding a surge winding of relatively very few turns and couple this high-frequency winding to the transformer terminals by condensers.

It is obvious that a shunt path of very low surge impedance is thus provided through the condenser and high-frequency few-turn surge winding. A surge of steep wave front will instantly pass through the condenser and pile up across the surge winding. The surge current will build up very fast in this surge winding and will induce a voltage in the low-frequency many-turn winding. This voltage in the low-frequency winding will serve to charge its turns and layers.

Mr. Palueff solves a problem in which the building up of the main magnetic flux in his primary winding is on the order of 2,000 microseconds. I propose to apply a surge winding about and close to a high-voltage winding having a corresponding time element on the order of a few tenths of one per cent as long; for example, having a time element on the order of say 10 microseconds instead of on the order say of 2,000 microseconds.

The time element required for voltage to build up on a secondary winding due to capacity effect between a primary and secondary winding is given for Mr. Palueff's transformer as on the order of 0.3 of one per cent of the long time element. With a surge winding such as I have outlined the distribution of voltage in the many-turn high-voltage winding, due both to inductance and capacitance, should be completed in a few hundredths of a microsecond.

A winding such as I have outlined should include in its circuit so great a series resistance that oscillation is completely damped. The success of such a surge winding requires:

1. The distribution of stress throughout the many-turn high-voltage winding so fast that substantially no oscillatory energy is stored in the high-voltage winding.

2. The natural period of oscillation of the surge winding designed so long that the many-turn high-voltage winding is able to maintain a substantially uniform voltage distribution as the voltage across the surge winding (relatively) slowly collapses to substantially zero.

3. Enough resistance in the surge winding to damp out oscillations. It would appear that considerable study could profitably be given to the use of resistance and reactance as a means of equalizing stresses in high-voltage transformers.

It is obvious that the above discussion applies primarily to those surge phenomena which Messrs. Palueff and Hagenguth classify under components 1 and 4.

In order to squelch components 2 and 3 more capacity must be used than can be justified for this purpose alone. It becomes necessary therefore to cause the capacity so employed to function

also for power factor correction and voltage regulation; and to still further distribute costs a method of transformer winding switching has been developed whereby a substantial saving can be made in transformer material, and transformer efficiencies can be materially improved, while a single control is common to all the above functions. This switching of transformer windings becomes more important as transformer iron is improved and the per pound cost of iron with respect to copper tends to increase while the total copper loss to total iron loss ratio simultaneously increases.

In practise what amounts to a six-phase transformer is used to accomplish all the above results but for simplicity in the following explanation and to avoid design discussion six single-phase transformers are assumed as follows:

1. The six single-phase transformers are assumed to be designed for 13.2-kv. to 4-kv. operation at a magnetic density of 15,000 gausses. Each transformer is assumed to have permanently connected across its 4-kv. terminals a capacitor of 60 per cent of the transformer kva. capacity.

2. The above six transformers are permanently connected in pairs in series on the high side and in series on the low side and these three pairs of transformers are then connected in delta on a 13.2-kv. circuit high side and in delta on a 4-kv. circuit low side. With this connection all six transformers and their associated capacitors are operating at one-half voltage. The magnetic density in the transformer iron will then be 7,500 gausses.

3. A separate two-pole single throw switch is provided for each pair of series connected transformers. Those two high-voltage windings connected in series across two 13.2-kv. phase wires have their common junction point connected to one pole of the switch so that on closing, the common junction point is joined to the third phase wire and the two high-tension windings are connected in open delta by the closing of the switch. In a similar manner the two corresponding low-voltage windings connected in series across a corresponding two of the 4-kv. phase wires have their common junction point connected to one pole of the switch so that on closing, the common junction point is joined to the third 4-kv. phase wire and the two low-voltage windings together with their connected capacitors are thus connected in open delta.

By successively closing the three two-pole single-throw switches the three transformers and their associated capacitors are successively changed from half-voltage to double-voltage operation until when all three switches are closed both 13.2-kv. and 4-kv. windings and their associated capacitors are connected in parallel delta.

The following results are obtained.

1. Surges under the Palueff-Hagenguth 2 and 3 classification are squelched.

2. For a given 80 per cent power factor load the active material required in the transformers is less than half what would be required in a normal transformer of high light load efficiency.

3. The transformer losses are reduced for very light loads and for overloads and are substantially the same for intermediate loads.

4. Transformer reactances are reduced at heavy loads.

5. 13.2-kv. line carrying capacity is increased by improved power factor.

6. Voltage regulation may be obtained by controlling the closing of the three two-pole switches.

7. Economy of material is obtained by combining the above functions in one device.

L. V. Bewley: The authors have shown that the transient transmitted to the secondary circuit consists of three distinct components:

1. The initial very fast electrostatic impulse.
2. The long slow electromagnetic transient.
3. The oscillatory component, of small amplitude, oscillating about (2) as an axis.

They have further shown that each of these components may be considered separately, due to the great disparity in their time constants, and have gone into considerable detail concerning the electromagnetic transient.

Some time after the above development was made by the authors, I undertook a mathematical analysis of the internal oscillations in the primary and secondary windings of a transformer. It was found that complete solutions were possible only for grounded or isolated terminal conditions; but that the initial distributions could be found for general terminal impedances, and that the equations which determine the axes of internal oscillations reduce to those given in Part II of the paper by Palueff and Hagenguth. It is interesting to attempt a verification of their conclusions by deductions from the general equations given in my paper.* As stated in the closing discussion thereto, one of the principal uses to which these general equations may be put is to examine the validity of approximate equivalent circuits. In submitting this discussion I do not wish to convey the idea that I am adding anything essentially new, but merely to correlate the work of the authors with the general equations mentioned above.

The Electrostatic Component

If both the primary and secondary neutrals are grounded, and the other secondary terminal is connected to a surge impedance z, then in the notation of my paper $Z_1 = Z_2 = O$, $Z_3 = z$, and by equations (26), (27), (28) and (29), there are:

$$\left.\begin{aligned}
E &= P\epsilon^{\alpha} + Q\epsilon^{-\alpha} + R\epsilon^{\beta} + S\epsilon^{-\beta} \\
O &= P + Q + R + S \\
O &= mP + mQ + nR + nS \\
O &= (1+a)\,mP\epsilon^{\alpha} + (1-a)\,mQ\epsilon^{-\alpha} + (1+b)\,nR\epsilon^{\beta} + (1-b)\,nS\epsilon^{-\beta}
\end{aligned}\right\}$$

in which $a = zK_2\alpha p$ and $b = zK_2\beta p$. Solving for the integration constants, and substituting in (24), there results for the secondary voltage:

$$e_2 = mn\left[\frac{\sinh \alpha x(\sinh\beta + b\cosh\beta) - \sinh\beta x(\sinh\alpha + a\cosh\alpha)}{n\sinh\alpha(\sinh\beta + b\cosh\beta) - m\sinh\beta(\sinh\alpha + a\cosh\alpha)}\right] E$$

At the terminal $x = 1$, the above equation reduces to

$$e_2 = mn\left[\frac{(\beta\sinh\alpha \,.\, \cosh\beta - \alpha\sinh\beta \,.\, \cosh\alpha)}{(n\beta\sinh\alpha \,.\, \cosh\beta - m\alpha\sinh\beta \,.\, \cosh\alpha)}\right]\frac{p}{p+\gamma}E$$

$$= \frac{E\,mn\,(\beta\sinh\alpha \,.\, \cosh\beta - \alpha\sinh\beta \,.\, \cosh\alpha)}{(n\beta\sinh\alpha \,.\, \cosh\beta - m\alpha\sinh\beta \,.\, \cosh\alpha)}\,\epsilon^{-\gamma t}$$

where

$$\gamma = \frac{(n-m)\sinh\alpha \,.\, \sinh\beta}{zK_2\,(n\beta\sinh\alpha \,.\, \cosh\beta - m\alpha\sinh\beta \,.\, \cosh\alpha)}$$

Now α and β are large enough in practical cases so that $\sinh\alpha \cong \cosh\alpha$ and $\sinh\beta \cong \cosh\beta$. Hereby the above equation simplifies to

$$e_2 \cong \frac{E\,mn\,(\beta - \alpha)}{(n\beta - m\alpha)}\,\epsilon^{-\frac{(n-m)t}{zK_2(n\beta - m\alpha)}}$$

Considering the case given in Mr. H. L. Rorden's discussion of my paper, and $z = 500$; there is $\alpha = 19.0$, $\beta = 11.7$, $m = -1.617$, $n = 0.618$, $K_2 = 1 \times 10^{-11}$. Then

$$e_2 \cong 0.193\,\epsilon^{-11.8t}$$

Thus the electrostatic component is an extremely short exponential impulse.

*A.I.E.E. Trans., June 1932, p. 299.

THE OSCILLATORY COMPONENT

For a grounded neutral and short-circuited secondary the general equations (53) and (54) become (dropping subscripts)

$$e_1 = E + \Sigma\,(A \cos \omega t + A' \cos \Omega t) \sin \lambda x$$
$$e_2 = O + \Sigma\,(rA \cos \omega t + r'A' \cos \Omega t) \sin \lambda x$$

and from equation (49)

$$(i_{L2} + i_{K2}) = I_2 + \Sigma\,[\omega A\,(rC_2 + rC_3 - C_3) \sin \omega t - \Omega A'\,(r'C_2 + r'C_3 - C_3) \sin \Omega t] \frac{\cos \lambda x}{\lambda}$$

where I_2 is an integration constant with respect to x and therefore a possible function of t. In Appendix II it is identified as the electromagnetic component of current. The oscillatory components are a maximum for $x = 0$ or $x = 1$ (the terminals). Now if the secondary is closed through an impedance the current will be *less* than that given above for a dead short circuit. Therefore the voltage transferred to the secondary by the oscillatory components can not exceed

$$e_2 = zi \leq z\,\Sigma \left[\frac{\omega A}{\lambda}\,(rC_2 + rC_3 - C_3) \sin \omega t - \frac{\Omega}{\lambda} A'\,(rC_2 + r'\,C_3 - C_3) \sin \Omega t\right]$$

In practical cases this is of the order of 1 per cent of E for $z = 500$ ohms. For example, in the case given in Mr. Rorden's discussion the fundamental component of the above equation becomes

$$e_2 \leq 0.0103\,E \text{ for the fundamental } (s = 1)$$

THE ELECTROMAGNETIC COMPONENT

The basic equations used by the authors in their investigation of the voltage and currents transmitted to the secondary are identical with the ordinary conventional equations of the transformer. However, they point out the difficulty of arriving at accurate numerical results using self and mutual inductances, and have circumvented this difficulty by showing how these equations for mutually coupled circuits may be replaced (on a 1:1 turn ratio basis) by a simple series inductance, which approximation tremendously simplifies the calculation of the electromagnetic transient. In Appendix II of my paper* it is shown that their basic equations are a natural consequence of the general differential equations of the complete circuit of the transformer (involving distributed inductances and capacitances). I would point out that while in my paper complete solutions are obtained only for zero and infinite terminal impedances, that the particular derivation given in Appendix II is general, and applies to any terminal conditions; so that the currents fixing the axes of oscillations are on conformity with the relationships

$$\left.\begin{aligned} E &= p\,(L_1'I_2 + M'I_2) + Z_1\,I_1 \\ O &= p\,(M'I_1 + L_2'I_2) + (Z_2 + Z_3)\,I_2 \end{aligned}\right\}$$

The secondary terminal voltage is a maximum when it is open-circuited ($I_2 = 0$), in which case for a grounded neutral it is in the turn ratio:

$$e_2 = \frac{M'}{L_1'} E \cong \frac{n_2}{n_1} E$$

But under this condition the electrostatic component persists for a much longer time, so that the secondary terminal voltage will be in excess of that given by the turn ratio, and should be insulated accordingly. When the secondary is open-circuited, the general equations describe the complete transient.

*Loc. cit.

THREE-PHASE BANKS

In order to determine the effect on the secondary voltage of the transformer bank connection, and the number of phases struck by lightning, the authors reduced each particular connection to an equivalent circuit and then solved these equivalent circuits. Two examples are given in their Fig. 9, one of which is discussed in Part I of their paper, and the equations for the other derived in Part II, equations (10), (11), (12) and (13). However, it is not at all necessary to reduce to equivalent circuits, and considerable time can be saved by writing the equations directly from the bank connections. The two cases which they discuss in detail will suffice as illustrations:

Fig. 9a. Writing the voltage drops from phase a to phase b directly from the bank connection, there is

$$(2E) = (Z_1 + pL_s + 0.5pL_s + 0.5Z_1 + 3Z_2 + 1.5Z_2)\,I$$

and the secondary voltage is

$$e_2 = \frac{1.5IZ_2}{r} = \frac{(2E)}{r} \frac{1.5Z_2}{1.5\,(Z_1 + pL_s + 3Z_2)} = \frac{(2E)}{r} \frac{q}{1 + 3q} \left(1 - \epsilon^{-\frac{Z_1}{L_s}(1+3q)t}\right)$$

Fig. 9b. Let the current in phase a be I and that in phase b be aI. Then writing the voltage drop from phase a to phase b directly from the bank connection, there is

$$\begin{aligned}(2E) &= [Z_1 + pL_s - aZ_1 - apL_s + (1 - a)\,2Z_2 + (1 - a)\,Z_2]\,I \\ &= (1 - a)\,(Z_1 + pL_s + 3Z_2)\,I\end{aligned}$$

and the secondary voltage is

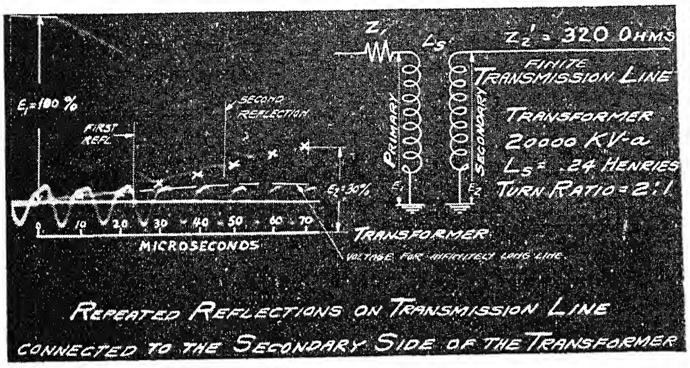
$$e_2 = \frac{IZ_2\,(1 - a)}{r} = \frac{(2E)}{r} \frac{(1 - a)\,Z_2}{(1 - a)\,(Z_1 + pL_s + 3Z_2)} = \frac{(2E)}{r} \frac{q}{1 + 3q} \left(1 - \epsilon^{-\frac{Z_1}{L_s}(1+3q)t}\right)$$

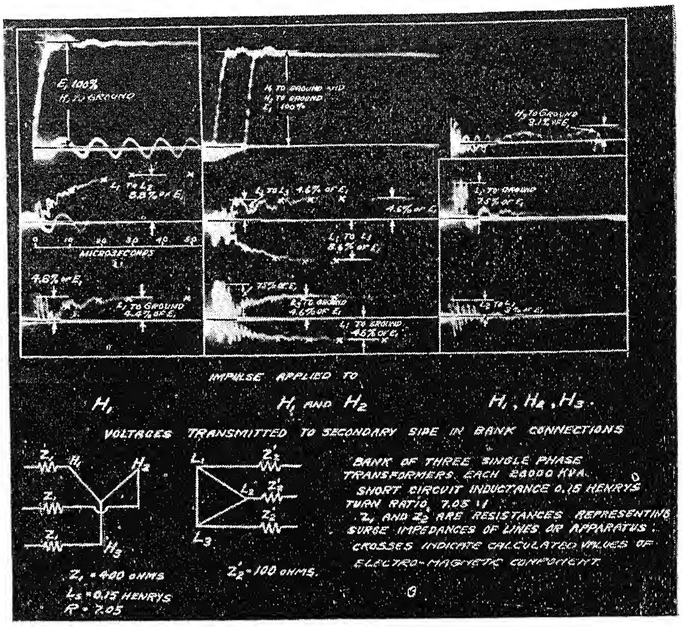
Incidentally, it may be mentioned that neither the procedure followed by the authors, nor that given above, takes cognizance of the fact that traveling waves on the transmission lines affect each other through the mutual surge impedances.

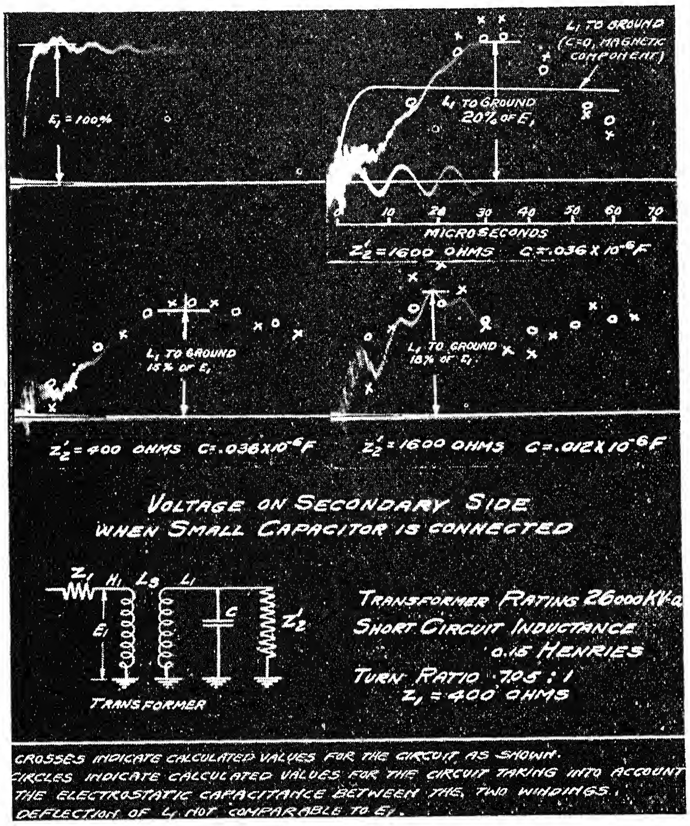
K. K. Palueff: Such tests as those suggested by Mr. Peek are very desirable indeed; they would weed out unsuitable transformer constructions. The principal difficulty in adopting an impulse test, as it appears to me, is to find reliable and positive methods of determining the effect of the test on all parts of transformer insulation. This is important because of the possibility of local damage resulting from such a test in transformers not properly designed or manufactured. Such damages, while of prime importance from the standpoint of the dielectric strength of the transformer, are often so extremely small physically that it is exceedingly difficult to locate them, or even to detect their presence.

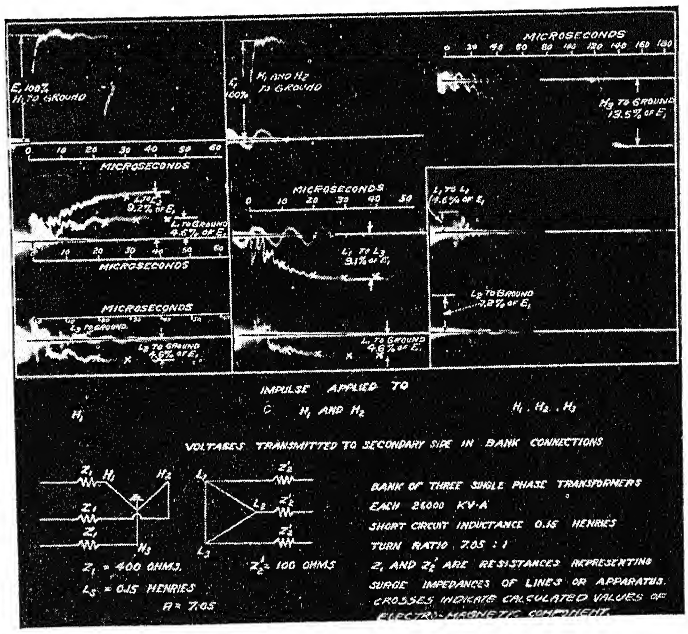
For example, on some occasions in the past upon examining transformers damaged in service by lightning, I observed, in addition to the principal damage, which is generally augmented by subsequent power flow through the short-circuit, numerous "pinhole" punctures of the insulation. These holes could not be detected without complete disassembly of the transformer and the use of an especially bright light.

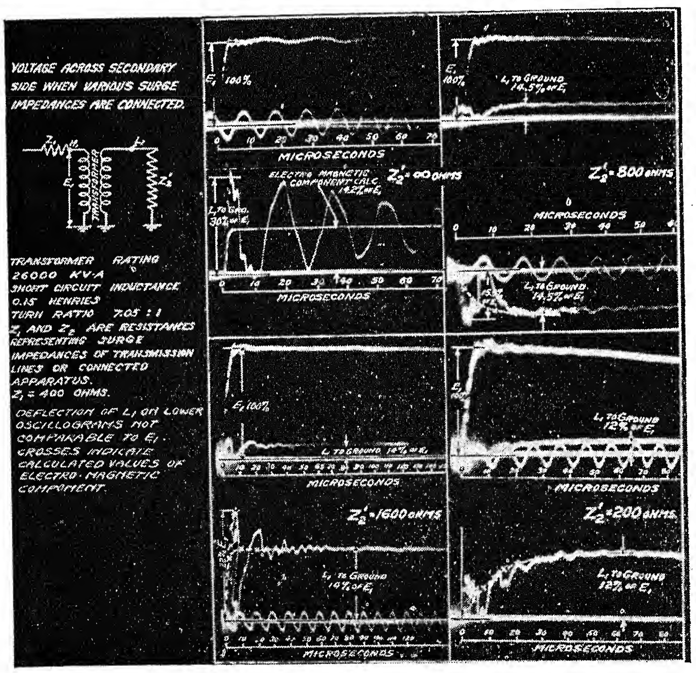
I fear that Mr. Gay's method of protection of the main transformer winding by means of an auxiliary or shunt winding connected to the terminals of the high-voltage winding through a capacitance, may prove to be not only impractical but also undesirable, as it would cause a substantial increase in the local voltage stresses in the main winding of the transformer. On account of this possibility I would suggest that Mr. Gay work out a numerical example of application of his device to an arbitrarily chosen transformer, showing at least the order of magnitude of the effect of the protective scheme.











Several years ago, we were confronted with the problem of the transmission of lightning through transformers and autotransformers. Considering the phenomenon which takes place within the transformer winding in its entire complexity, it became obvious that a general solution, permitting computations with engineering accuracy, was not possible. Consequently, we directed our efforts toward evaluation of the relative practical importance of the four components discussed in the first part of the present paper. The considerations presented led to the conclusion that the electromagnetic component dominates the phenomenon. Once this was understood, the solution of the problem became not only mathematically elementary but sufficiently accurate numerically as to be used in practise as shown in the paper. The results of this solution were presented by me in January 1931. (See reference 5 of the bibliography.) On account of space limitations it was impossible at that time to include the material of the present paper.

While Mr. Bewley's paper is devoted to the study of transient voltages *within* transformer windings, our present paper deals with transient voltages that get *through* a transformer to secon-

dary circuit. For this reason, Mr. Bewley's discussion is particularly interesting as it ties these two papers together.

The chief limitations of Mr. Bewley's solutions for oscillatory and electromagnetic components lie in the practical impossibility of sufficiently accurate evaluations of some principal constants (like L_1, L_2, M). This essentially limits the practical applications of these solutions.

His solution for electrostatic components, I believe is quite satisfactory for windings concentrically located, but not for windings that are interleaved. In the latter case capacity constants like K_1, K_2, C, C_2, C_3 can not be analytically determined. For a number of years, therefore, we were obliged to use an entirely different method for the calculation of this component. The results of this calculation were published in my previous papers and discussions.

It is my conviction that while equivalent circuits are rarely necessary for the solution of a problem, they are extremely helpful in visualizing and describing the phenomenon. It is for this reason that the equivalent circuits are given in this paper.

The oscillograms of this discussion illustrate the relative importance of the four components. They also show the degree of agreement between the calculations made in accordance with simple equations given in our paper and the actual values measured.

The Proximity Effect

Its Application to the Concentration of Heating Currents in Predetermined Strips

BY EDWARD BENNETT*

Fellow, A.I.E.E.

Synopsis.—*This paper describes methods of concentrating heating currents used for any industrial operation, such as welding, in predetermined strips of conducting plates, pipes, or other shapes. The method is to so place the shapes in close proximity to each other, or to auxiliary conductors, and to so interconnect the shapes with sources of alternating, or oscillatory, current of moderately high frequency that the heating current concentrates largely in adjacent strips in close proximity, in which the current flows in opposite directions in the two adjacent strips. Briefly stated, the method is to use an enhanced "proximity effect" to control the distribution of heating currents in bodies.*

The paper contains curves which have been worked out to illustrate the control of the distribution of the heating current densities by means of the proximity effect.

I. The Proximity Effect

Any variation in the value of the electric current in a wire is attended by, or has associated with it, a definite distribution of induced *electric intensities* at all points of the surrounding space. The value of the induced electric intensity at the point P which is associated with variations in the current in the wire W of Fig. 1 may be computed as follows. Divide the wire

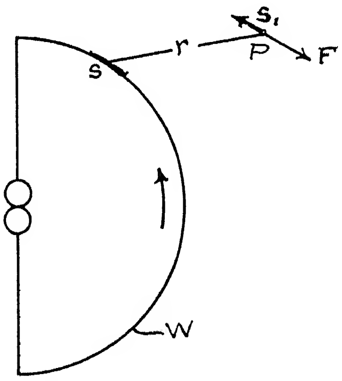


FIG. 1—ELECTRIC INTENSITY DUE TO A SEGMENT OF CURRENT

into segments, each so short that it may be regarded as a short straight filament. Take the vector contribution, F, to the electric intensity at P which is made by each short segment of wire of length s to be as expressed by equation (1), and find the vector sum of the contributions of all the segments which make up the wire W.†

$$\mathbf{F} \text{ (volts per cm.)} = -\, \mathbf{s}_1 \frac{\mu_o s}{4\pi r} \frac{di}{dt} \quad \text{(amperes) (sec.)} \tag{1}$$

μ_o represents the permeability of free space in weber ampere-turn cm. units. $\mu_o = 4\pi 10^{-9}$

$\mathbf{s}_1$ represents a unit vector parallel to the segment s and pointing in the arrow direction along s.

The significant features of this relation are that the electric intensity at P associated with the current in the segment is directly proportional to the *rate of increase* of the current, *inversely proportional to the distance* r from the segment to the point P and is parallel with but in a direction *opposite to the direction of acceleration* of positive charge in the segment.

By applying equation (1) to compute the electric intensity at a point P, (Fig. 2) lying close to a long straight filament of current and in its mid plane, the following expression is derived for the electric intensity:

$$\mathbf{F} \text{ (volts per cm.)} = -\, \mathbf{s}_1 \frac{\mu_o}{2\pi} \log \frac{s}{h} \frac{di}{dt} \quad \text{(approx.)} \tag{2}$$

If the current flowing in these filaments is the sinusoidal current,

$$i = I \sin 2\pi f t \tag{3}$$

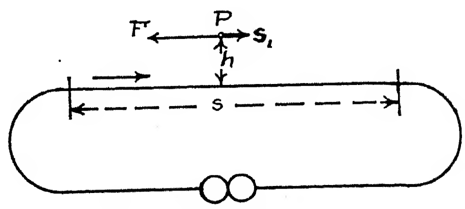


FIG. 2—ELECTRIC INTENSITY DUE TO A LONG STRAIGHT FILAMENT OF CURRENT

the expressions for the induced electric intensities accompanying this current become,

$$\mathbf{F} \text{ (volts per cm.)} = -\, \mathbf{s}_1 \frac{\mu_o s f I}{2r} \cos 2\pi f t \tag{1a}$$

and

$$\mathbf{F} \text{ (volts per cm.)} = -\, \mathbf{s}_1 \mu_o f I \log \frac{s}{h} \cos 2\pi f t \text{ (approx.)} \tag{2a}$$

*Professor of Electrical Engineering, University of Wisconsin, Madison, Wis.

†The fact that the electric intensities existing at P at the instant t are to be found from the value of di/dt in the segment at an instant r/c seconds earlier than the instant t, has been omitted from this statement. It is not material to the situation to be discussed.

Presented at the Great Lakes District Meeting of the A.I.E.E., Milwaukee, Wis., March 14-16, 1932.

In other words, a long straight filament of alternating current is attended by induced electric intensities, or electric forces, which tend to cause current to flow in all neighboring conductors in a direction *opposite* to the direction of increase of the current in the filament. These forces increase in direct proportion to the increase in the frequency of alternation of the current, and increase in logarithmic fashion as the point P is taken closer to the inducing current filament.

The induced electric intensities, whose relation to the current is expressed by equation (**1**), serve to account in a quantitative way for such electrical effects as transformer action between primary and secondary coils, the *skin effect*, the *edge effect*, and the *proximity effect* in conductors carrying alternating currents.

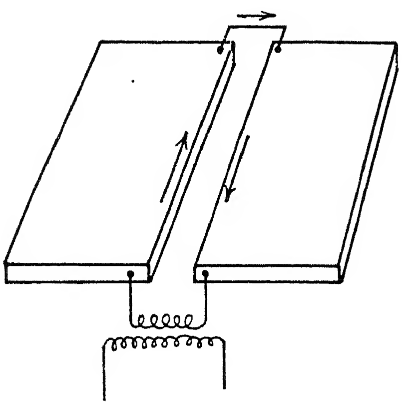


FIG. 3—CONNECTIONS FOR HEATING EDGES OF PLATES

Consider, for example, a long straight wire carrying an alternating current and let us suppose that the alternating current is uniformly distributed over the circular cross-section of the wire. For the purpose of computing the induced electric intensities within the body of the wire which attend the variations in the current, the entire current may be viewed as consisting of many long fine straight filaments of current all in parallel. The value at any point within the wire of the electric intensities associated with each filament is that given by equation (**2a**). It will be evident that these intensities if summed up for all filaments will yield a greater sum for points in the copper near the axis of the wire than for points more remote from the axis. That is to say, filaments near the axis are subject to greater induced electric intensities than filaments more remote, and these intensities are directly proportional to the frequency of alternation and at each instant are in a direction tending to lessen the rate of increase or decrease of the current. It follows that the current densities will be higher in filaments near the outside surface of the wire than in filaments near the axis. At high frequencies, this effect is so pronounced in conductors of large cross-section that an alternating current is confined to the filaments lying in a thin surface layer or *skin*, and the effect is called the *skin effect*.

If the alternating current flows in a conductor whose cross-section is a long narrow rectangle, as in a bus-bar of thin wide strap copper, it will be evident from the above considerations that the current densities will be much higher near the two edges of the rectangle than near the central portion of the strap. This increase in the current density toward the edges of rectangular sections is called the *edge effect*.

If now two long wires parallel each other, and if the wires constitute the outgoing and return conductors of an alternating current circuit, then at each instant of time the currents in the two conductors are opposite in direction. Accordingly, an increase in the current in conductor A results in induced electric forces in the filaments of conductor B which are in the direction in which the current in B tends to increase. But equation (**2a**) shows that these forces are greatest in the filaments of B which lie closest to A. In like manner the current B gives rise to assisting electric forces in the filaments of A. It follows that the current densities will be higher in the closely adjacent filaments of two circular conductors than in the more remotely separated filaments. The less the separation of the conductors and the higher the frequency, the greater will be the variation of the current density from the closely adjacent to the more remote filaments. This redistribution of the current densities over the cross-section of conductors which occurs when two conductors, at first remote from each other, are brought in close proximity is called the *proximity effect*.

These three effects,—the skin effect, the edge effect, and the proximity effect,—all lead to a departure from a uniform distribution of the current over the cross-section of conductors. Now in the conduction of a current of a given value, any departure from uniform

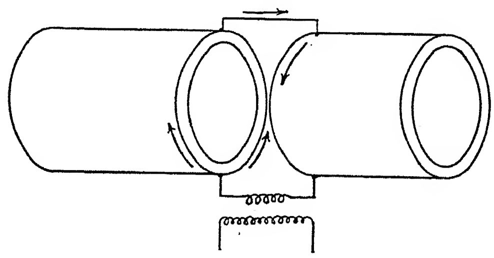


FIG. 4—CONNECTIONS FOR HEATING ENDS OF PIPES

current densities over the cross-section of homogeneous conductors means that the so-called I^2R loss, or the Joulean power expenditure, in the conductors is greater than for uniform densities. It is this wasteful aspect of these effects which seems to have led to many of the mathematical and experimental studies which have been made of the distribution of alternating current densities in conductors. It would seem that, in the main, the proximity effect has hitherto been viewed as an undesirable wasteful effect, and that thought and effort has been principally directed toward the minimizing of the effect.

II. The Use of the Proximity Effect to Concentrate Heating Currents in Predetermined Strips

While considering existing methods of welding pipe, it occurred to the author that the proximity effect could readily be magnified or enhanced to the point at which

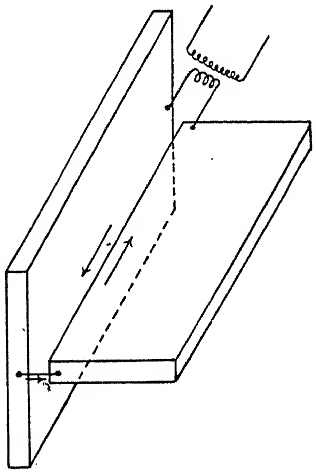


Fig. 5—Connections for Heating an Edge and a Strip

it might serve a very useful purpose in such industrial heating operations as those involved in welding, forming, hardening, etc. In heating operations requiring the heating of predetermined strips of conducting bodies or structural shapes, the proximity effect may be used to confine an electric heating current to these strips.

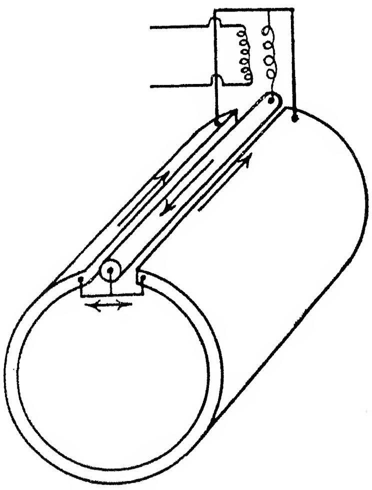


Fig. 6—Connections for an Auxiliary Conductor Which Determines the Path of the Heating Current

The confining of the heating currents to predetermined strips is accomplished by using heating currents whose frequency is in the range known as the audio range and by the electrical connections and the space arrangements illustrated in Figs. 3 to 8. An examination of these arrangements will show that they may be grouped under the following classification.

Classification of Connections for Applying the Proximity Effect

I. Current conductively conveyed to the body to be heated.
   A. The shapes and the connections of the heated bodies alone determine the strips in which the current concentrates. See Figs. 3, 4, and 5.
   B. An auxiliary water-cooled conductor is used to determine the strips of concentration. See Figs. 6 and 7.

II. The current is *induced* in the strip to be heated. (A water-cooled inducing coil lies in close proximity to the strip and there is no conductive connection with the body to be heated.) See Fig. 8.

The connections and the relations illustrated by the figures are so clear that they require little explanation. In each diagram, the transformer symbol represents the source of audio frequency current. The arrows represent the directions of the currents at a given instant of time in the different parts of the path. For the sake of clearness, it has been necessary to draw the parts with some considerable distance between the adjacent edges,

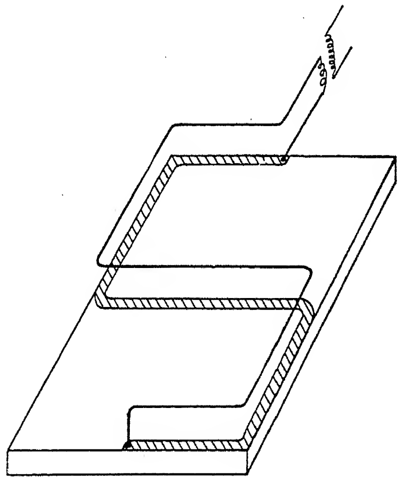


Fig. 7—Connections for an Auxiliary Conductor Which Determines the Path of the Heating Current

but in the actual equipment this gap is of the order of a few millimeters or centimeters depending upon the thickness of the edges. Fig. 3 shows an arrangement in which the heating current is confined to layers a few millimeters to several centimeters deep along the adjacent edges of two steel plates. In Fig. 4, the heating current is confined to similar layers on the ends of two steel pipes. In Fig. 5, the heating current is confined to the edge of one plate and to a strip extending along the middle of the second plate.

Figs. 6 and 7 show arrangements in which an auxiliary water-cooled tubular conductor is connected in series with the body to be heated. This auxiliary conductor parallels the strip to be heated in close proximity to it and thus serves to bring about the desired concentration of the heating current in the strip. The first sight of the red-hot strip of steel conforming to the sinuosities of the water-cooled copper tube mounted above the steel plate illustrated in Fig. 7 is found to be somewhat startling

even to engineers who have witnessed the heating effects obtained with the Figs. 3, 4, and 5 arrangements.

The two views in Fig. 8 serve to illustrate the group of arrangements in which no conductive connection is made with the body to be heated. In this group an inducing current of suitable frequency flows in a tubular water-cooled inducing coil. By mounting this coil so that the entire coil or one leg or segment of the coil lies in *close proximity* to the strip, heating currents are induced along the selected strip.

It will be seen that the features common to all of these arrangements are as follows:

a. The strip of the conducting body in which heating current is to be caused to concentrate is selected and predetermined by so shaping, proportioning, and mounting another electric conductor that some portion of this second conductor extends along the selected strip in close proximity to it.

b. The bodies (and the auxiliary conductor, if any) are so connected to the source of alternating current that the current flows in opposite directions in the two closely adjacent edges, conductors, or strips. (In the inductive case, the direction of the induced current is opposite to that of the inducing current.)

c. The control over the pattern according to which the current densities are to decrease from the center line to the edges of the selected strip and from the surface of the strip to the interior of the body is had by the joint adaptation of three things to bring about the desired pattern. These three things are, the frequency of alternation of the heating currents, the distance from the surface of the strip to the second conductor, and the width and cross-sectional shape of this second conductor. A high frequency, a short distance between the surface of the strip to be heated and the second conductor, and a narrow second conductor lead to the concentration of the heating current in a narrow strip.

III. Illustrations of the Control Over the Distribution of the Current Densities

In dealing with the distribution of alternating currents over the cross-section of conducting bodies, it is very helpful to express all linear dimensions, not in centimeters, but in terms of a unit of length appropriate to this problem. We will call this unit of length the *penetration unit* for the material at the frequency in question. The advantage of using this unit is that it enables us to represent the distribution of the current for a wide range of conducting materials, spacings, and frequencies by a few curves.

The length of the *penetration unit* is,

$$1 \text{ P. unit (penetration unit) (in cm.)} = \frac{1}{\sqrt{\pi f \mu \gamma}} \tag{4}$$

f represents the frequency.

γ represents the conductivity of the conductor in mho-cm.

μ represents the permeability of the conductor in weber ampere-turns cm. $\mu = 4\pi \; 10^{-9}$ for non-magnetic materials.

The values of the penetration unit for different materials and frequencies are as shown in the following table.

TABLE I—VALUES OF THE PENETRATION UNIT

Material	Relative Resistivity	Relative Permeability	Frequency	Conductivity mho-cm.	P. unit cm.
Copper 20 deg. cent.	1	1	960	580,000	0.214
Steel 20 deg. cent.	11	100	960	52,700	0.070
Steel 800 deg. cent.	94	1	960	6,240	2.06
Steel 1,300 deg. cent.	100	1	960	5,800	2.14
Earth	5.8×10^9	1	60	10^{-4}	65,000

In order that a physical significance may attach to the penetration unit as defined in equation (4), the following statements may be noted. The effective resistance to an alternating current of frequency f of a conductor of circular cross-section whose radius is five or more times the penetration unit of the material at the frequency f, is approximately equal to the ordinary ohmic resistance of a surface layer of the conductor having a depth equal to the penetration unit. Again, at a depth of one penetration unit below the surface of such a large conductor, the current density is approximately 36.8 per cent of the density at the surface, and it lags by one radian behind the current density at the surface.

To illustrate the extent to which heating currents can be caused to concentrate in narrow strips by enhancing the proximity effect, the current densities have been worked out and plotted for the arrangement illustrated in Fig. 7, except that the auxiliary conductor has no sinuosities but is in the form of a long straight wire mounted over a wide conducting slab whose thickness is five or more times the penetration unit of the material at the operating frequency. The radius of the auxiliary conductor has been assumed to be very small in comparison with its mounting height h, say 5 per cent or less.

The current density C at any point A in the slab may be computed from the following Fourier integral.

$$C = \frac{j\,2\,I}{\pi} \int_o^\infty \frac{1}{\sqrt{b^2 + 2\,j} + b\,\mu_r} \cos xb \; exp\,(-\,hb - y\,\sqrt{b^2 + 2\,j})\, db \tag{5}$$

in which

C represents the current density at A in amperes per square penetration unit.

b represents the variable of the integrand. It disappears from the integral upon substituting the two limits 0 and ∞.

I represents the value of the heating current in amperes.

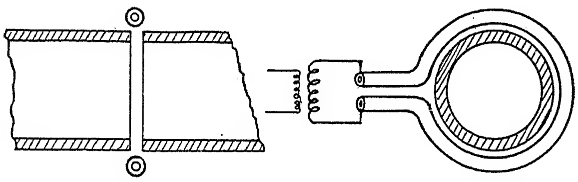


FIG. 8—STRIP HEATING BY AN INDUCING COIL

h represents the height of the auxiliary conductor above the surface.

y represents the depth of the point A below the surface.

x represents the x coordinate of the point A, as shown in Fig. 9.

μ_r represents the relative permeability of the conducting slab.

j represents $\sqrt{-1}$

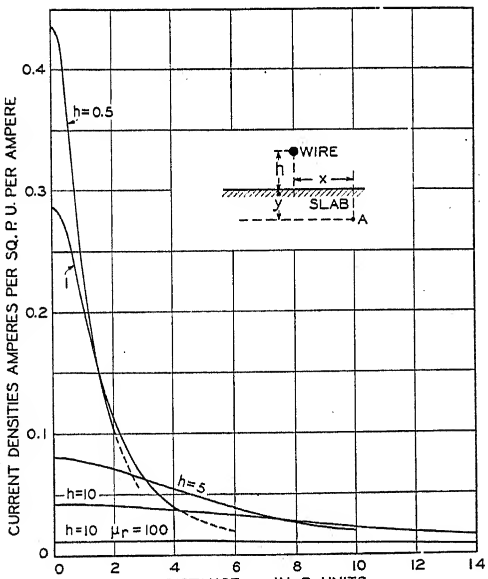


FIG. 9—CURRENT DENSITIES AT THE SURFACE OF A SLAB FOR DIFFERENT MOUNTING HEIGHTS

Current densities are expressed in amperes per sq. penetration unit per ampere of heating current
The distances x and h are expressed in penetration units

$exp\ (-hb)$ represents the exponential series in $(-hb)$. (The values of x, y, and h are to be expressed in P units).

Equation (5) is valid for materials and frequencies in which the conduction current density is 100 or more times as great as the displacement current density. For copper at a frequency of 1,000 cycles per second, the conduction current density is 10^{15} times the displacement current density.

If the slab is of non-magnetic material or of ferromagnetic material at temperatures above the point at which the material becomes non-magnetic, μ_r has the value unity, and equation (5) reduces to a form equivalent to that given by Carson.*

$$C = \frac{I}{\pi}\cdot\int_0^\infty [\sqrt{b^2+2j}-b] \cos xb \ exp\ (-hb-y\sqrt{b^2+2j})db \tag{6}$$

For mounting heights for the auxiliary conductor in which h is greater than 20 penetration units, equation (6) for non-magnetic materials evaluates to the simple form

$$C = \frac{1+j}{\pi}\ \frac{h}{h^2+x^2}\ (exp - y)\ (\cos y - j \sin y)\ I \tag{7}$$

An examination of Table I will show that the value of the penetration unit for steel at a frequency of 960 cycles per second lies between 0.07 cm. for steel with an assumed relative permeability of 100 and 2.14 cm. for steel at a temperature of 1,300 deg. cent. For an auxiliary conductor mounted at a height of one cm. above the conducting slab, the important range of mounting heights for this steel would lie between $h = 0.5$ and $h = 20$ penetration units.

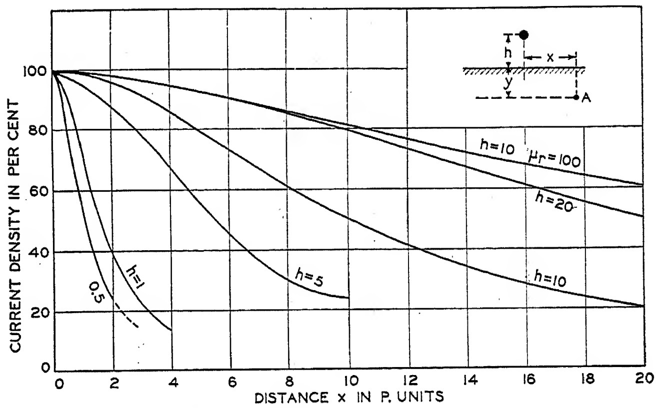


FIG. 10—CURRENT DENSITIES AT THE SURFACE OF A SLAB

For each height of wire, the current densities are expressed in per cent of the current density existing immediately under the wire at that height
The distances x and h are expressed in penetration units

Fig. 9 contains curves, each worked out for a different mounting height of the auxiliary conductor, which show the manner in which the current density at the surface of the slab falls off on each side of the center line of the heated strip. The values shown by the curves are the current densities in amperes per square penetration unit per ampere of heating current flowing in the slab. All the curves have been plotted for material in the non-magnetic state save the curve marked $\mu_r = 100$, in which the relative permeability has been taken to be 100. The curves in Fig. 10 likewise show the manner in which the current density at the surface of the slab falls

*"Wave Propagation in Overhead Wires with Ground Return," John R. Carson, *Bell System Tech. Jour.*, Oct. 1926.

off on each side of the center line, the curves in each case showing the current densities expressed in per cent of the current density immediately below the auxiliary conductor. In Fig. 10 the effect of the permeability of the steel in confining the current to a thinner surface layer of the steel and thereby causing a wider spread of the current is to be noted.

A mounting height of one penetration unit means that the height of the auxiliary conductor above the slab would be about two cm. for a steel slab with a strip at its welding temperature and a 960 cycle heating current. For this important case, Figs. 11 and 12 show the distribution of current densities at the surface and at points in planes located at depths of one and two penetration units below the surface. These figures also show the

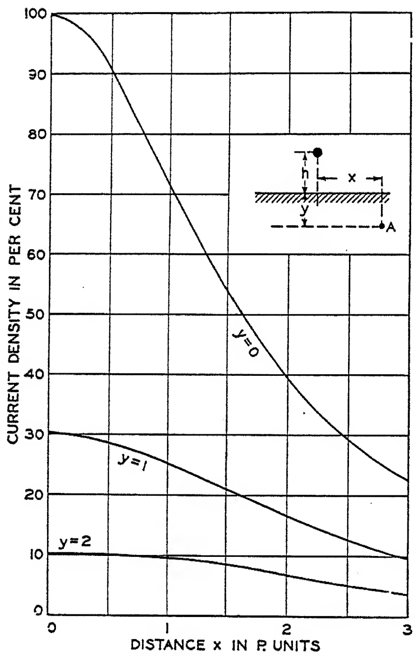


FIG. 11—CURRENT DENSITIES FOR A MOUNTING HEIGHT OF ONE PENETRATION UNIT

The current densities are given in per cent of the current density directly below the wire. x, y, and h are in penetration units

phase of the current density at these points, relative to the phase of the (total) heating current. The current density along the center line of the strip is seen to be 60 deg. in advance of the total current. At points in planes one and two penetration units beneath the surface, the current density lags substantially one and two radians behind the current density at corresponding points on the surface, and has decreased to roughly $(1/\epsilon)$th and $(1/\epsilon)^2$ of its value at the surface. In the surface at a distance of two penetration units from the center line of the strip, the current density is seen to be 40 per cent of the value on the center line, and thus the power expenditure per unit volume is only 16 per cent as great as along the center line.

The relative effect of close proximity of the auxiliary conductor and of a high frequency of alternation in contributing toward the concentration of the heating currents in narrow strips is brought out more clearly by plotting the current densities at the surface against the ratio of the x distance of the filament to the mounting height h of the auxiliary conductor, as in Fig. 13.

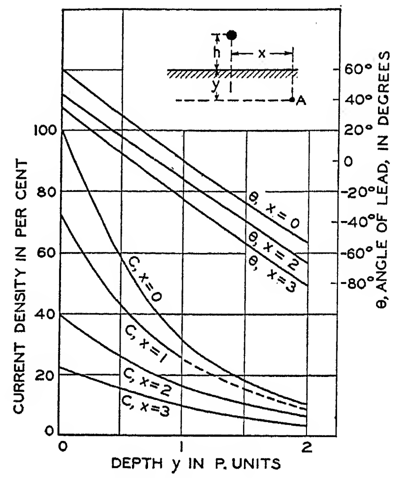


FIG. 12—CURRENT DENSITIES AND PHASE ANGLES FOR A MOUNTING HEIGHT OF ONE PENETRATION UNIT

The current densities are given in per cent of the current density directly below the wire

x, y, and h are in penetration units

The phase angles are the angles by which the current density leads the heating current

ECONOMIC AND PRACTICAL FEATURES

No attempt will be made in this paper to discuss the important questions of an economic and practical nature which remain to be solved before the practise of selective heating by means of the proximity effect can be established as a shop process. In demonstrating the

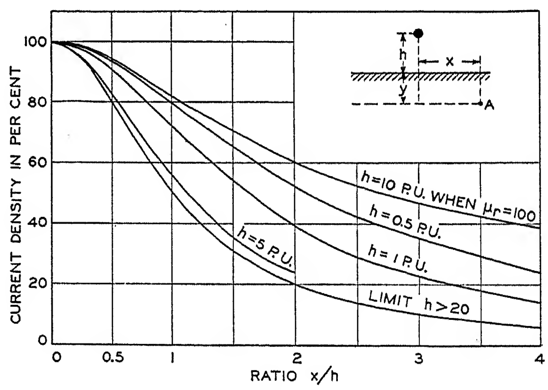


FIG. 13—CURRENT DENSITIES AT THE SURFACE OF A SLAB

For each height of wire, the current densities are expressed in per cent of the current density existing immediately under the wire at that height

The distance h is expressed in penetration units

method, the author has been limited to the experience to be had with a condenser-mercury-spark-gap source of high-frequency currents having a rating of 35 kw.

The most economical range of frequencies for use in the process remains to be determined. Many factors which vary from operation to operation, such as the kind and size of material to be heated, the nature of the

desired temperature pattern, and the relative cost of generating energy at the different frequencies will serve to determine the range of frequencies which will be the most economical. Present experience would indicate that the economic frequencies may be expected to center upon 960 cycles per second.

Unquestionably the control of the distribution of heating currents which is made possible by the application of the proximity effect opens up the possibility of a new range of industrial heating effects. The method of selectivity heating bodies by causing the heat energy to be generated in the body in strips of predetermined width and depth would seem to make possible a hitherto unattainable nicety of control of the distribution of temperatures in bodies. The control of the temperature patterns is obtained by adapting both the rate at which energy is supplied and the current density pattern to the desired end. For example, if in the welding of plates edge to edge, the induced heating current is for all practical purposes confined to a strip a few millimeters deep on the edge of the plate or plates, and if the power delivery is made large by using large inducing currents, portions of the plate a few centimeters from the edge may be substantially at room temperature when the edge reaches a welding temperature. On the other hand, if the energy is delivered at a more moderate rate, that is if smaller heating currents are used, the temperature gradient in the plate at the time the edge reaches a welding temperature may be made much more gradual. It will be seen that this control of the temperature patterns results from the fact that the application of the proximity effect makes it possible to confine the delivery of energy in rather definite fashion to predetermined strips of the body.

Bibliography

Bibliographies of the literature dealing with the distribution of current densities over the cross-section of conductors will be found at the end of the following papers. The second bibliography contains references to papers appearing since the preparation of the first bibliography.

Experimental Researches on Skin Effect in Conductors, A. E. Kennelly, F. A. Laws, P. H. Pierce, TRANS. A.I.E.E., 1915, p. 1953.

Bessel Functions for Alternating Current Problems, by H. B. Dwight, TRANS. A.I.E.E., 1929, p. 820.

Toll Switching Plan for Wisconsin

BY W. C. LALLIER[1]
Associate, A.I.E.E.

Synopsis.—This paper outlines the general plan which is being employed in the handling of intrastate toll traffic of the Wisconsin Telephone Company, including many of the transmission features involved and touching upon the relationship of the plan to that for handling countrywide toll connections. A brief discussion is included of the present and proposed toll cable network in Wisconsin.

THE growth in long distance telephone traffic has been rapid from the inception of the telephone business. Developments of telephone plant and technique have constantly extended the distance over which it is technically possible and commercially practicable to furnish toll telephone service.

The result of this extension in the range of transmission has been to increase the number of possible commercial toll connections and to stimulate growth in toll service. A graphic picture of the past and expected near future growth in toll business by five year periods is given in Fig. 1.

The enlarging field of toll telephone operations, together with the particularly rapid rate at which the long haul toll traffic has grown, requires that circuits be provided of such numbers, arrangements, and design as to permit the prompt establishment of the desired connections with a satisfactory grade of transmission. The general basis under which this is being accomplished is known as a toll switching plan. The toll switching plan for Wisconsin is outlined briefly in this paper, including some features of the transmission design requirements and the present and proposed toll cable network and the relationship of the plan to that for handling countrywide toll connections. A description of a toll switching plan for the Bell System was given in a paper* presented at the summer convention of the A.I.E.E., 1930.

The intrastate toll traffic in Wisconsin is composed of over 3,500 individual items, that is, traffic between two specific toll centers. Many of these traffic items are not of sufficient volume to justify direct circuits between the terminal exchanges and the traffic must therefore be switched at one or more points. In accordance with the toll switching plan, the switching of multiswitched toll traffic is generally restricted to selected toll centers arranged for introducing transmission gains into the switched connections and having toll circuits of such transmission characteristics connecting them to other toll centers as to facilitate the provision of reasonably low transmission losses between toll switchboards on connections within the State and on connections to points reached via the Bell System network.

When we speak of the net transmission loss (or net equivalent) of a toll circuit or a complete toll connection, we mean the net difference between the sum of the attenuation losses in the circuit or connection and the total transmission gain introduced by telephone repeaters. It may not at first be obvious why transmission gains are introduced at the switching points in switched toll connections, that is, why each of the links involved in the switched connections is not normally adjusted to have sufficiently low losses as to result in satisfactory overall losses on the complete toll connections without introducing transmission gains at the points of switching. The answer to this question lies in the fact that it is generally impracticable for noise, crosstalk or repeater singing reasons, as will be explained later, to work individual repeatered toll circuits to sufficiently low transmission losses. As an example, it would not be advisable to work a 2-wire cable circuit loaded with 88 millihenry coils spaced 6,000 ft. apart from Milwaukee to Madison to less than about 5 or 6 db. on terminal business for singing and crosstalk reasons, whereas it is practicable and desirable to work such a circuit to a loss of 1 or 2 db. when used as an intermediate link in multiswitched toll connections.

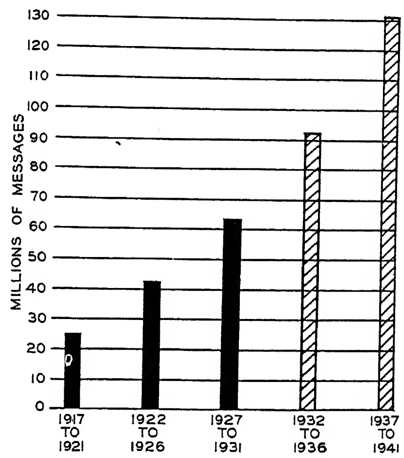


FIG. 1—TOTAL TOLL MESSAGES ORIGINATING IN WISCONSIN INVOLVING BELL SYSTEM CIRCUITS

There are two methods which are commonly used as a means of introducing transmission gain into switched toll connections, that is, the cord circuit repeater method and the pad control method.

The cord circuit repeater method, which employs special cord circuits containing telephone repeaters in connecting the various circuits together to form complete toll connections, has been used extensively in the past in providing improved transmission on switched

1. Wisconsin Telephone Co., Milwaukee, Wis.

**A General Switching Plan for Telephone Toll Service*, by H. S. Osborne, TRANS. A.I.E.E., Oct. 1930, p. 1549.

Presented at the Great Lakes District Meeting of the A.I.E.E., Milwaukee, Wis., March 14-16, 1932.

connections. This is illustrated in Fig. 2. It has the disadvantage that the routine whereby the cord circuit repeaters are inserted into the switched connections by the toll operators is necessarily somewhat cumbersome, involving considerable expense for operators' labor and an increase in the time required to set up the switched toll connections. Furthermore, due to the human element involved, it is not possible to insure that the cord circuit repeaters will always be used when and as required by the routing instructions.

The method which is now being employed in connection with all new repeater switching offices, and which is also superseding the cord circuit repeater method of operation in many existing cord circuit repeater offices, is the pad control method of introducing transmission gain into switched toll connections. In this method, terminal repeaters and suitable loss pads are provided in the toll circuits at the pad control switching offices and are so associated with relay equipment that the loss pads are automatically cut out of the toll circuits at the pad control switching office when two circuits are connected together in a switched connection. This method of introducing gain into switched toll connections is also illustrated in Fig. 2.

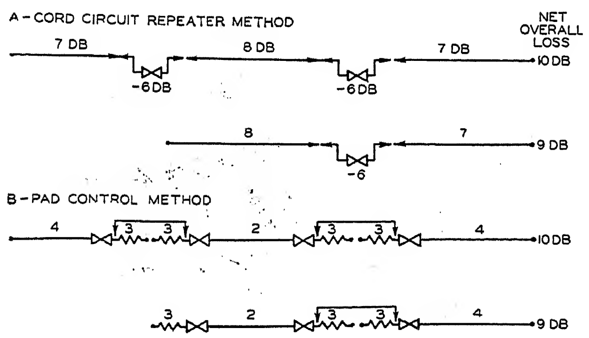


Fig. 2—Illustrations of Cord Circuit Repeater and Switching Pad Methods of Introducing Gain in Switched Connections

Two factors which have played an important part in justifying the extensive application of the pad control method of introducing transmission gains into switched toll connections are (a) important reductions which have occurred in the cost of terminal repeaters and (b) the substantial increase in the number of terminal repeaters required on toll circuits for other reasons such as, for example, in connection with the increased application of cable to the toll plant and the improved transmission efficiencies to which toll circuits are now generally being worked.

When the switching pad arrangement for gain control is employed at a switching office, pads are generally provided on all toll circuits which may be switched at this office in order to avoid repeater singing or undesirable noise or crosstalk effects when the circuits are used for terminating business. These pads are generally adjusted to have a loss of 3 db. except as higher loss values appear desirable to avoid noise, crosstalk or repeater singing or except as lower loss values are desirable to meet satisfactory transmission loss requirements and are satisfactory from the standpoint of noise, crosstalk, and repeater singing.

Switching Arrangements of Plan for Intrastate Traffic

The next item which will be discussed is the arrangement of switching offices and circuit groups employed in accordance with the toll switching plan for intrastate traffic.

Several primary switching points called "primary outlets" have been selected which are completely interconnected with direct circuit groups. Every toll center is to be provided with direct circuits to at least one of these primary outlets. With this arrangement, connections between any two toll centers can be handled with not more than two switches. As indicated in Fig. 3, the primary outlets which have been selected in Wisconsin are located at the following offices, each of which is a telephone repeater office and is arranged for introducing transmission gain into switched toll connections: Appleton, Eau Claire, Madison, Milwaukee, Stevens Point.

A number of "secondary switching points" has been selected for handling items of traffic where the length of haul can be shortened or other economies obtained by switching traffic at other than the above primary outlets, and for alternate and emergency routes. Each of these offices is also required to be a telephone repeater office arranged for introducing transmission gain into switched toll connections and to have such direct circuits to primary outlets, secondary switching points, and other toll centers as are warranted. Referring to Fig. 3, the following offices are now secondary switching offices: Ashland, Green Bay, La Crosse, Marinette, Rhinelander, Rice Lake.

In addition to the points just mentioned, it will be noted that Duluth, Minnesota; Ironwood, Michigan; and Dubuque, Iowa, all of which are repeater offices equipped with switching pads, are considered as secondary switching points in the toll switching plan for Wisconsin.

The following additional offices are planned to be secondary switching points as soon as the necessary telephone repeater and switching pad arrangements are provided, and are shown as such in Fig. 3: Fond du Lac, Janesville, Lake Geneva, Prentice.

The toll centers which are neither primary outlets nor secondary switching points are called terminating toll centers. Switching at these toll centers is limited to such single switched connections as can be made without exceeding certain transmission limitations as illustrated in Fig. 4. It will be noted from this figure that the permissible overall loss between terminal toll switchboards is 10 db. where the toll terminal losses of the toll centers are each 7 db., and that correspondingly

higher overall toll circuit losses are permitted in cases where the toll terminal losses are less than 7 db.

Relation to Switching Arrangements for Countrywide Connections

Each of the five primary outlets in Wisconsin is provided with direct circuit groups to at least one "regional center," which is a backbone switching point in the one of the regional centers and every toll center in the country is to be connected to at least one primary outlet. With the complete realization of this plan, therefore, it will be possible to talk between any two toll centers in the country with a total of not more than four switches involving five toll circuit links.

The regional center to which all of the primary outlets in this portion of the country are connected is Chicago.

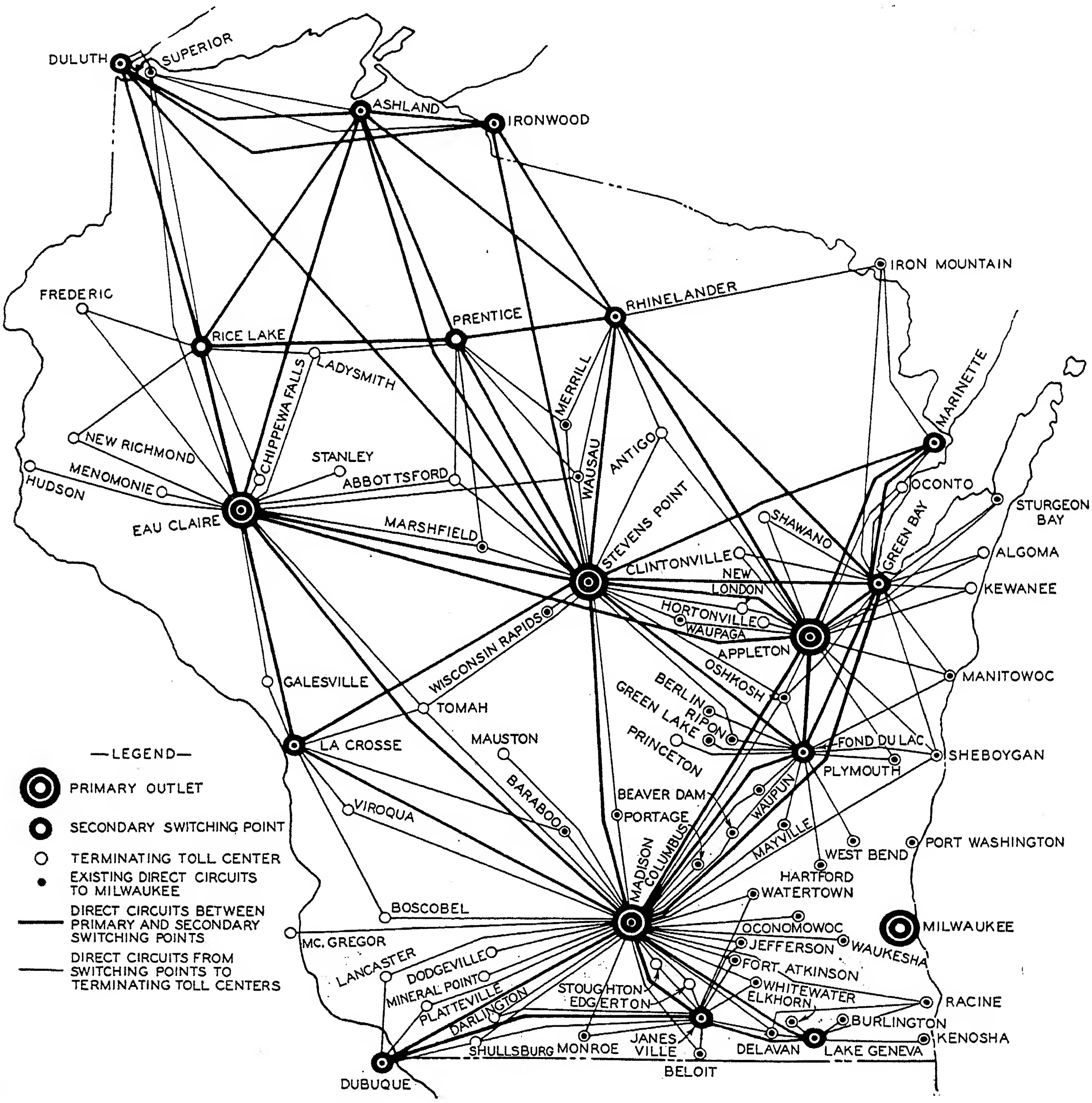


Fig. 3—Toll Switching Plan for Wisconsin

Note: As shown in the code, circuit groups to Milwaukee are indicated by dots at the distant terminals

nationwide toll traffic network. The regional centers are analogous in the countrywide toll switching plan to the primary outlets in the plan for Wisconsin in that they are arranged for introducing transmission gains into switched toll connections and are to be completely interconnected by direct circuit groups. Fig. 5 shows the present arrangement of regional centers and primary outlets in the Bell System. Every primary outlet in the country is to be provided with direct circuits to at least Most of the secondary switching points in Wisconsin have direct circuits to Chicago, in which case they may be considered as "secondary outlets" in the countrywide toll switching plan.

Transmission Requirements of Toll Switching Plan

As mentioned previously, the transmission loss of a toll circuit or complete toll connection is the net differ-

ence between the total attenuation loss in the circuit or connection and the total transmission gain introduced by telephone repeaters. In practise, the amount by which it is possible to decrease the transmission losses

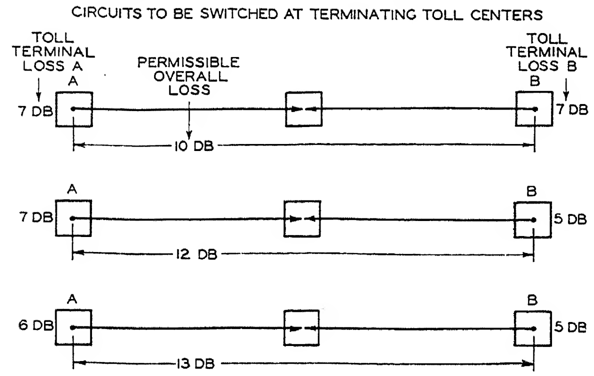


FIG. 4—TRANSMISSION REQUIREMENTS OF TOLL SWITCHING PLAN FOR WISCONSIN

of toll circuits or connections by introducing repeater gains is limited by the following factors:

a. Distortion or, in extreme cases, sustained oscillation or singing may result in the circuit if the repeaters introduce too great an amplification.

b. Undesirable amounts of crosstalk between telephone circuits or noise induced in telephone circuits from outside sources may result by introducing too much repeater amplification.

c. Disturbing effects may be caused by the amplification of echo currents, *i. e.*, those portions of the speech currents which are reflected back from the distant end of the connection or from intermediate points, if the repeater gains are such as to result in too low an overall transmission loss.

On the longer toll connections, echo considerations are almost always the controlling factor in determining the minimum overall transmission loss which may be employed, whereas on the shorter toll connections, repeater singing, crosstalk or noise considerations generally are controlling. This is due to the fact that the echo effects on individual circuits increase more rapidly with length than do crosstalk and noise, the disturbing effect of echo currents being governed to a large extent by the amount of time elapsing between the transmittal of the speaker's voice and the return of the echo currents. This time interval is directly proportional to the length of the circuit. Singing tendencies also increase at a rapid rate with increase in length on two-wire circuits but tend to be independent of length on four-wire cable and carrier telephone circuits, which are used to a large extent in circuits between the primary outlets, between primary outlets and regional centers, and between the regional centers. Furthermore, when two or more toll circuits are connected together, the echo effects of the individual circuits add together almost directly, whereas the effects of crosstalk, singing and noise in-

FIG. 5—GENERAL TOLL SWITCHING PLAN
Location of points at present employed as regional centers and primary outlets in the United States and Canada

crease at a much less rapid rate. It follows, therefore, that when toll circuits are connected together in multiswitched connections, the overall combinations can, in general, be operated at transmission losses as limited by echo effects. Therefore, in establishing satisfactory transmission efficiencies for overall toll connections in accordance with the toll switching plan, each link must be designed not only with a consideration of its contribution to the overall transmission loss on the connection, but also with a consideration of its "echo minimum

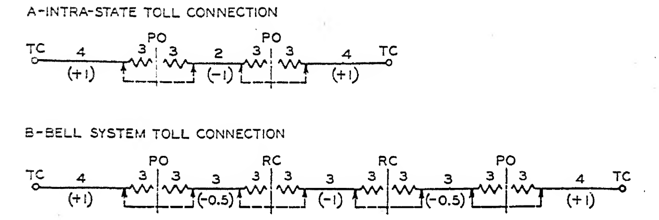


FIG. 6—TRANSMISSION REQUIREMENTS OF GENERAL TOLL SWITCHING PLAN TOLL CIRCUITS USED IN MULTISWITCH TOLL CONNECTIONS

A. Figures above line indicate maximum transmission loss in db.
B. Figures below line in parenthesis indicate minimum echo net loss margins

working net loss," *i. e.*, its contribution to the minimum working net loss of the overall connection from the echo current standpoint.

It is apparent that some of the circuits in a complete toll connection may be operated at net equivalents less than their echo minimum working net losses, providing other circuits in the connection are worked at net equivalents greater than their echo minimum working net losses. In each case the algebraic difference between

Fig. 6 shows the net equivalent and echo net loss margin requirements which have been set up for the various classes of toll circuits commonly used in multiswitched toll connections. These requirements were

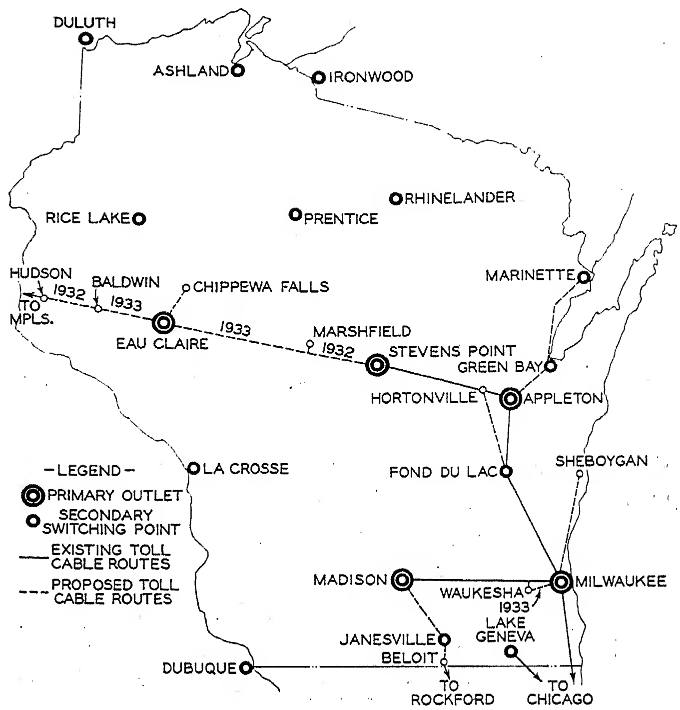


FIG. 7—TOLL CABLE LAYOUT IN WISCONSIN

selected after a considerable study of the effects of various values of overall transmission loss upon the grade of service provided and upon the cost of providing the service, including comprehensive analyses to determine

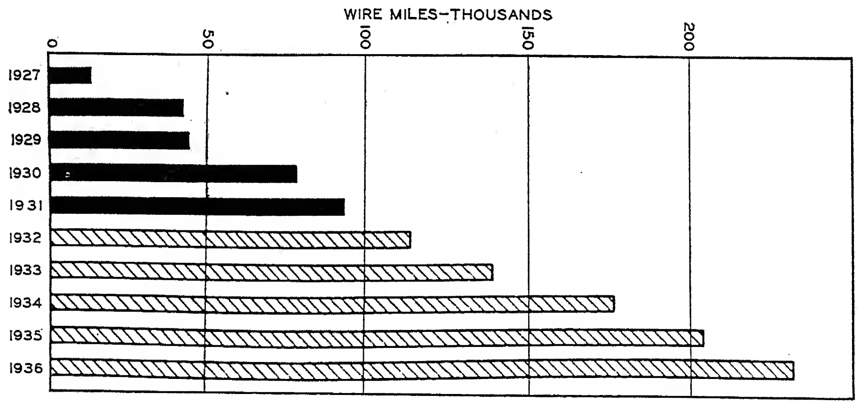


FIG. 8—WIRE MILES OF TOLL CABLE IN PLANT OF WISCONSIN TELEPHONE COMPANY

the net loss contributed by the circuit in the connection, and the echo minimum working net loss of the circuit, may be referred to as the "echo net loss margin" of the circuit. Thus, if certain circuit links in a complete toll connection have positive echo net loss margins, then other links may be permitted to have negative echo net loss margins in that connection, as long as the sum of the echo net loss margins of all of the links in the connection are in no case less than zero.

the most economic allocation from the Bell System standpoint of transmission losses and echo net loss margins among the various classes of toll circuits used in multiswitched toll connections.

With the transmission limitations indicated on Fig. 6, the maximum transmission loss between terminal toll switchboards on overall toll connections should not exceed 10 db. on intrastate traffic in Wisconsin involving a maximum of three toll circuits, or 17 db. on traffic

involving a maximum of five toll circuits in the Bell System network.

Toll Cable Network in Wisconsin

Fig. 7 shows the present and contemplated toll cable network in Wisconsin. It will be noted that many of the important cities are now connected by toll cable facilities. Within the next few years, it is expected that the cable network will be extended to provide an all

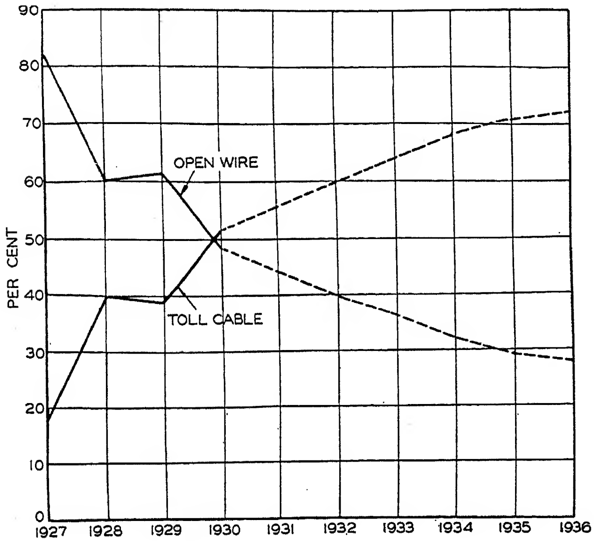


Fig. 9—Percentage of Toll Wire Miles in Toll Cable and Open Wire—Wisconsin Telephone Company

cable route from Chicago and Milwaukee to St. Paul and Minneapolis over the Fox River Valley route to Appleton and thence west through Stevens Point and Eau Claire. In addition, future requirements will warrant extending this cable north from Appleton to Green Bay and Marinette and tying Sheboygan into the toll cable network by constructing a cable from Milwaukee along the lake shore route to Sheboygan. The future extension of toll cable south from Madison through Janesville and Beloit to Rockford, Illinois will link the southern portion of Wisconsin and the northern portion of Illinois to the toll cable network in Wisconsin.

It is apparent that the toll cable type of facilities is gradually supplementing or replacing the open wire type of facilities along many of the important backbone routes of the toll system. Ultimately, it is expected that the use of open wire lines will be limited largely to the feeder and branch routes to cities and towns located some distance from the main toll cable routes.

An idea of the rate at which the toll cable program has been going forward in the past few years is indicated graphically by Figs. 8 and 9. Fig. 8 shows the total wire-miles of toll cable in plant in Wisconsin and Fig. 9 shows the trend in the percentage of the total wire-miles in the toll plant which are in cable and in open wire.

The application of superimposed carrier systems to open wire lines has also been an important factor in providing for the increasing toll traffic during the last few years. Thus it may be seen from an inspection of Fig. 10 that a large majority of the principal open wire lines in the state now are arranged to have superimposed carrier systems operating on them.

Discussion

E. O. Neubauer: One of the important results of the application of the toll switching plan is the improved toll service to small communities located somewhat away from the larger cities. While the volume of toll traffic to and from these communities is comparatively small, it is necessary to include them in any plans for universal telephone service. As brought out in Mr. Lallier's paper a maximum of two switches and three toll lines is provided in the switching plans for complete intrastate service so that a satisfactory limit is automatically placed on the grade of toll service given to the smallest and most remote telephone exchange.

The large cities originate and receive such large volumes of toll traffic that large numbers of direct circuits are justified but the small community may have only a one circuit outlet. In Illinois under the toll switching plan, 87 per cent of the intrastate messages are completed over direct circuits, 12 per cent with one switch and only 1 per cent with two switches. It is this last 1 per cent of the traffic that is the largest beneficiary of the toll switching plan because previous to the switching plan there was no maximum limit to the number of switches and, therefore, no satisfactory service limit to the small community.

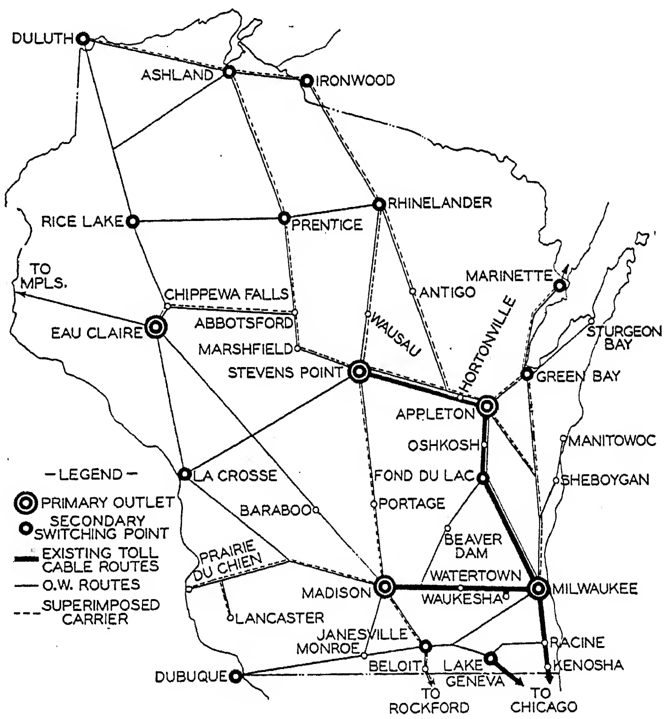


Fig. 10—Principal Toll Cable, Open Wire and Carrier Routes of 1931—Wisconsin Telephone Company

To make the switching plan fully operative, it is necessary only to have one connection from each terminal toll center to a primary outlet and to have the primary outlet interconnected. With these minimum requirements only 126 circuit groups would be needed to accommodate the 116 toll centers in Illinois. Actually there are over 500 such groups in service; the surplus groups affording more than one route between toll centers. The justification of these surplus groups is upon the basis of economy; *i. e.*, additional direct groups beyond the 126 must show economies in operation. Incidentally they afford a better grade of toll

service than sought for under the switching plan. As the volume of toll traffic increases more of these surplus groups will be installed resulting in a gradual reduction in the number of messages requiring switches and a continuous improvement in toll service. However, to reach the other extreme would be to provide direct circuits between all toll centers but with 116 toll centers in Illinois, this would require nearly 6,700 circuit groups as against a little over 500 in service. The need for switching arrangements such as discussed in Mr. Lallier's paper is, therefore, a permanent one.

In regard to the replacement of cord circuit repeaters with the terminal repeaters and pads, this change in many situations has required a major investment and toll board rearrangement. Some offices showed ultimate savings by the conversion. These savings are brought about by the reduced number of toll positions required with a reduction in operating costs. Where ultimate economies could not be shown the conversion was made at Illinois switching points on the basis of the toll service improvements obtained with the pad system. Since the operation of the pads is automatic the operator is not required to determine the conditions under which pads are left in or removed as is the case with cord repeaters.

There are 4 primary outlets and 1 regional center in the switching plan for Illinois and all of these except one have already been converted. Centralia will be completed in the near future.

Glen Ireland: Although the general toll switching plan has only been in effect for about two years, sufficient progress has already been made and sufficient experience obtained to show conclusively that the advantages of such a plan are fully as great as expected. Only a few per cent of the circuit groups required to complete the routing arrangements of the general toll switching plan within the Bell System remain to be placed and these represent less than one per cent of the circuit miles now in service. Also, excellent progress has been made in providing terminal repeater-switching pad arrangements, which are mentioned in Mr. Lallier's paper, at the more important switching centers throughout the country, as is evidenced by the fact that all of the regional centers and over half of the primary outlets are now equipped with the improved arrangements. This provision throughout the system of the general switching plan arrangements, including the improved terminal repeater-switching pad arrangements, has been accompanied by material improvements in the quality of service on multi-switched connections which are, of course, the most difficult types of calls on which to provide satisfactory service. For example, the average speed of service interval on the multi-switched calls, that is, the time required from the placing of a toll call to the response of the called party or until a definite report is made by the operator, has been reduced by 45 per cent. Likewise there has been a marked improvement in transmission involving toll connections, particularly on the multi-switched toll connection.

In the latter part of Mr. Lallier's paper he points out a very interesting trend in the design of toll plant in Wisconsin which is also generally true throughout the Bell System, that is, that there has been a considerable increase in the use of toll cable and open wire carrier facilities. In this connection it is interesting to note that consideration has been given to and development is actively under way on the proposition of applying telephone carrier to long distance cables. For large groups of long distance circuits it appears that a carrier frequency range can be advantageously used in toll cables employing frequencies at least as high as those in the open wire. The circuits to be obtained from cable carrier are expected to have excellent transmission characteristics combining many of the most desirable features of both the present toll cable and open wire carrier circuits.

R. C. Siegel: The toll switching plan, by limiting the switching to fewer toll centers, results in the concentration of larger numbers of toll circuits on a smaller number of main routes and the concentration of equipment in fewer central offices. This makes it possible economically to use toll cable more extensively and to secure the advantages of greater circuit stability and protection which this type of plant affords. With the installation of toll cable from Milwaukee to Madison in 1928, from Milwaukee north via the Fox River Valley to Appleton in 1930 and from Appleton West to Stevens Point in 1931, four of the five primary outlets in Wisconsin are now interconnected by toll cable and have cable connection with the regional center at Chicago.

It is planned that within the next few years toll cable will be extended from Stevens Point west to Minneapolis, thereby completing the cable route from Chicago to Minneapolis and providing complete interconnection by means of toll cable of all primary outlets in Wisconsin. The construction of additional toll cables extending from these primary outlets to other important cities in Wisconsin is also contemplated in the future.

The procedure in reaching the objectives of the toll switching plan has been a gradual one and does not contemplate major replacements or rearrangement of plant, except those which may be required for other reasons. It is necessary, however, that the toll cable plant, central office buildings and telephone repeater equipment be so planned, constructed and arranged to permit of flexibly and economically caring for the requirements over a long period in the future.

Weather Resistant Covering for Line Wires

BY C. F. HARDING,*
Fellow, A.I.E.E.

L. L. CARTER,*
Associate, A.I.E.E.

and

J. W. OLSON*
Non-member

Introduction

THE problems involved in producing insulation for high-voltage cables have been of much interest to all electrical engineers for many years. Great advances have been seen in the art of producing insulation for higher voltage cables but it is only within the last two years that any unified and organized attempt has been made to improve the weathering and insulating qualities of the covering of weather resistant line wire. Previous to this time the development work lay in the hands of the manufacturer and was centered upon cutting costs to meet the demand for a cheaper product, which would meet the very meager specifications available.

Origin of Investigation

The concentration of effort upon the reduction of weather resistant wire costs during the years following 1918 or 1920 began to bear fruit in the early loss of saturating compound and the shedding of braids, in many cases, within a period of two to three years after the erection of the lines.

As more and more early failures of weather resistant wire coverings were seen to occur in extremely short lengths of time, the sentiments of many operating engineers swung toward discontinuing a weather resistant covering on overhead lines and adopting bare conductors for all service except where tree wire was necessary or where highly insulated conductors were required at points of inadequate spacing or clearance. Certain engineers opposed the policy of abandoning the use of weather resistant coverings on overhead lines of 4,000 volts line to line and less, for the reason that such a covering might reduce the number of service interruptions by preventing grounds and short circuits. This group believed that an attempt should be made to improve weather resistant coverings before making a decision to discontinue using such coverings.

At the insistence of the engineers among the companies represented by the Utilities Research Commission, that weather resistant wire coverings should be developed to provide longer life and higher insulating values, the Commission initiated a study of this problem at Purdue University. This study, Case 28, of the Utilities Research Commission was started October 1, 1929 under the direction of the Electrical Division of the Engineering Experiment Station with the Chemical Division cooperating.

Preliminary Investigation of Problem

Shortly after the work was started a visit was made to the majority of the weather resistant wire manufacturing plants in the United States to observe the processes of manufacture, inspect the wire plant laboratories, and to discuss the problem with the men engaged in the production of weather resistant wire. This preliminary visit and discussion disclosed the fact that there was no adequate equipment available in the wire plants to test the comparative life of different weather resistant wire coverings and that there was only a meager knowledge of the reasons for early failure of weather resistant coverings and of means to increase the life of such coverings.

This preliminary survey emphasized the need for the purchase or construction of complete testing equipment at Purdue University.

Organization and Equipment of Laboratories

The equipment purchased or constructed for this work includes apparatus for both accelerated and natural weathering of finished wires or wire covering materials and for complete physical and chemical tests of such materials. This equipment is being used to study the faults of present materials and to test new coverings and new materials.

The accelerated weathering equipment for testing the comparative length of life of completed wires, asphalts and other saturants, yarns, etc., exposes the samples to the light from a carbon arc and to a water spray at a constant wire temperature of 175 deg. fahr. during one part of the daily weathering cycle. The samples are subjected to freezing by moving the wire holding drum to a large refrigerator and to thawing and drying by moving back to the light exposure chamber. Distilled water is circulated to the sprays from a storage tank and pressure provided by an auxiliary pump.

A natural weathering rack for completed wires is located on the roof of the Electrical Engineering Building and provides continuous current loading and application of line voltage to the wire samples.

Service testing equipment for duplicating installation and service conditions on weather resistant wire includes apparatus for subjecting the wire to abrasion such as occurs when drawn over a cross arm, or that caused by rubbing on insulators, wrapping and bending at freezing temperatures, and for producing decentralization of the conductor in the covering such as occurs in service.

All the standard physical tests of asphalts, such as the determinations of softening or fusing point, viscosity at saturating temperatures, hardness at wire

*Purdue University, Lafayette, Indiana.

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operating temperatures, and pliability at low temperatures are applied to weather resistant wire saturants.

The fusing point is determined by the standard ball and ring method used by the asphalt industry. According to this test the fusing point is the temperature at which a steel ball of specified weight drops through a mold of the material under test, supported on a brass ring and immersed in water, the temperature of which is raised at a slow and definite rate.

The viscosity of the asphalt at saturating temperatures is determined by the Stormer method. Viscosity, by this method, is expressed as the number of seconds required for 100 revolutions of a cylinder propelled by a 100-gram weight and immersed in the material to be tested. It may be seen that the higher the time required for 100 revolutions the higher is the viscosity.

The consistency or hardness is determined by use of the Abraham consistometer. This equipment measures the pressure required to force various size plungers into the material under test, at a constant rate. The ends of these plungers are flat circular disks of larger size than the body of the plunger so that there is no friction between the asphalt and the shank of the plunger. The higher the consistometer hardness number the harder is the material under test. This hardness number is defined as the third root of the number of grams which must be applied to a circular flat plate, one square centimeter in area, to cause it to displace the substance under test at the rate of one centimeter per minute.

The pliability at low temperatures is determined by bending a mold, of dimensions ¼ in. by 1 in. by 4 in., broadside about a round mandrel ⅜ in. in diameter while the sample and apparatus are held at the low temperature of test. The angle of bend is increased at the rate of five degrees per minute and the angle of bending required to break the mold is taken as a measure of the pliability.

Major Faults of Present Weather Resistant Wire Coverings Lie in the Saturants

The work at Purdue University indicates positively that the poor physical properties of the saturants used in weather resistant wire coverings in the past have been the cause of the extremely early failure of weather resistant wire. The coverings of wires which last well are eventually caused to fail by a combination of physical changes, by the saturants shifting in the coverings, etc., and chemical changes in the saturant caused by exposure to light, oxygen of the air, etc., but in very early failures the effects of light and oxygen are of less importance than the effects of poor physical properties which cause the saturant to shift in the braid when hot and to crack and split when cold.

These poor physical properties of the saturants are, namely, too low fusing points, extreme softness when warm, great changes in hardness with changes in temperature, extreme brittleness at even moderately low temperatures, and a marked tendency to evaporate. The majority of saturants used have had softening points (as determined by the ball and ring method) ranging from 130 to 150 deg. fahr., lower than maximum wire operating temperatures. These materials shift in the braids leaving the upper braids unprotected at ordinary summer temperatures, which are well below the fusing point. The extreme brittleness of these saturants, which results in cracking and breaking of the braids in cold weather, may be illustrated by the fact that less than two degrees bending is required to break molds of these saturants in the pliability test at 0 deg. fahr.

Many of the poor physical properties, particularly the disappearance or evaporation of the saturant from the braids, have been caused by the addition of excessive amounts of waxes, wax tailings or low melting fluxes to obtain fluidity and ease of saturation. This addition was usually made by the workman operating the saturating tank and resulted in a saturant of variable properties. These low boiling materials soon leave the saturant, causing the body of the asphalt to disintegrate. The saturants vary in their resistance to the weathering effects of light and oxygen and the worst are subject to excessive surface deterioration and erosion.

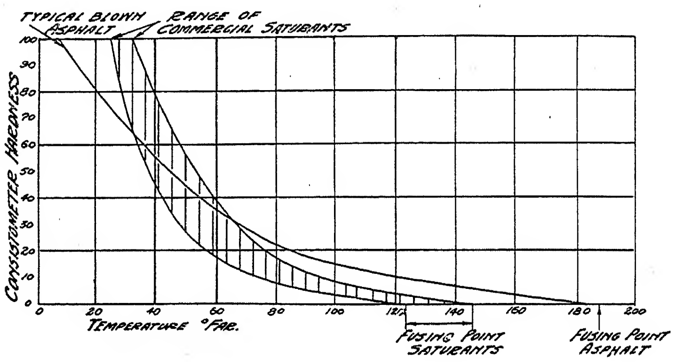

Fig. 1—Comparison of Initial Hardness

The result of all these factors is the loss of compound, particularly from the upper side of the braids, with the resultant breaking of the braids on the upper side and the familiar festooning.

The use of materials subject to such faults has been permitted by specifications which did not specify the source nor physical properties of saturants. Proper specifications have not been written because of a lack of knowledge essential to the writing of such specifications. The work at Purdue University is to provide such a specification.

The commonly used drip and stain tests cannot be regarded as tests of the physical properties of the saturants. The high melting wax used as an outer coating prevents staining and the yarns hold the asphalt, preventing dripping, even though the saturant may be quite soft. The drip test may also favor an undersaturated wire.

Finishing waxes cannot be regarded as of great protection to the covering because they evaporate or crack

and flake off within a short time or are damaged in installation. The wax has cracked, chipped off, or simply disappeared from the surface of the wire, on the various samples in the natural exposure rack, in from three to eight months.

Cotton Yarns

Have Minor Effect Upon Length of Life of Wire Coverings

It has been found that the cotton yarns are not so important in producing a long lived covering as are the saturants used. It is important, however, that sufficient cotton be present in the two inner braids to absorb and hold the saturant and that the outer braid should be closely woven to resist abrasion and erosion.

The one fault of the braided construction is that it does not, consistently, afford insulation beyond that of the air spacing provided, because of the interstices always present. It is desirable to have a more solid and homogeneous material next to the conductor.

Improved Saturants

Intensive experimental work, in the past year and a half, with many asphalts of different types from different sources, tested in both accelerated and natural weathering exposures, has shown conclusively that the length of life of weather resistant wire coverings may be materially increased by using, for a saturant, blown petroleum asphalts having a ball and ring softening point of 180 to 200 deg. fahr., free of low boiling materials and without the addition of wax tailings in the wire plant.

Several of the weather resistant wire makers have offered to produce a quantity of experimental weather resistant wire on a commercial scale. It has been decided, therefore, to produce a carload of wire, using as a saturant, the asphalts which have been found to provide the longest lived wire coverings. This saturant will be a blown petroleum asphalt having the above range of ball and ring softening points; a hardness, as determined by the needle penetrometer, of 35-25 at 77 deg. fahr. with 100 grams loading for 5 seconds; chosen for the greatest pliability at low temperatures and for freedom from low boiling materials. The experience gained in the production of this wire will be an aid in the writing of a specification for an improved weather resistant wire covering.

Such an asphalt, of greater stability at higher wire temperatures, will greatly reduce the shift of the saturant in the braids and will be less subject to evaporation and erosion. In addition, these asphalts are less susceptible to temperature changes and are much less brittle at low temperatures. Such asphalts may be bent many degrees in the pliability test while commercial saturants cannot be bent more than two degrees.

The differences in hardness of these materials may be noted by referring to the hardness curves in Fig. 1. Table I gives comparative values for fusing point, hardness, pliability, and temperature susceptibility.

TABLE I—PROPERTIES OF COMMERCIAL SATURANTS COMPARED TO A TYPICAL BLOWN ASPHALT

	Range of saturants	Blown asphalt
Ball and ring fusing point	120–150 deg. fahr.	192 deg. fahr.
Pliability at 0 deg. fahr.	½–1½ deg.	23 deg.
Consistency-Abraham hardness:		
20 deg. fahr.	Above 100	80
60 deg. fahr.	17–40	35
100 deg. fahr.	4–9	15
Loss by evaporation 100 hr. at 325 deg. fahr.	0.25–4.0	none
Ball and ring fusing point after heating	190–230 deg. fahr.	224 deg. fahr.
Consistency after heating:		
20 deg. fahr.	above 100	95
60 deg. fahr.	43–90	42
100 deg. fahr.	18–55	18
Viscosity: Temperature at which a viscosity of 100 sec. per 100 rev. is obtained	260–278	357

The presence of undesirable low boiling fluxes and waxes is detected by testing the changes in the physical properties of the asphalts and measuring the evaporation after they have been heated at 325 deg. fahr. for 100 hours. It is expected that this test gives, in a short time, a measure of the changes which take place in the saturants in the wire covering, over a period of years. The changes taking place during heating have been of much help in eliminating the poorer saturants. The comparative changes with heating, for commercial saturants and a typical blown asphalt, may be noted by comparing Fig. 2 with Fig. 1.

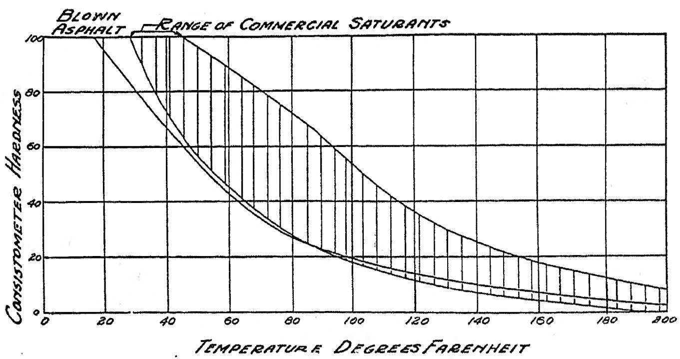

FIG. 2—COMPARISON OF HARDNESS AFTER HEATING

Heated 100 hours at 325 deg. fahr.

While the shifting of the saturants in the braids can be greatly reduced by choosing asphalts of the proper physical properties, such physical properties as may be checked in the laboratory in a few hours, and while saturants containing low boiling point materials can be eliminated by the heating test requiring several days, the accelerated weathering test is still required as a guide in choosing saturants least subject to weathering when exposed to light and air.

An exhaustive study of the manner in which asphalts

weather, which have been selected from all sources and of different physical properties, is to be conducted to learn if it is possible to use only physical tests and the changes in their physical properties with heating as checks upon the life of such asphalts. If this is possible, then a specification for weather resistant wire which sets forth the properties of the saturant used will provide long life for such coverings. The asphalts can be tested

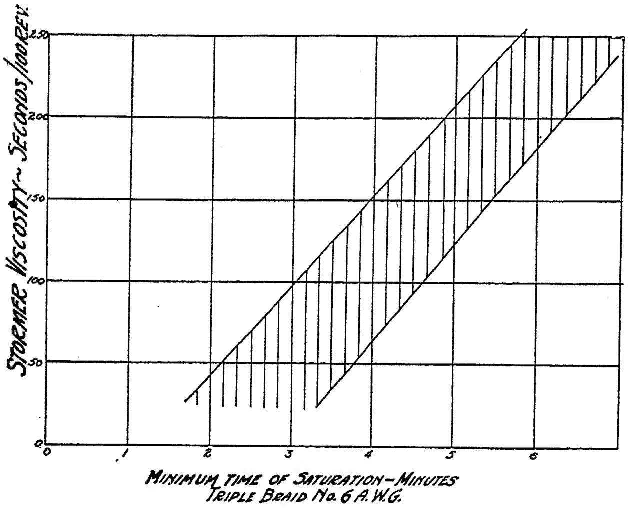

Fig. 3—Relation of Viscosity and Saturating Time

within a reasonably short time to see that they meet the specification.

Modification of Asphalts

After asphalts of the proper physical properties have been chosen it will be found that these asphalts may be further improved, both in regard to their stability when hot and their pliability when cold, by the addition of certain modifying agents. It is expected that complete accelerated weathering tests will show that the differences in weather resistance between asphalts of similar physical properties will be reduced by the proper addition of fillers and modifying agents.

Any finely divided mineral filler such as slate dust, fly ash, dolomite, etc., may be used. The metallic salts used have included copper and aluminum sulfate; bakelite and rubber have also been tested for this purpose. In general, these fillers and modifiers decrease the changes in physical properties of the asphalts with changes in temperature and improve their resistance to the effects of weathering.

Temperatures of Saturating Braided Coverings *Must be Increased if Saturants of More Satisfactory Qualities are to be Used*

To obtain thorough saturation of triple braid coverings with saturants of a fusing point of 180 to 200 deg. fahr. at present saturating speeds it is necessary to use higher saturating temperatures than are used today. Laboratory tests have shown that to obtain saturation of three braids on No. 6 A. W. G. conductors in five minutes requires a saturant viscosity of 100 deg. Stormer or less. Fig. 3 shows graphically the time required to saturate the triple braid covering on No. 6 A. W. G. conductors with asphalts of a very wide range, at any given viscosity. It should be noted that the time of saturation is not the same for all saturants at a given viscosity but varies with the asphalt, some requiring more and some less time, as indicated by the shaded area.

Reference to Fig. 4 discloses that temperatures of 375 to 400 deg. fahr. are required to provide this viscosity in the case of a typical blown asphalt having a fusing point of 190 deg. fahr. A change would be required in the heating equipment for the saturating tanks in the wire plants, if such saturants are to be used.

Saturating Temperatures of 400 Deg. Fahr. *Are Not Harmful to Cotton Yarns or Copper Conductors*

To determine definitely that saturating temperatures as high as 400 deg. fahr. can be used without damaging the cotton yarns nor too greatly annealing the copper conductors, a number of tests was made to determine the effects of this temperature.

The cotton yarns were tested for tensile strength, heated in air for different times, allowed to regain normal moisture content and again tested. The tensile strength of oil soaked yarns was also tested and compared with that of similar yarns heated in hot oil for varying times. No. 6 A. W. G. copper conductors, both medium hard drawn and hard drawn, were tested for tensile strength after heating in hot oil for varying times and compared with tests made on unheated conductors.

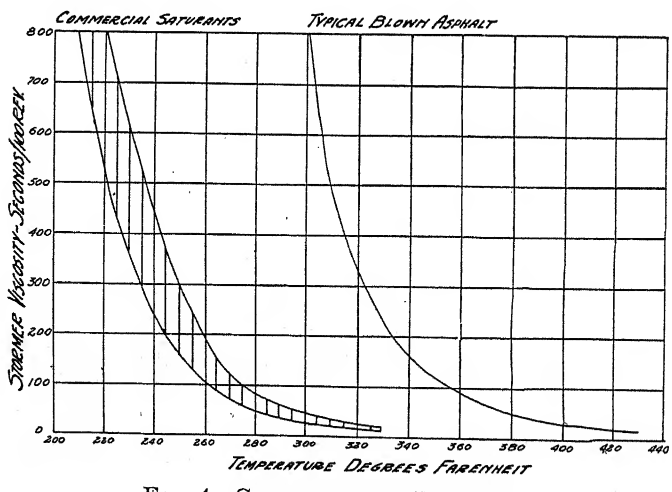

Fig. 4—Comparison of Viscosity

The results of the cotton and copper tests are presented in Table II.

It may be seen that the loss of strength of the cotton yarn is of no importance in even the longest saturating time. There is no appreciable annealing of the copper conductors in the saturating time required in the modern continuous saturating tanks which are now generally used. In the case of the older type "reel in and reel out" saturating tank the time required for saturation

TABLE II—EFFECT OF SATURATING TEMPERATURE ON THE TENSILE STRENGTH OF COTTON YARN

Yarn heated in air (constant temperature oven)

Sample	Tensile strength	Loss	Per cent loss
Original yarn	2,001 gm.		
15 min. at 392 deg. fahr.	1,760 gm.	241 gm.	12.0
30 min. at 392 deg. fahr.	1,526 gm.	475 gm.	23.7
90 min. at 392 deg. fahr.	1,360 gm.	641 gm.	32.0

Yarn heated in oil

Sample	Average tensile strength	Per cent loss A	B	C
Dry yarn	1908 gm.			
Dipped in cold oil	1917 gm.			
Heated 15 min. 212 deg. fahr.	1809 gm.	5.62		
Heated 30 min. 212 deg. fahr.	1731 gm.	14.5	4.32	
Heated 15 min. 302 deg. fahr.	1683 gm.	12.2	7.00	
Heated 30 min. 302 deg. fahr.	1623 gm.	15.3	10.30	3.57
Heated 7½ min. 392 deg. fahr.	1637 gm.	14.6	9.55	2.52
Heated 15 min. 392 deg. fahr.	1593 gm.	16.85	11.98	5.35
Heated 30 min. 392 deg. fahr.	1546 gm.	19.3	14.55	8.13

A Loss in strength considering the cold oiled sample as a standard.
B loss in strength considering the sample heated 15 min. at 212 deg. fahr. as a standard.
C loss in strength considering the sample heated 15 min. at 302 deg. fahr. as a standard.

All samples heated at 392 deg. fahr.

Kind of sample	Tensile strength lb. per sq. in.	Loss in strength lb. per sq. in.	Per cent loss
Hard drawn No. 6 copper wire			
Unheated wire	63,560		
Heated 10 minutes	62,590	970	1.53
Heated 30 minutes	61,140	2,420	3.96
Medium hard drawn No. 6 copper wire			
Unheated wire	51,430		
Heated 10 minutes	50,780	650	1.27
Heated 30 minutes	49,490	1,940	3.93

may be long enough to anneal the conductor slightly, requiring that the conductors be drawn somewhat harder initially.

These tests, it is believed, have definitely cleared the way for the use of higher saturating temperatures in so far as likelihood of any damage to yarns or conductors is concerned.

New and Different Types of Coverings

While the development work carried on thus far has been concentrated upon providing an improved asphaltic base saturant which could be applied to the present triple braid covering without great changes in the weather resistant wire plant, experimental work with other types of weather resistant saturants and with coverings other than triple braid, has been carried on.

Bakelite and other synthetic resins, Harvel compounds and varnishes, and other materials have been applied to weather resistant wire coverings. The use of these materials as saturants for a single outer braid, applied over an insulating cover on the conductor, provides a highly protective coating because of the excellent weathering quality of such materials.

Experiments with conductors having a paper tape covering next to the conductor and a single outer braid weatherproofed with these very high quality saturants have shown that coverings of consistently high insulating value can be obtained and that the paper next to the conductor does not deteriorate appreciably in the time of one accelerated weathering test.

Most of these high grade saturants can only be applied to one braid because of their high cost.

It is expected that future work will provide a weather resistant wire covering of higher insulating value and longer life than the present triple braid construction but of smaller size and weight. There are many interesting possibilities of improvement in this type of covering.

Conclusion

The present triple braid weather resistant wire covering may be materially improved by the use of higher fusing point blown asphalts, free of waxes and low boiling fluxes, as saturants. Such a saturant will provide a covering of uniformly long life.

It has been proved that the higher saturating temperatures required to apply such saturants may be used without damage to the cotton yarns or excessive annealing of the copper conductors.

Specifications for weather resistant coverings will soon be written as a result of this work, the terms of which will guarantee to the utility, weather resistant wire of long life.

Future work will provide a covering of lighter weight and smaller size but of higher insulating value and longer life than present construction.

Acknowledgment

Acknowledgment is gratefully extended to Mr. G. W. Hamilton, Chairman, and to the members of the committee of the Utilities Research Commission in charge of this project, in recognition of the encouragement and the valuable suggestions and advice which have made possible the performance of this work.

Professors H. C. Peffer and R. N. Shreve of Purdue University have rendered invaluable aid in acting as consultants on chemical problems involved in the work.

Many of the weather resistant wire manufacturers have cooperated in furnishing materials and in frankly discussing the work and its application to the plant.

To the students who have assisted in the construction of equipment and in routine testing many thanks are due.

Discussion

C. D. Brown: The development of an improved saturant for weather resistant covering of line wires is a decided step in the right direction and Mr. Harding is to be complimented for his research work on this subject.

In some cases, triple braid weatherproof insulation has little insulating qualities after it has been up for four or five years, and a great many companies do not place any confidence in this type of insulation on primary distribution feeders. The result is that many companies only use weatherproof insulation where it is required by the code, and their construction is such that reliable service can be maintained even though the insulation drops off entirely, except where tree conditions are encountered.

Tree wire is used under much less favorable conditions than the ordinary weatherproof wire and the weather resistant covering

of tree wire must not only be depended upon to protect the rubber insulation against weather conditions but also against abrasion. If the insulation of tree wire fails, it often results in an interruption to service. Our experience has indicated that the outer covering of tree wire has in many cases the same deficiencies with respect to impregnating compound, as are brought out in the paper regarding weatherproof insulation.

Has any investigation been made of the properties of the weather resistant covering for tree wire, and if so, what have been the results of the investigation?

James H. Foote: It is indicated in the paper that seven years' life of weatherproofing on overhead wire would seem to be reasonable, and certain economic considerations given in the paper are based upon this life. In my experience this would seem to be an abnormally short period of time. I have in mind a distribution system which was entirely rebuilt in 1914 using triple braid weatherproof wire manufactured to no special specifications, which does not today after 18 years show any appreciable amount of raggedness or deterioration easily visible from the ground. In another city the entire street lighting system was rebuilt in 1917 using triple braid weatherproof wire obtained on the market, which today shows no deterioation visible from the ground after 15 years of life.

The question may arise as to whether the results obtained with the lighter braids ordinarily used on the smaller sizes of wire used by the authors of the paper in their experiments will also apply to the considerably thicker cotton covering usually applied to the larger sizes of wire ordinarily used in distribution, such as 0 or 0000 A.W.G.

It is mentioned in the paper that many engineers have swung towards discontinuing the weather resistant covering on overhead lines. While it would seem that the omission of insulating covering may well be justified on the higher voltage conductors, especially primary lines of over 5,000 volts, still the weatherproofing has a real value as insulation in the case of overhead secondary lines run on close spacing such as is common where secondary racks are used instead of crossarms, and especially where the secondary lines are run through trees where the tree branches may push the wires together during storms. It would seem entirely reasonable to omit the weatherproofing from the neutral wire of a three-wire secondary, leaving the weatherproofing on the outside wires. Service wires should also have weatherproofing or other insulation since they very frequently are so strung as to swing together under certain conditions. Thus, there appears to be an adequate market for weatherproofed wire in the future and it would seem that the investigation carried on by the authors is certainly very well worth while.

Frank Kingburne: While some of the findings may be revolutionary, largely because of the dearth of available information, this centralized, unbiased research should excite the manufacturers toward improving the general character of this phase of their business.

The contention that the saturant is of first importance is quite correct, but at the same time the final improved product will depend on both the saturant and the braid construction. To that end a better knowledge of braids and braiding construction should be important; a braiding schedule specifying the following: size of cotton; number of plys; number of ends; picks per inch to attain a so-called 100 per cent braid covering according to size or diameter of wire. The wording "closely woven braid" in present specifications is not very definite and lends itself readily to controversy. Again, in normal times, cotton is quite an item in the quoted price in comparison with the present saturant, although it may not be with a high quality saturant.

The statement of having "a more solid and homogeneous material next to the conductor" should also indicate that flexibility in the material is desirable. The product of the Peerless Wire and Cable Company at Pennington, N. J. may be somewhat along this line, also the experimental work done by the American Steel & Wire Co., Worcester, Mass., on a similar type of line wire, *viz.:* paper tape next to the conductor and a single cotton braid overall, weatherproofed with a high quality saturant, recorded in the paper.

The matter of using a high saturating temperature, 400 deg. fahr., with a high quality saturant should not act as an impediment, or be detrimental to the tensile strength of the cotton braid, provided proper precautions and refinements are used during the saturating process. Locomotive headlight wires for steam engines are saturated under conditions quite similar with good results. Standardization of this practise, high saturating temperatures, would undoubtedly mean revamping the present apparatus found in most wire plants.

It may be of interest to know that the railroads purchase large amounts of line wire, usually in accordance with A.R.A. specifications. These specifications require two cotton braids over the conductor, the conductor being either iron wire, copper, or copper alloy. This leads to the question as to why the utilities should require three braids, or their equivalent, for their purposes. True, the operating conditions, voltage for instance, are different, yet from contact with operating men it has been my impression that the utilities in replacing festooned line wire were not worried about the lowered insulating value, but to comply with municipal ordinances, or public opinion, or perhaps to establish an underground system. So from this why would not a similar wire be just as good, regardless of whether rural or urban use was contemplated.

C. F. Harding, L. L. Carter, J. W. Olson: Many reports are received of weatherproof wires that have been in service for periods of 15 to 35 years without shedding the braided covering. In marked contrast to this there are many reports of weatherproof wire coverings that have failed in periods as short as two years, particularly in hot climates. It should be remembered that the majority of the wires that have failed quickly have been produced in the last 10 or 12 years whereas the longer lived wires were produced in the years previous to 1920 or previous to the war period.

A graphic illustration of the differences in the weathering qualities of the recent product and the older weatherproof wire is the fact that certain commercial wires placed in the outdoor weathering exposure at Purdue in September of 1930 appear to be in poorer condition than wires installed in an outdoor weathering rack in Chicago in 1916.

Brief History of Weatherproof Wire Production Methods

To explain the reasons for this variation in life and the reasons for the changes in weatherproof wire covering materials and production methods, which have caused this change in life, it seems desirable to give a brief history of weatherproof wire production methods and materials.

Before the start of the last decade, weatherproof wire coverings were usually built up of braids made from high strength, long fiber, cotton yarn, often saturated after each braid was applied and usually saturated with natural asphalts rather than petroleum base asphalts. Production methods were slow and the materials used more expensive than it is now possible to use.

With the start of the last decade the increased demand for electric service caused a very great increase in demand for weatherproof wire. Coupled with this demand for more wire was an insistence that the cost of the wire be lowered. The increased demand alone, if the earlier materials and production methods had been used, would probably have resulted in an increased cost of the product. We thus had two factors, namely, the increased demand and the demand for lower prices, that forced the substitution of cheaper materials and speedier production methods in the weatherproof wire industry.

The weatherproof wire producers began to use cheaper grades of cotton yarn and looser braids and they turned to petroleum

by-product asphalts for saturants. They further changed from saturation of each braid after its application to the application of all three braids in one step and the saturation of these braids as a second step. To obtain the production speeds required and to saturate three braids in one step at the temperatures available in the wire plants (that of low pressure steam) the use of low melting point petroleum asphalts of low viscosity at saturating temperatures became general.

In this short description of the change on the industry we have the background for the short-lived weatherproof wire coverings produced in the past 10 or 12 years. This background of the changes in the weatherproof wire industry is of much importance to anyone interested in developing a longer lived covering.

Fundamental Problem Involved in Research at Purdue

The real problem in the research work at Purdue has been that of finding a material which could be used as a saturant, that would provide equal or better life than was had by the weatherproof wire coverings produced years ago but at a very slight increase over the present cost of production.

The answer to the question "if long lived wire coverings were produced 15 to 20 years ago, why cannot they readily be produced again" is very simple. Such coverings could not be produced with the old materials and the old methods at a price that could, in most cases be economically justified.

Improvement in Wire Coverings Can be Made at Little Increased Cost

The use, as weatherproof wire saturants, of the air blown asphalts meeting the preliminary specification which has been developed, has been proven in the laboratory to yield a greatly increased length of life of the covering on any size wire, over that provided by the usual commercial saturants and to provide a greatly increased flexibility of the finished wire. The physical properties of these asphalts and the present commercial saturants are given in Table I. The preliminary specification follows:

The saturating compound shall consist of a pure blown petroleum asphalt and shall not be mixed with any material such as paraffin, wax tailings or fluxes. It shall have the following physical properties:

Ball and Ring Fusing Point: 180-200 deg. fahr.

Penetration: 77 deg. fahr. 100 g., 5 sec. 35 or harder, tenths of mm.

Pliability: A mold of the asphalt ¼ in. by 0.1 in. by 0.4 in. shall not break when subjected to an angle of bending of 15 deg. around a mandrel of 3/16 in. radius at a temperature of 0 deg. fahr. This test shall be made in the Reeve and Yeager pliability testing equipment, at a rate of bending of 10 deg. per minute.

Loss By Evaporation: A 100 gram sample of the compound heated for 100 hr. at a temperature of 325 deg. fahr. shall not lose more than 1.0 g. in weight.

Rise in Fusing Point: The difference in the ball and ring fusing point determined before and after the above heating test shall not be greater than 40 deg. fahr.

Change in Penetration After Heating: The penetration of the sample subjected to the above heating test shall not become harder by more than 10 points. The crust formed on the sample during the heating test may be removed before determination of the penetration after heating.

Adequate proof of the improvement obtained by the use of these materials is given by comparing the wires in Fig. 1 with those in Fig. 2, all having had the same exposure in the accelerated weathering test. Fig. 1 shows the condition of wires saturated with the present commercial saturants after having been exposed in the accelerated weathering test at Purdue.

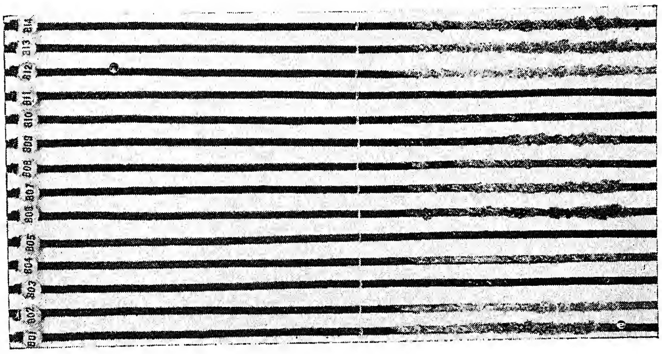

Fig. 1—Condition after Accelerated Weathering Test of Wires Saturated with Commercial Solvents

Fig. 2—Condition after Accelerated Weathering Test of Wires Meeting Various Parts of Preliminary Specifications

Wires 815-8, 820-6, and 830 meet these specifications in every detail

The four wire coverings that have held up well on this test are saturated with very hard and brittle materials. The asphalts with which these four wires are saturated bend less than two degrees in the pliability test. This emphasizes the importance of applying both the laboratory pliability test and the accelerated weathering test in choosing the most satisfactory

TABLE I—COMPARISON OF PHYSICAL PROPERTIES, COMMERCIAL SATURANTS AND ASPHALTS PROPOSED AS SATURANTS

	Range of physical properties		Average of physical properties	
	Commercial saturants	Proposed saturants	Commercial saturants	Proposed saturants
B & R fusing point				
Before heating	119-158°F.	186-208°F.	143.2°F	196.1°F.
After heating	176-245	209-232	198.4	220.7
Penetration: 77°F., 100 grams, 5 seconds				
Before heating	3-66	21-33	25.5	27.5
After heating				
With crust	0-23	0-4	8.7	1.0
Crust removed	0-26	17-29	11.8	22.3
Loss in weight during heating	0.12% gain, 7.17% loss	0.03-0.97% loss	1.63%	0.36%
Pliability	0.41° -13.3°	16.2°-48°	3.21°	26.5°
Specific gravity				
Before heating	1.001-1.200	0.987-1.044	1.071	1.011
After heating	1.021-1.208	0.988-1.042	1.080	1.013

weather resistant coverings. The wires in Fig. 2 have coverings saturated with air blown asphalts meeting the specification for melting point and penetration, but in the case of samples 819, 826, 827, 828, 829 and 831 not meeting other parts of the specification such as that for pliability or changes in physical properties with long continued heating. It may be readily seen that the wires meeting the preliminary specification, in every detail, provide a vast improvement in weather resistance over the commercial saturants. These wires are numbers 815-818, 820-826 and 830. All of these asphalts bend more than 15 deg. in the pliability test and wires saturated with these materials may be bent about their own diameter at freezing, or lower in some cases, without breaking any of the braids.

Sample 819 is saturated with a material that has shown peculiar breakdown when exposed to light and heat and that has been used in the lower melting point ranges in very great quantities of weatherproof wire in the past ten years.

The use of the asphalts meeting the preliminary specification requires only a change of saturating temperature and does provide a remarkable improvement in the weather resistance of the covering.

Fig. 4 illustrates the smaller change in hardness of air blown asphalts as compared with the commercial saturants and also the much more stable asphalt of lower temperature susceptibility. The asphalts illustrated in Fig. 2 are stock products. They are not special materials in any sense. There is no reason to think that the increased use of air blown asphalts meeting these specifications should result in a marked increase in cost of the materials (which now cost very little more than the steam reduced asphalts) because the petroleum residue is just as readily blown to meet this specification as to meet one of a lower melting point.

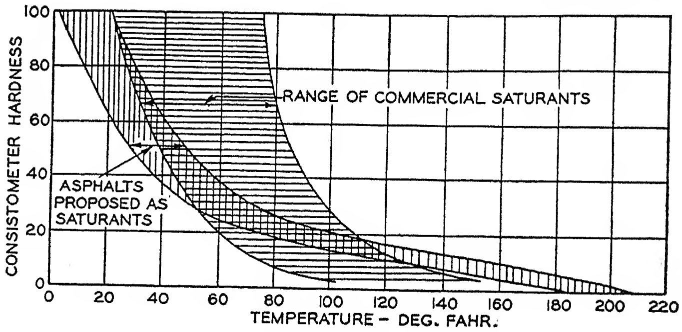

Fig. 4—Comparison of Initial Hardness

Commercial saturants and asphalts proposed as saturants

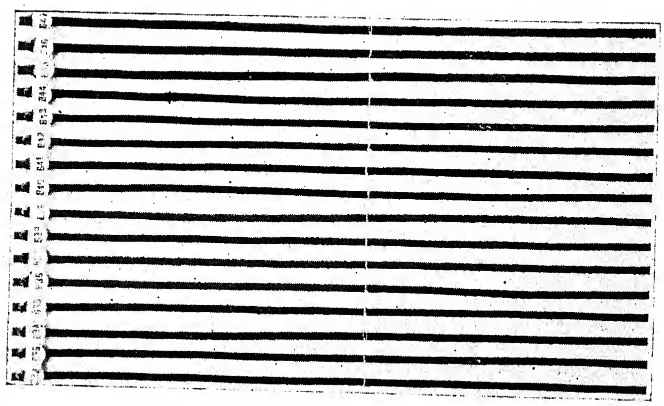

Fig. 3—Wires Illustrating the Improved Weather Resistance of Air

Blown asphalts to which proper inert mineral fillers have been added

greater stability at the higher operating temperatures. It may be seen that the air blown asphalts are harder, as an average, at temperatures of 60 deg. fahr. and higher, but are softer than the commercial saturants below this temperature.

Fig. 5 shows the smaller change in the hardness of the air blown asphalts as compared with the commercial saturants after being heated for 100 hours at 325 deg. fahr. It will be noted that the blown asphalts have not changed greatly while the commercial saturants have become much harder. This lack of change with long heating has been proven to correlate rather closely with the resistance of the wire covering to weathering.

Fig. 6 shows that higher temperatures are required to obtain saturation of the braided coverings with the air blown asphalts than are required in the case of the commercial saturants. Viscosities lower than 100 seconds per 100 revolutions are required to obtain saturation of triple braid coverings at present saturating speeds and temperatures of 360 deg. and higher are required for saturation with these asphalts.

These "air blown asphalts" are produced by blowing air through the heated petroleum residue, as compared to the production of the usual commercial saturant by blowing steam through the petroleum residue. Certain changes in the structure of the asphalt are produced by the air blowing which results in a

Modification of Asphalts

Fig. 3 shows the improvement that may be made in the weather resistance of the air blown asphalts by the use of the proper inert mineral filler. Wires numbers 832-835 are saturated with the same materials as wires 815-818 in Fig. 2 except that 30 per cent by weight of 325 mesh slate dust was added to the asphalt before saturation. Wires 840-842 in Fig. 4 are also asphalts to which certain modifying agents have been added. The use of such modifying materials provides an additional and very desirable improvement in the weather resistance of the wire covering and also may slightly reduce the cost of the saturant because of the cheapness of the slate dust. The use of the filler is particularly helpful in the case of the asphalt on the surface of the wire and in the case of the finishing materials.

Improved Finishing Materials

Finishing materials that are more flexible than the present waxes may be had and are much more desirable, particularly

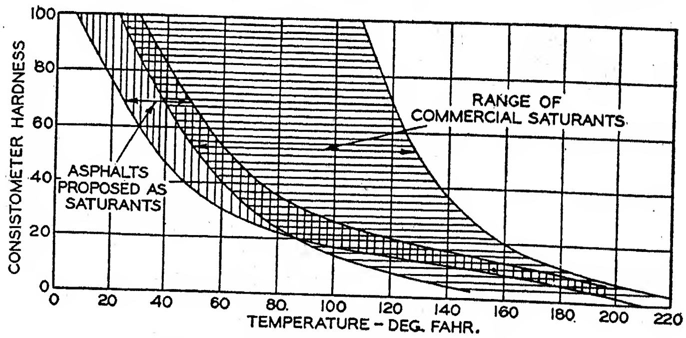

Fig. 5—Comparison of Hardness

Heated 100 hr. at 325 deg. fahr. Commercial saturants and asphalts proposed as saturants

when some filler is used as a coating or an anti-stick in these finishes. Mica flake is particularly valuable as an outer finish both to prevent sticking and to improve weathering, reduce cracking, etc.

Accelerated Weathering Test is Still Required in Choosing the Saturants and Finishers

While all of the asphalts which weather poorly in the wires in Fig. 2 would have been eliminated by requiring them to meet

the preliminary specification, we cannot be certain as yet that this will always be true and we must still depend upon the accelerated weathering test to eliminate the poor materials. In the operation of the weather-ometer on the standard cycle used by the Bureau of Standards the commercial saturants all show extreme checking or cracking in only one day whereas the asphalts recommended show no signs of deterioration in this time, except in one instance, thus checking the tests of the asphalts in wires as illustrated in Figs. 1, 2, and 3.

Asphalt Analysis

The analysis of asphalt and the determination of chemical unsaturation are now being carried on to learn if it is possible

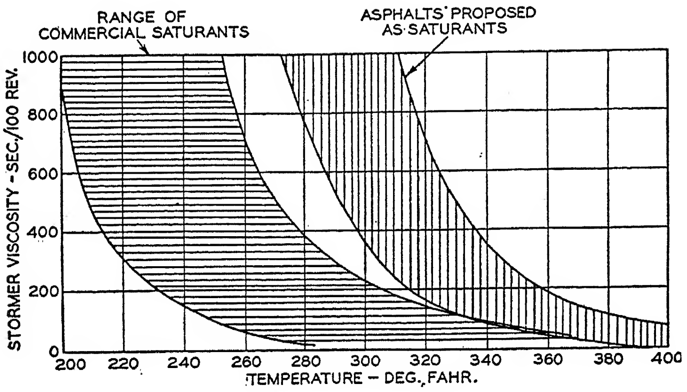

Fig. 6—Comparison of Viscosity

Commercial saturants and asphalts proposed as saturants

to correlate some result of this analysis with the variation in weathering of the asphalts.

Additional Testing Equipment Designed or Constructed

The design for an improved Abraham consistometer and a Reeve & Yeager pliability test equipment especially adapted to this work has been completed. The rate of penetration in the consistometer and the rate of bending in the pliability tester will be quite constant as both are to be electrically driven and the degree of bending in the pliability test will be determined by automatic electric timing of the start and the finish of the bending period. The consistometer hardness will be read from a pressure gage which indicates the pressure on the table of the consistometer supporting the asphalt sample under test.

In the testing of finished wire a small electrically heated braid press and a pliability tester in which very small samples of asphalt may be tested have both been completed. The braid press is used to extract the saturant from the two inner braids of a finished wire. Sufficient asphalt is obtained from a two foot length of No. 6 wire for the standard ball and ring melt point test and the penetration determination. The pliability test can be made with this new equipment upon the small sample extracted from the braids.

The accelerated weathering equipment is still depended upon to indicate certain types of failures in the asphaltic saturant.

New and Improved Coverings

The much superior insulation of paper tape covered samples having a single weatherproofed outer braid has been shown repeatedly by the slower decrease in insulation resistance of this covering when immersed in water, as compared with the triple braid construction. The use of this paper tape and single braid covering provides a conductor of lighter weight and much smaller overall diameter, which means that sleet and wind loads will be lower, and also of consistently high dielectric strength even with considerable exposure of the braids to the weather.

Conclusion

This discussion is intended to answer the questions raised at the time of presenting this paper and also to summarize the work done since the writing of the original paper.

The work done since writing the paper has given ample proof that the materials recommended will provide longer life for weatherproof wire coverings and also that these materials may be obtained within a reasonable distance of any weatherproof wire plant in the United States.

It is felt that the experience and equipment built up at Purdue as a result of this work should prove of value in the development of improved weatherproof braids as applied to other types of wire than the ordinary weatherproof wire. This is especially apparent when it is pointed out that in looking for improved materials, the coverings of tree wires and other insulated overhead line wires were investigated to learn if the weatherproofing compounds used in the braids on these wires were superior to the ordinary weatherproofing compounds, and it was found that while these coverings were made of exceptionally strong and high grade braids they were not saturated with compounds in any way superior to the present commercial weatherproof saturants.

Electrical Design Features of Waukegan Station

BY E. C. WILLIAMS*

Member, A.I.E.E.

THE Waukegan Station of the Public Service Company of No. Illinois is located at the north limit of the city of Waukegan on the west shore of Lake Michigan. This steam generating station is the chief energy supply point between the Illinois-Wisconsin State line and Chicago. It also forms one of the supply points for the Company's 132-kv. transmission system which forms an outer belt around Chicago. The major substations and interconnections with other companies are shown in Fig. 1. The main one-line diagram of the station is presented in Fig. 2.

Some features of the design anticipated the general use of similar equipment in other stations. The general design of the station, however, paralleled the advance of the art, as the station has developed from a single 25,000-kw., 12,000-volt unit in 1923 to five units having a total capacity of 290,000 kw. in 1931. The fifth of these is a 115,000-kw. unit generating at 18,000 volts.

Waukegan Station is unique in a number of its design features, some of which result in a material capital saving, some in a greater safety to personnel, and others in greater operating reliability. Probably the best way of presenting these features is to review the development of the station in three fairly distinct periods.

Initial Installation

In the first period (units 1 and 2), a double 12-kv. bus was started using 15-kv. truck type oil circuit breakers and cell-type construction to enclose the buses, disconnecting switches, and instrument transformers, as well as the oil circuit breakers. A Cory lock system was provided to prevent improper operating sequence of the disconnecting switches. The auxiliary equipment operated at 480 volts, and an ordinary open bus was provided for this purpose. All cables were run in conduits from the single bus used for the turbine and boiler-room auxiliaries. Reserve for the auxiliaries was provided by a tie between the unit 1 and 2 sections of the auxiliary bus. Feeders were run from the 480-volt auxiliary bus to *VV* switch groups located at various load centers along the coal-preparation and conveying system.

Metal-Clad Switchgear Used in Second Period

The second period started with the installation of a unit twice the size of the original one, and a radical departure from conventional design was introduced by the use of metal-clad switchgear for the 12,000-volt equipment, and for the 2,400-volt and 480-volt auxiliary equipment. Fig. 3 shows the 12,000-volt, type M switchgear.

The bus arrangement of the 12,000-volt metal-clad switchgear used for units 3 and 4 is somewhat of a departure from the conventional design in that two buses have been provided, but there is only one oil circuit breaker per position connecting to the two buses. Selector switches are provided for making it possible to

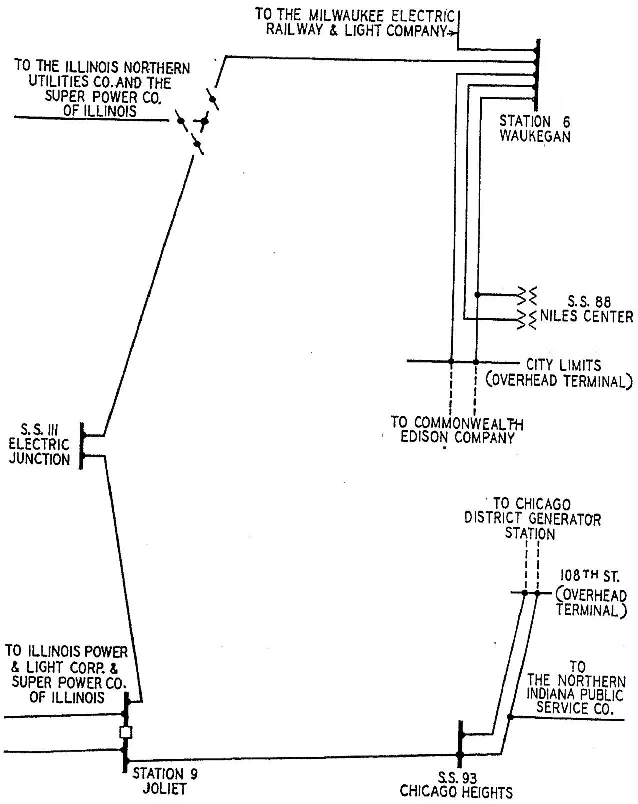

Fig. 1—132-Kv. System and Interconnections of the Public Service Company of Northern Illinois

place the oil switch on either the main or the reserve bus, as desired. There are fourteen of these units, which range in carrying capacity from 600 to 4,000 amperes. The interrupting capacity of all these switchgear units is 1,500,000 kva.

There is a number of interlocks on this switchgear which prevent incorrect operation due to errors, oversights, or carelessness when switching operations are performed. For instance, the isolating or selector

*Elec. Engr., Public Service Co. of No. Illinois.

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switches cannot be opened or closed unless the main oil circuit breaker is open. Also, the oil circuit breakers cannot be lifted from their tanks, nor can the selector and isolating switch tanks be lowered, when current carrying parts are alive.

Auxiliary Equipment Size Causes Operating Voltage Change

The use of larger turbo-generator units required boilers of greater steaming capacity and larger auxiliary equipment. As a result of the increased size of auxiliary equipment, larger motors were used; so the operating voltage was changed from 480 to 2,400 volts for these motors. Also, the larger capacity of the boilers made it necessary to provide more flexible operation of the stoker-feed mechanism on the boilers of unit 3. To provide this flexibility, a-c. brush-shifting motors were used. This arrangement provides almost as much flexibility as could be obtained by the use of the customary d-c. motors and control.

Unit 4 Design Features

Although the same general electrical design was used for unit 3 and 4 installations, there are a few features of the unit 4 installation that are of interest. Since unit 4 has a capacity of 65,000 kilowatts, the carrying capacity of its metal-clad switchgear increased; the size and number of auxiliary motors increased because of the use of larger auxiliaries and also because unit-type coal-pulverizing mills were installed. Centralized control of the boiler coal and air supply auxiliary motors was provided on panels similar to the one shown in Fig. 4; and the electrical control system was designed for either manual or automatic operation, in response to the demands of the automatic combustion-control equipment which regulates the coal and air supply for the boilers.

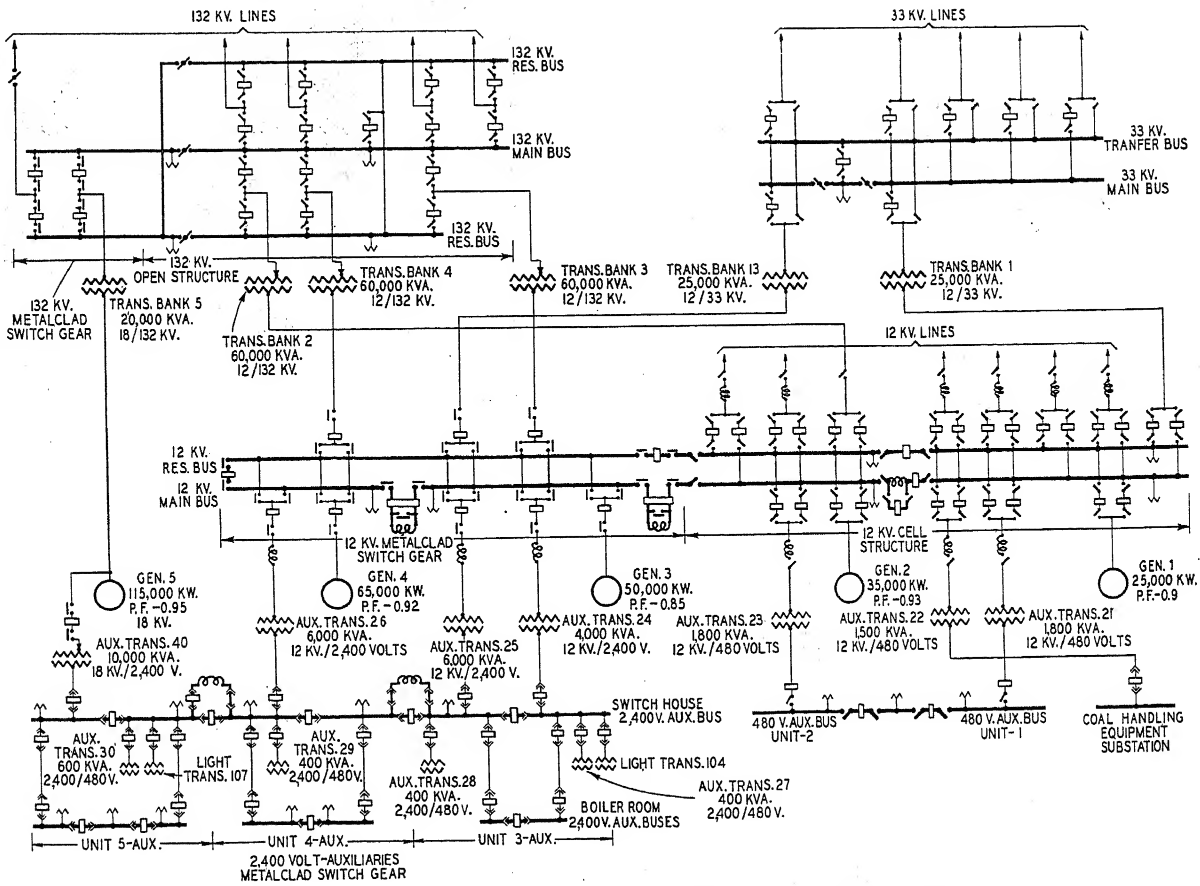


Fig. 2—Single-Line Diagram of the Waukegan Power Station

Main one-line diagram of connections

Protection on Pulverized Fuel System

The use of automatic combustion-control equipment and of pulverized coal made it necessary to exercise unusual care in designing the control and protective system. By designing the system so that a negative pressure or suction must be present inside the furnace when it is in service, the possibility of the pulverized coal's igniting and flashing back into a pulverizing mill and causing an explosion, are eliminated. This is the principle followed in designing, for the boiler-feed auxiliary equipment, an electrical interlock system which prevents any deviation from the prescribed operating procedure.

Starting Sequence

The prescribed starting sequence of the equipment that must be followed whenever a boiler is fired up is as follows:

1. Start low-speed, induced-draft fan, wound-rotor motor. When the resistance is all out of the secondary circuit, the drum controller automatically closes the high-speed squirrel-cage motor oil switch.
2. When the induced draft fan is up to speed, the forced-draft fan oil switch can be closed.
3. After the forced-draft fan motor is up to speed, the exhauster-fan motor can be started, and this can be followed, in order, by the pulverizing mills and the coal feeders.

Method of Making Prescribed Operating Sequence Effective. Each fan, exhauster, pulverizing mill, etc., has its own oil switch, and the closing and trip circuits of these oil switches are so interconnected that it is impossible for an operator to close an oil switch unless the equipment ahead of it in the sequence is running; that is, if an operator attempts to run a pulverizing-mill motor before placing the draft fans and the exhauster fan in service, the oil circuit breaker controlling the pulverizing-mill motor cannot be closed.

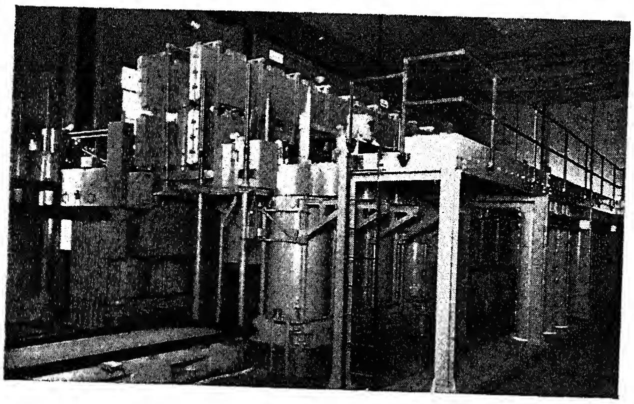


Fig. 3—12,000-Volt, Type M, Metal-Clad Switchgear

Two-Motor Drive for Induced-Draft Fans

To provide the maximum operating flexibility and reliability, two motors are used on each induced-draft fan. The fan starts on the low-speed wound rotor motor; after it is up to its maximum speed, the drum controller completes a circuit which closes the high-speed motor oil switch and trips out the low-speed motor oil switch. If the high-speed motor should fail to start or fail in service, the operation of the overload relays would function to place the low-speed motor in service. If the low-speed motor should fail, the protective relay operation would cause the high-speed motor oil circuit breaker to close. This arrangement, of course, complicates the interlock scheme, but this problem was met by using centrifugal switches on the motor shafts to complete or open auxiliary switch circuits on the oil switches as the motor speed increases or decreases.

Unit 5 Installation Constitutes Third Period of Development

In the third period, the unit illustrated in Fig. 5, which is over four times the capacity of the original 25,000-kw. unit, was placed in service. Since the greater part of the output of this unit is transmitted at 132,000 volts, it was decided to omit the generator bus and arrange for switching at the transmission voltage. This decision permitted the use of any convenient generating voltage, and the manufacturer selected 18,000 volts as the most feasible from the manufacturing standpoint. Subsequent units, of course, may generate at other voltages. As the generator and its associated 120,000-kva. transformer bank constitute an operating unit with no intervening bus, the generator leads had to be tapped to supply the 10,000-kva. auxiliary transformer. The high-voltage and low-voltage switchgear units provided for this transformer are

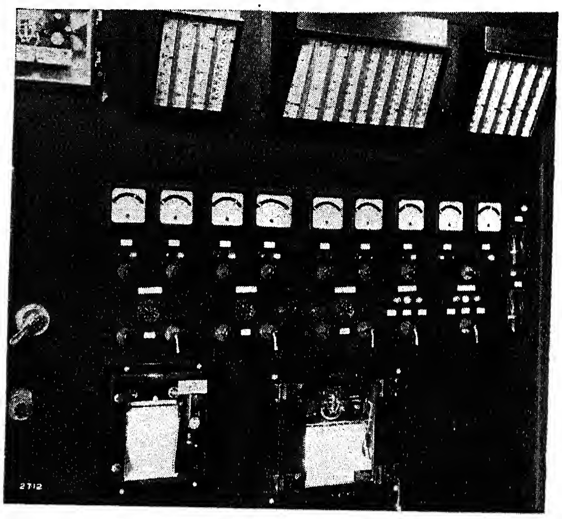


Fig. 4—Electrical Portion of Boiler-Control Panel

Showing the indicating instruments in the motor circuits, the oil circuit-breaker control switches, and the motor-operated boiler inlet valves feeding the main and auxiliary steam headers

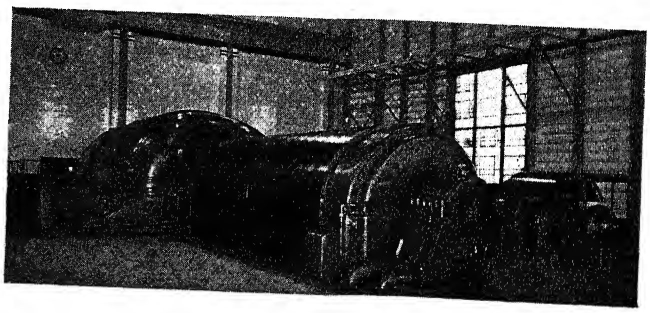


Fig. 5—115,000-Kw., 18,000-Volt Generator 5

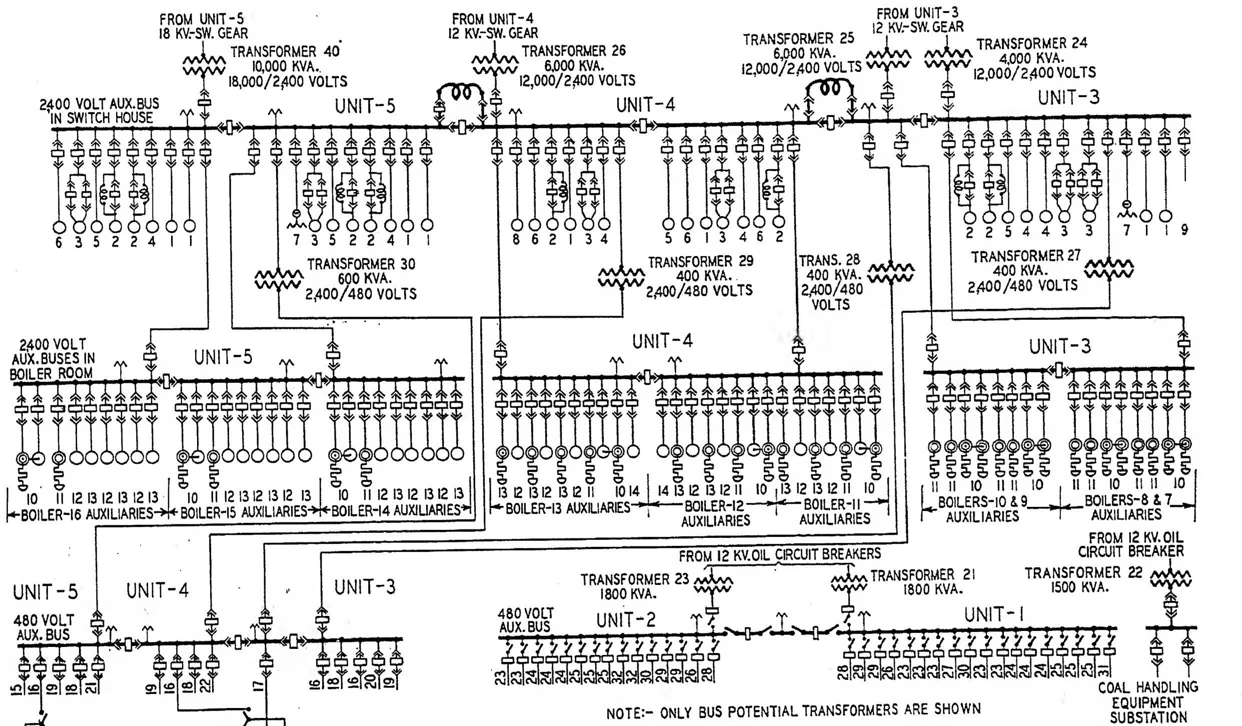
tripped by differential relays in case internal trouble develops. In this way, interference with the operation of the generator is avoided.

Auxiliary System

The low side of this 10,000-kva., 18,000/2,400-volt transformer is connected to the main 2,400-volt station auxiliary bus. This bus can be sectionalized by units if desired, or it can be operated as one bus. Reactors have been provided between sections of the auxiliary bus for fault current control. Power for starting the auxiliaries on units 3, 4, or 5, or for emergency operation

1. For equipment not included in the original design but which appears desirable to install later.
2. A reduction of the investment in reserve transformers, thus saving foundations, switching and control equipment, and installation costs for an "unproductive" transformer.
3. The excess capacity required for station overloads.

The general arrangement, design, and type of auxiliary switchgear used for units 1 and 2 and for the succeeding three units offers quite a contrast, as well as showing the development of the art. The auxiliary sys-



UNITS-3, 4 AND 5 SWITCH HOUSE 2,400 VOLT AUXILIARY MOTORS		UNITS-3, 4 AND 5 BOILER ROOM 2,400 VOLT AUXILIARY MOTORS		UNITS-3, 4 AND 5 480 VOLT AUXILIARY DISTRIBUTION GROUPS		UNITS-1 AND 2 480 VOLT AUXILIARY MOTORS AND DISTRIB. GROUPS	
1	GENERATOR AIR FAN	10	INDUCED DRAFT FAN	15	SPARE SWITCH GEAR UNIT	23	DISTRIBUTION GROUP
2	BOILER FEED PUMP	11	FORCED DRAFT FAN	16	BOILER ROOM DISTRIB. GROUP	24	FORCED DRAFT FAN
3	CIRCULAT. WATER PUMP	12	EXHAUSTER	17	BOILER ROOM EMERG. FEED	25	INDUCED DRAFT FAN
4	CONDENSATE PUMP	13	PULVERIZING MILL	18	TURB. ROOM DISTRIB. GROUP	26	CIRCULATING WATER PUMP
5	SERVICE PUMP	14	ASH SLUDGE PUMP	19	CRIB HOUSE DISTRIB. GROUP	27	BOILER FEED PUMP
6	ASH SLUICE PUMP			20	SWITCH HOUSE DISTRIB. GROUP	28	SERVICE PUMP
7	LIGHT TRANS. WITH V. REG.			21	MOTOR GEN. SET, BATTERY CH.	29	CONDENSATE PUMP
8	MULE HAUL MOTOR			22	TURB. ROOM CRANE MOTORS	30	BOOSTER PUMP
9	SPARE SW. GEAR UNIT					31	LIGHTING TRANSFORMER
						32	SPARE SWITCH GEAR UNIT

Fig. 6—Single-Line Diagram of the Station Auxiliary System of Waukegan Power Station

in case one of the station auxiliary transformers is out of service, is obtained from the station auxiliary bus. Fig. 6 gives a one-line diagram of the station auxiliary system.

The present auxiliary power installation provides 9.4 per cent of the generator capacity in auxiliary transformers. The active and reserve transformer capacities are 7.4 per cent and 2 per cent respectively. These transformer capacities have been selected so that we have 20 to 30 per cent unused capacity in each transformer under normal demand conditions. This normally unused capacity provides:

tem used for units 1 and 2 was of an open-bus type with its connections extending from the oil switches up to the disconnecting switches and then to the overhead bus, and considerable space was occupied for a comparatively small number of pieces of auxiliary equipment. All of the switching equipment for the turbine and the boiler-room was supplied from a single bus in the switch-house. On the other hand, for units 3, 4, and 5, 2,400-volt metal-clad switchgear has been used for the main auxiliaries, and separate buses have been provided for the equipment in the turbine-room and in the boiler-room. Two feeders are provided from the main bus

over to the boiler-room auxiliary bus. These feeders are protected differentially, and the bus in the boiler-room, as well as the main bus in the switch-house basement, can be sectionalized.

Generator Field Control

Another point in which the change in practise is apparent is in the method of controlling the generator field. Instead of using generator field rheostats, as had been done for the preceding four units, a main and a pilot exciter have been provided on the shaft of generator 5. The exciters and the generator field are so interconnected that variations of the main exciter field rheostat provide the means of controlling the generator voltage. This arrangement results in a considerable saving due to the fact that there are no generator field rheostat losses; in addition, a much more rapid and flexible method of controlling the generator is provided. In case automatic voltage control is considered advisable, this feature can very easily be provided at a comparatively small additional cost.

Excitation. All the generator field exciters, except on unit 5, are connected to a common excitation bus. As all these exciters have excess capacity, this design provides reserve for any of the exciters on the first four generators. Consideration of the facts, however, that generator 5 must be stopped to repair either exciter, because of their locations; that the main exciter will operate self-excited, thus making it possible to operate without the pilot exciter, and also the reliability of present day exciters, led to the conclusion that the expense of reserve excitation was unjustified.

Instrument Transformers. One of the features of the generator installation for which it has always been rather awkward to make a satisfactory design is the instrument-transformer installation for metering and protective purposes. On the first four units installed, the current and potential transformers have been installed in the generator foundations in rather inaccessible places, or in rather unsightly and inaccessible transite enclosures on the side of the generator foundations. On the unit 5 installation, however, an extension of the metal-clad switchgear idea was made. The metering and protective transformers were placed in metal-clad petrolatum-filled compartments in a recess in the side of the generator foundation in such a way as to make a safe, compact, accessible, and well-appearing installation. The metal-clad instrument-transformer compartments were designed with potheads on the lower side ready for the installation of the generator cables.

Generator Cable and Pothead Installations and Insulation

On the unit 5 installation four single-conductor, 3,000,000-circular mil, paper-core, 25-kv. paper-insulated, lead-covered cables are used for each phase. These cables extend from the generator to the 120,000-kva. transformer bank, in fiber ducts. This 16-duct bank was designed in the form of a square surrounding a large, square tile placed in the center of the duct bank to permit air circulation thus equalizing the internal and external temperatures of the ducts.

Special care in design was used to make the cables carry equal load currents and to avoid injurious sheath voltages. The cable phases in the duct have been so arranged that the electromagnetic fields surrounding the cables tend to neutralize each other, thus equalizing reactances and avoiding reductions in carrying capacity.

As it was considered more economical to have induced voltages on the cable sheaths than to suffer the losses due to induced sheath currents, cable-sheath insulators were placed at the ends and at about the midpoint of the 550-ft. fiber-duct run, thus dividing the cable sheaths into two sections. These two cable-sheath sections are grounded at their midpoints, thus making the induced sheath voltage too small to affect the lead-cable sheaths injuriously.

On the other generator installations, short cable runs have been installed between the generators and the oil circuit breakers connecting to the 12-kv. station bus. On unit 4, for instance, this run is installed mainly in the open air in split aluminum tubing suspended from the turbine-room floor. Since conditions were somewhat different for the preceding three units, an ordinary fiber-duct and concrete cable run was used.

As an added safety factor, 17,000-volt insulation on the cable has been used for the units which operate at 12,000 volts between phases, or 6,900 volts to ground. On unit 5, which operates at 18,000 volts or approximately 11,000 volts to ground, 25-kv. insulation has been provided on the cable. Not only has higher insulation than required for rated voltage been used, but very liberal conductor areas have been provided also. On all the 12,000-volt cables, compound-filled potheads were used. On the 18,000-volt unit 5 installation, oil-filled potheads, as well as oil-filled joints, have been used as an added safety factor which should insure maximum reliability in service.

Small Power Cable. As a result of the development of various types of armored cable which have proved quite satisfactory in service, it has been possible to eliminate the very expensive conduit runs which were needed to protect cable of the ordinary type from mechanical injury. Instead, troughs suspended from the ceiling are provided. The armored cables are laid in these troughs much more quickly than cable could be pulled through conduit. As a result of this change in practise, the cost of our cable installations has been materially reduced because of the saving in drafting-room and field work.

Protective Relays

Various types of relays have been used to protect the equipment. The generators, cables, and transformers are protected with differential relays. The transformers

are also equipped with overcurrent-relay protection. Some of the motors are equipped with phase-balancing relay protection and others with overcurrent protection, depending on the particular duty. The 132-kv. line protection consists of impedance relays which operate in conjunction with overcurrent and undervoltage auxiliary relays. This arrangement makes it unnecessary to change current-transformer ratios in light or heavy-load periods.

Outdoor Installations

The outdoor installations at Waukegan have gone through two successive stages of design similar to those involving the indoor equipment. At the time unit 1 was installed, it supplied a 25,000-kva. 12/33-kv. transformer bank, and an open-type main and transfer-bus switching structure. At this time, also, the first 132,000-volt installation was made. This consisted of a 30,000-kva. water-cooled transformer bank provided with an air-break switch only, on the high-voltage side. The associated transmission line extended about 28 miles to a step-down substation.

The second stage occurred in 1927 as a result of the installation of the second 132-kv. line. This necessitated the 132-kv. switching structure and bus, which was installed in conjunction with this unit. At this time, a 60,000-kva. water-cooled transformer equipped with load-ratio control was installed. The steel switching structure was added to as additional lines were needed, and for unit 4, a 60,000-kva. oil air-cooled transformer bank was installed. This bank is also equipped with load-ratio control equipment.

132-Kv. Switching Structure

The 132,000-volt structure has a number of rather unusual features. To conserve space in a north-and-south direction, it was necessary to use more in an east-and-west direction. This permitted placing the equipment for a line and a transformer in a bay 38 ft. 6 in. long from north to south, but much wider than this from east to west. As the main bus is located above the other equipment in the middle of the structure, the structure appears to have three buses. The line and transformer buses, however, are connected so that they form a rectangular "ring" bus with the main bus in the middle. The line and transformer or reserve bus may be sectionalized by means of an oil switch, or by means of load-break disconnecting switches.

132-Kv. Metal-Clad Switchgear

The first 132,000-volt switchgear ever used has been placed in service as part of the unit 5 installation. Two double bus units were installed as shown in Fig. 7, one for switching the high-voltage side of the 120,000-kva. transformer bank supplied by generator 5, and one for an additional 132,000-volt transmission line. The buses of these two units are connected to, and form an extension of, the main and reserve buses in the open-type steel structure. These two double bus switchgear units are arranged as indicated in Fig. 8, and consist of oil-filled cable buses and interconnections, special motor-driven disconnecting switches enclosed in metal housings, oil switches equipped with potheads instead of outdoor bushings, and special bus connections and disconnecting devices for the instrument transformers.

As this switchgear is the first of its kind, the details of its capacity and design features are of interest. The switchgear is rated to carry 1,200 amperes and the oil circuit breakers will interrupt 2,500,000-kva. at rated voltage. The capacity of the buses may be increased to 2,400 amperes by installing another set of buses. The present buses consist of 3,020,000-cir. mil., oil-filled, paper-insulated, lead-covered, copper-armored cable. Isolated racks support the bus cable about nine feet above the switchgear foundations as it loops from switchgear unit to switchgear unit. The ends of each of these bus cable loops are insulated, and the midpoint of each cable sheath in each loop is grounded to keep the induced voltage in the cable sheath at a minimum.

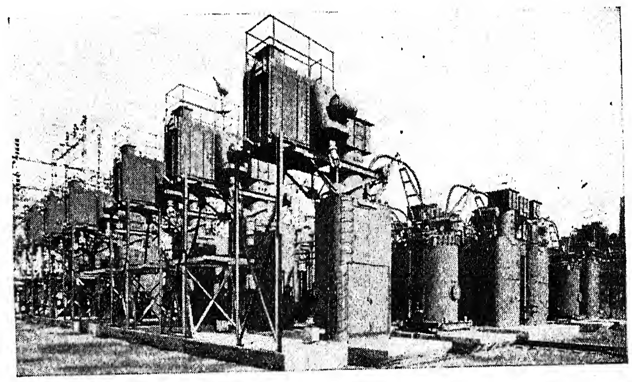


Fig. 7—The 132-Kv. Metal-Clad Switchgear at Waukegan

The disconnecting switch consists of contacts in an oil-filled tank equipped with entrance bushings which are, in turn, enclosed in an outer housing. The tank, as well as the bushing on the bus side, is raised and lowered by a motor-driven mechanism, and overtravel is prevented by limit switches. When the tank is lowered, the circuit is opened under oil and the bus contact on the entrance bushing withdraws from its bus receptacle, thus providing a visual separation, which may be observed from the outside of the housing through a sight glass. The bus receptacle consists of a hollow porcelain shell that forms a part of the main structure into which the entrance bushing on the disconnecting switch tank enters. The opening in the lower end of this porcelain shell is automatically covered by a metal cover when the disconnecting switch is withdrawn. These switches are operated in groups from the oil circuit breaker control panel, or individually at the disconnecting unit.

As it is seldom necessary to disconnect a potential transformer, this disconnecting device is not motor

operated. A portable motor has been provided, however, for raising or lowering the potential transformer "coupler" tank whenever necessary.

132-Kv. Switchgear Safety Features. There is a number of interlocks provided to prevent operating errors and to protect personnel. For instance, one of the interlocks prevents opening the upper door in the disconnecting switch house until the disconnecting switch is closed. An interlock also prevents operating any of the line disconnecting switches unless the line cable test switches are open. Conversely, no line cable test switch can be operated until line disconnecting switches have been opened. An interlock also prevents the lowering of any potential transformer coupler tank unless the ground switch and test switch on that phase are closed. An interlock prevents opening any ground switch or

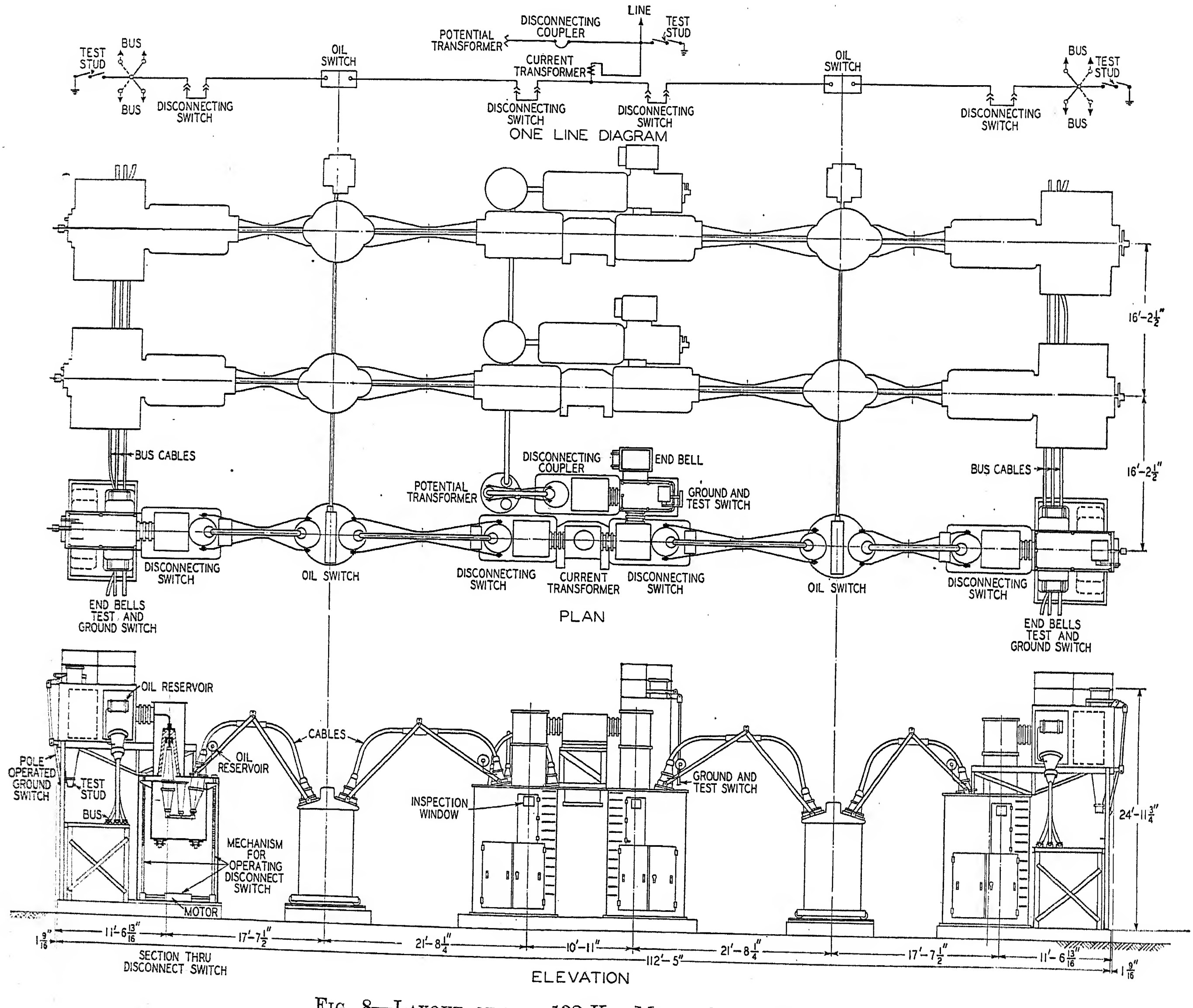


Fig. 8—Layout of the 132-Kv. Metal-Clad Switchgear

Showing a plan and elevation, the internal features of the disconnecting switches, and a one-line diagram of connections which conforms to the physical arrangement of the equipment

opened, which causes the shutter preventing access to live parts to close. Another interlock prevents operation of any or all disconnecting switches on both sides of the breaker unless the breaker is open. This interlock is effective on each disconnecting switch on individual operation, as well as on group control. The breaker cannot be closed unless all the disconnecting switches on both sides of the breaker are fully open or completely test switch until the potential transformer coupler tank is in the connected position. This is accomplished by means of a system of Cory locks.

A-C. Car-Mule Drive

One of the recent additions to the coal-handling system at this station is the coal-car mule, which pushes the loaded cars up an incline to the rotary car dumper. The

special service requirements of the car-mule drive would normally dictate the use of d-c. motors, but, as direct current has not been necessary for the other auxiliaries, except for control purposes, it was decided to avoid its use for this device, if possible. Accordingly an a-c. system was developed which is, perhaps, the first of its type used in this country. Solution of the problems of securing large variations in torque with large variations in speed, of obtaining a flexible control system, and of taking the slack out of the hoisting cable to prevent breakage when the load is suddenly applied, have made the use of a-c. motors possible.

The essential feature of this installation is the use of a slow, two-speed, 440-volt, high-torque motor. Coupled directly to the torque motor is the 500-hp., 2,300-volt, wound-rotor, main driving motor. The drive shaft to which the motors are coupled is geared to the car-mule cable-hoisting and lowering drum.

The control system of the equipment which pushes a loaded coal car up the incline is complex, but to hoist a car automatically, it is only necessary for the operator to move a master control switch to the final point. This closes contactors which put the torque motor on its high-speed winding and thus bring the car mule out of the pit at a speed of 150 ft. per minute. As the car mule approaches the coal-car which has rolled over the hump, a limit switch operates and energizes intermediate circuits that put the torque motor on its low-speed winding, so that the mule engages the coal-car at a speed of 50 ft. per minute. This operation also places the main motor on the line with all its starting resistance inserted. After the car has been picked up, another limit switch operates to energize a contactor which accelerates the main motor. This process continues in six steps until the main motor reaches its maximum speed of about 300 ft. per minute. As the car approaches the car-dumper cradle, the car is slowed down to 150 ft. per minute by inserting automatically all of the main motor resistance, and the torque motor is put on its high-speed winding. If the cradle is properly seated, the car and mule will proceed to the final mule limit; at this point the motors stop automatically and the brakes set. By reversal of the master switch the motors operating the track gates, through which the car mule leaves or enters the pit, change the position of the gates and the car mule is lowered automatically.

Discussion

A. H. Lovell: To one who is not a resident of this territory of Chicago and its adjacent western surroundings, it is very puzzling that Waukegan, built for heavy energy production, in relation to the Chicago load, should have chosen units of only 25,000 kw. in 1923 and 65,000 kw. in 1930. Will the author discuss this in relation to the size of the load in Chicago at those times?

W. S. Monroe: The history of the Waukegan power station from the time the first unit was ordered in 1921, represented, in a large measure, the development of the power station industry in the Chicago District so far as steam pressure and steam temperature are concerned, and the change of boiler furnaces from the chain grate stoker to pulverized coal. Referring to the size of the units, the first two represented the largest units that it seemed desirable to put into the station, considering the load available in the district, and the last three units, Nos. 3, 4 and 5, of 60,000, 75,000 and 105,000 kw. respectively, were each the largest single shaft unit available at the time they were ordered.

115,000-Kw. Turbo-Alternator

BY R. B. WILLIAMSON*

Fellow, A.I.E.E.

Synopsis.—Due to the increasing use of large single-shaft steam turbine units, it has been necessary to design very large generators operating at 1,800 r.p.m. The present paper describes a 115,000-kw. unit recently placed in operation. The generator is wound for 18,000 volts and the whole output of the machine, with the exception of a small part used for the operation of auxiliaries, is stepped up to 132 kv. and all switching is done on the high-voltage side of the transformers. It was possible, therefore, to design the generator for a voltage that would permit a simple two-conductor per slot winding.

On account of the great axial length of the machine, ventilation required careful study and a new scheme of rotor ventilation was developed to secure uniform cooling throughout the length of the long rotor body. The stator ventilation is of the inward-outward parallel flow type with fourteen parallel paths in each half of the stator.

Air circulation is provided by four single inlet motor-driven blowers mounted under the generator. A fin type radiator cooler is mounted under the generator yoke and condensate is circulated through the cooler. Excitation is furnished from a 350-kw., 250-volt coupled, shunt-wound exciter and a 7½-kw. overhung pilot exciter is used to excite the fields of the main exciter.

THE development of large turbo-alternators has in recent years been characterized by the great increase in size of machines for operation at 1,800 r.p.m. The later designs of large steam turbines tend toward single shaft units and consequently the generator has to be large enough to carry the full output of the steam turbine. With units of the compound or triple-

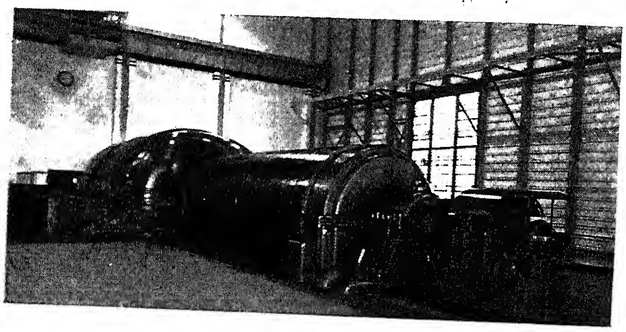


FIG. 1--115,000-KW. UNIT INSTALLED

expansion type, the total output was divided between two or three generators and the maximum size of generator required was considerably smaller than with single shaft units. There has, therefore, been a demand for very large generators operating at 1,800 r.p.m. and the designer has had to solve a number of problems, mechanical and electrical, because of the great axial length of these machines and the large weights involved, particularly in the rotor.

Following is a description of a 115,000-kw., 0.95-power factor, 121,000-kva., 1,800-r.p.m. generator put into operation in September, 1931, in the Waukegan Station of the Public Service Company of Northern Illinois. This generator is wound for 18,000 volts, 3,884 amperes, three-phase, 60 cycles. A general view of the unit as installed in the station is shown in Fig. 1.

*Engineer-in-Charge, A-C. Design, Allis-Chalmers Mfg. Co., Milwaukee, Wis.

Presented at the Great Lakes District Meeting of the A.I.E.E., Milwaukee, Wisconsin, March 14-16, 1932.

STATOR CONSTRUCTION

The stator frame is of welded construction and made in two sections for convenience in handling, the halves being bolted together in a plane, normal to the shaft, at the center of the machine. Fig. 2 shows an end view of the stator core and yoke with the stator laminations in place, and Fig. 3 shows the completed winding. The axial length of the laminated core is 281 in. (714 cm.) between end heads. The punchings are clamped between heavy finger plates of non-magnetic cast steel. In stacking the core, the laminations were pressed down, by means of hydraulic jacks, as the work progressed. The core punchings are of silicon steel having a loss not to exceed 0.75 watt per lb., at a density of 10,000 c.g.s. lines per sq. cm., 60 cycles. A sectional view of the generator is shown in Fig. 4.

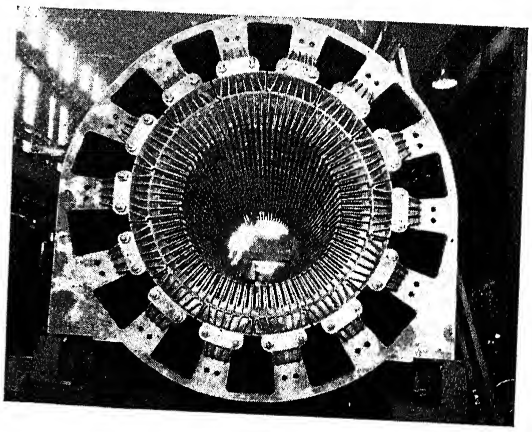


FIG. 2—END VIEW OF STATOR WITH LAMINATIONS IN PLACE

The stator winding is placed in 84 open slots, 21 per pole, and connected in two parallel circuits. In selecting the voltage for this generator, 18,000 volts was decided on after careful study, both from the operating point of view and from considerations of generator design. The whole output of the machine with the exception of a small part used for the operation of station auxiliaries, is stepped up and all switching done on the

high-voltage side of the transformers, hence the voltage of this unit did not have to correspond with that of the older units in the station, and it was possible to wind the generator for 18,000 volts in order to obtain the best type of stator winding. Two conductors per slot give the simplest form of winding and eliminate practically all danger of insulation breakdown between turns. The coils are made in halves, each conductor being built up of asbestos-covered, varnish-treated strands transposed to eliminate stray currents. The half coils are connected together at each end of the stator with solid clips, to form one-turn coils. Since the winding is in two parallel circuits, each conductor carries one-half of the total current output.

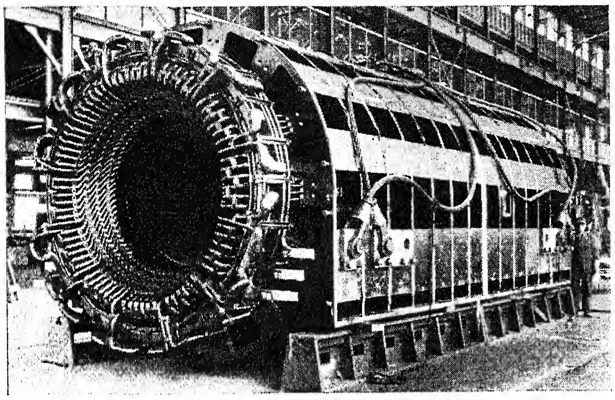


Fig. 3—Stator with Completed Winding

The coils are insulated with mica tape, with asphaltum base binder. The insulation is molded in place in such manner as to secure a dense insulation free from air inclusions. A layer of asbestos tape treated with varnish is placed on the outside of the coils to form a corona sheath. The whole winding was subjected to the standard A.I.E.E. test of 37,000 volts for one minute and there was very little evidence of corona at this voltage. Fig. 5 shows the stator slot and arrangement of conductors. The projecting ends of the coils, Fig. 6, are supported by lashing to insulated, non-magnetic brackets which form ring supports and at the same time avoid closed circuits for stray currents induced by leakage fluxes.

The terminals of the winding are brought out at each side of the end housing so as not to interfere with the air ducts, and also to render them accessible above the floor line of the unit. Line terminals are brought out on one side and neutrals on the other.

Fig. 7 shows the arrangement of the special compound filled terminal boxes on the line side of the generator. These are mounted on the foundation wall directly below the outlet on the generator. Each of the leads is provided with a terminal box in which current transformers for differential protection are mounted. A tap for auxiliary power supply is taken off directly from the main leads as indicated, and a current transformer in the

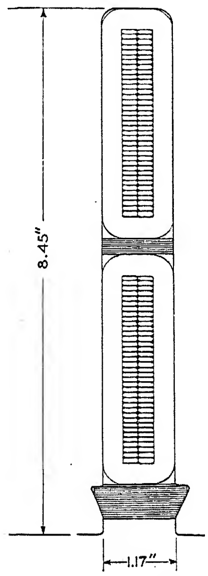


Fig. 5—Stator Slot Section Showing Conductor Arrangement

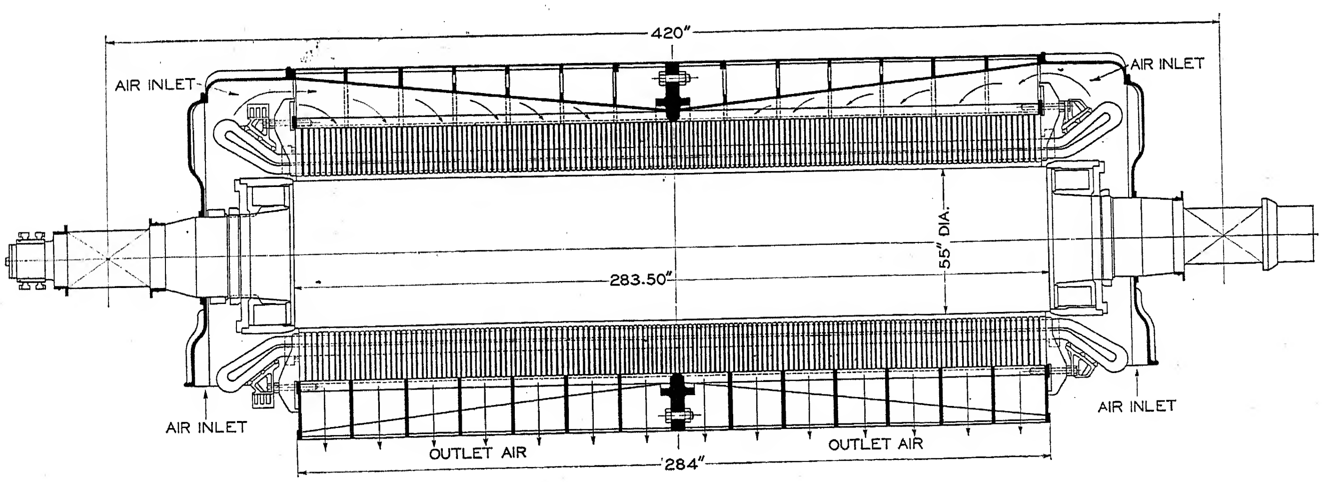


Fig. 4—Sectional View of 115,000-Kw., 18,000-Volt Generator

auxiliary tap is provided in the terminal box together with potential transformers for the machine. The whole arrangement is compact and affords a high degree of insulation. The boxes are filled with soft compound and the potential transformers are oil-insulated. The cable potheads on the terminal boxes are oil-filled, and lead-covered cables are carried down the side wall of the foundation and protected by a casing extending to the basement floor. This casing is made of aluminum plates bolted together so as to be readily removable. The stator end enclosures are of non-magnetic cast iron. The stator with core and windings complete weighs 350,800 lb. net, (159,000 kg.) and was shipped on a special car designed for 400,000 lb. (181,000 kg.) load.

Rotor Construction

The construction of the rotor for such a large high-

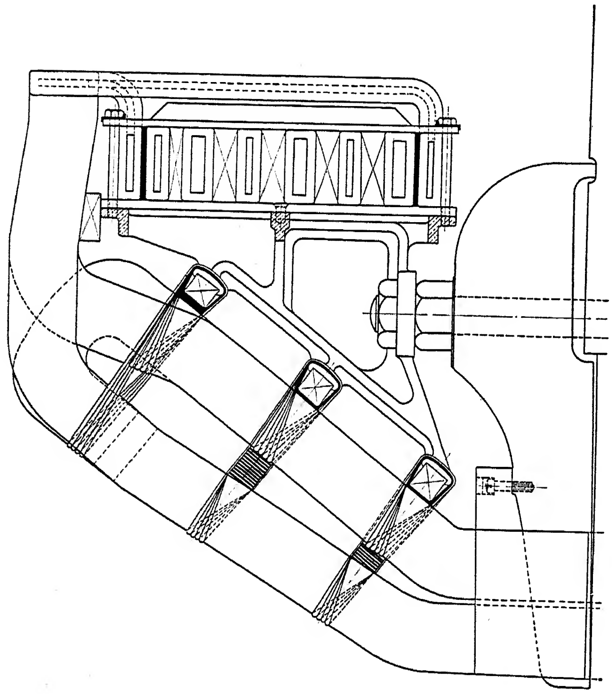


Fig. 6—Method of Supporting Projecting Ends of Stator Coils

speed machine, required a great deal of study as the weight of the finished rotor is 228,650 lb. (103,600 kg.). The weight of the rough turned forging was 240,000 lb. (109,000 kg.) and, before rough turning, 264,000 lb. (119,500 kg.). The forging was made from an ingot 108 in. (274 cm.) diameter, weighing approximately 480,000 lb. (218,000 kg.). The finished diameter of the rotor body is 55 in. (139.6 cm.) and the peripheral speed 26,000 ft. per min. (132 m. per sec.). There was considerable discussion with the steel makers as to the possibility of securing a sound forging of such large size and weight in a single piece. Several alternatives were considered, and it was finally decided to make the forging in one piece, but to arrange the design such that if difficulties were encountered, the stub-shafts could be cut off and stubs bolted to the body, thus making a three-piece construction. Fortunately, this was not necessary as the steel maker produced a single-piece forging which, after careful physical tests and inspection by boring, proved satisfactory in every respect.

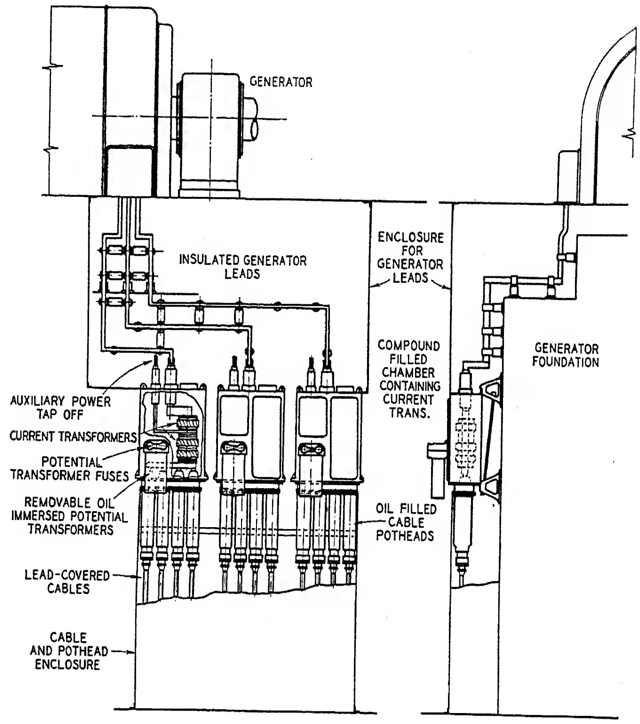


Fig. 7—Terminal Boxes for Stator Leads

The material in the rotor forging is open hearth, carbon steel, with a small percentage of vanadium, thoroughly annealed. The body is approximately 24 ft. (732 cm.) long and the distance between bearing centers 420 in. (1,070 cm.). Fig. 8 shows the rotor body after

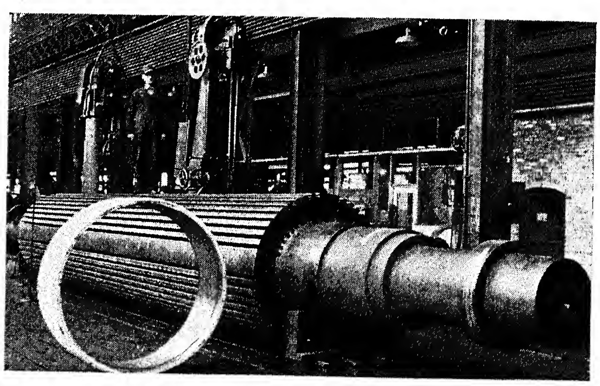


Fig. 8—Rotor Core and Shaft After Slotting

slotting; also one of the coil retaining rings. The rotor is bored throughout its length with an 8 in. (20.3 cm.) hole, the surface of which is polished. The stub shafts are given a smooth finish, the same as on the bearing journals, to remove all tool marks and avoid localized

stresses that might start cracks during long-continued service. For the same reason, all fillets on the rotor and end rings are carefully polished. The rotor end rings are of high tensile strength, non-magnetic alloy steel in order to reduce stray loss.

In general, the rotor construction does not differ materially from that used for similar large rotors. All insulation is of mica and asbestos. On account of the large diameter of the stub shafts it was not advisable to mount the collector rings between the outboard bearing and the generator, as the rings would have to be of such large diameter that the rubbing speed at the brushes would be too high. The rings were, therefore, mounted outside the bearing and leads brought through the shaft.

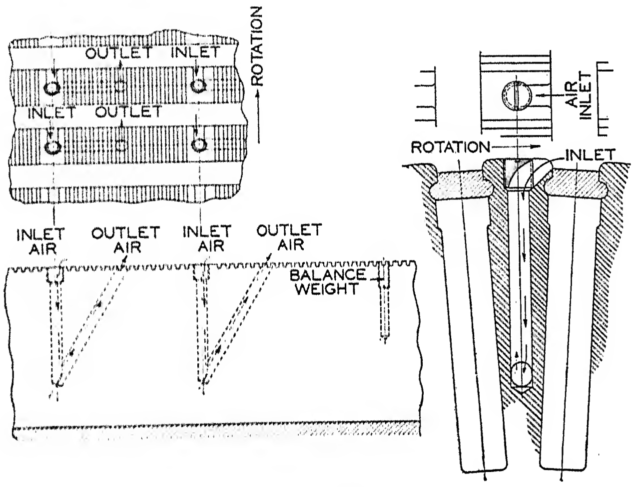


FIG. 9—METHOD OF ROTOR VENTILATION

A new method of ventilation was used on this rotor because, on account of its great axial length, the methods heretofore used would not be effective in ventilating the central part. The usual method of rotor ventilation has been to provide longitudinal channels under the coils, into which air can be drawn from each end of the rotor. Holes drilled down through the teeth connect with these channels and when the machine is in operation, air is drawn in from each end through the channels under the rotor coils and thrown out of the holes by centrifugal action. This method works well for rotors of medium length, but for very long machines it is difficult to secure enough inlet duct area under the coils to supply adequate ventilation, and what ventilation is obtained is not uniform along the length of the rotor body.

In the present case, instead of using the centrifugal effect and feeding air in at the bottom of the slots from each end, a method was devised of taking up air out of the air gap at the circumference of the rotor and utilizing the impact head to force air down through the ventilating holes and return it to the air gap. Air from separate ventilating fans is supplied under pressure to the air chambers at each end of the stator and from there passes into 14 inlets (Fig. 3) spaced equally around the back of the stator punchings. From these chambers it blows inward in a radial direction against the rotor, makes a U-turn and passes out radially through the yoke. Thus, air is blown in against the rotor at 14 points and this air is comparatively cool, as it has traveled only a short distance from the inlet. A small portion of the air is by-passed back of the coils in order to cool the part of the stator core behind the teeth, but the greater part blows in against the rotor. There is thus a strong circulation of air through the air gap at all points throughout the length of the rotor.

The scrubbing action of the surface and the ventilation at the ends of the rotor removes a large part of the heat, but it is also necessary to provide additional ventilation as stated above. Fig. 9 shows the method used in this case. Radial holes are drilled to within a short distance of the roots of the teeth and holes are drilled at an angle as shown, so as to connect with holes at the bottom. Plugs cut away on the under side are screwed into the tops of holes as indicated, and a short channel is cut in the tooth lip as shown, to connect with the slot in the under side of the plug. The plugs are of course locked in position so as to prevent any possibility of turning. When the rotor is rotating, at a peripheral speed in this case of 26,000 ft. per min. (132 m. per sec.) the impact head is sufficient to force air down through the radial holes and out through the holes at an angle. Numerous tests made on models, showed that the air velocity through the ventilating holes is at least 6,000-8,000 ft. per min. (30.5-40.7 m. per sec.). Also tests made with an experimental rotor, with and without these ventilating holes, showed a decided gain in cooling effect with the rotor run in a machine under exactly similar conditions. The ventilation does not depend on any centrifugal action, as the centrifugal effects on the air in the inlet and outlet holes are balanced. With this arrangement, as the length of the rotor is increased, the number of ventilating units and the air carrying capacity are also increased in proportion, and a long rotor can be ventilated as effectively as a short one. It might

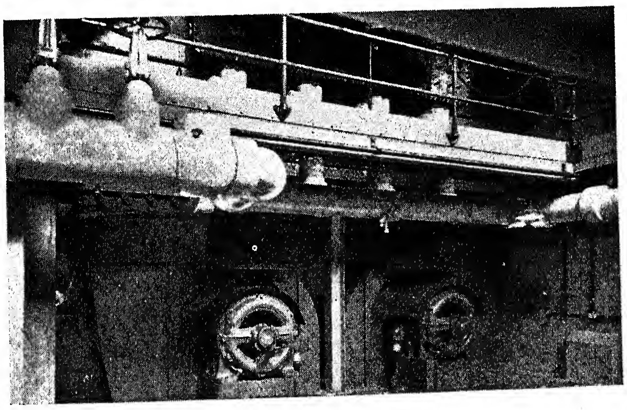


FIG. 10—TWO OF THE FOUR MOTOR-DRIVEN BLOWERS

be thought that this method of rotor ventilation would be noisy, but such has not proved to be the case as the rotor makes no more noise than one provided with the usual centrifugal ventilation.

Air Supply

Air is supplied to the generator by four separate motor-driven blowers mounted within the foundation space, directly beneath the generator and operating on a closed system. Fig. 10 shows two of the blowers as installed, the other two being similarly located on the opposite side of the foundation. Separate fans were used for several reasons. With a rotor of such large weight and length of body, it was necessary to keep the stub shafts as short as possible and the use of self-contained fans with their air inlets adds considerably to the length of the shaft. Moreover, the large shaft restricts the air inlets to such an extent that self-contained fans could not deliver the amount of air required, without excessive inlet velocities and consequent loss in efficiency. Separate fans are more efficient and permit better mechanical design of the rotor, besides being more quiet in operation. Each fan can deliver 50,000 cu. ft. (1,420 cu. m.) of air per minute against 9 in. (22.8 cu. m.) static pressure. The fans operate at 1,160 r.p.m. and each requires a maximum of 104 hp. To allow ample margin, 150 hp. line-start, squirrel cage motors are provided. The fans are single inlet type with runners mounted directly on a shaft extension of the motor. All four fan inlets open directly into the chamber under the coolers into which the air is discharged. With the generator delivering 70,000-80,000 kw. it is found that two fans furnish sufficient air, and four are operated only under full load conditions. Outlet dampers are provided on each fan so that it can be cut off when not in use. Fig. 11 shows the general arrangement of the blowers in relation to the generator and air coolers.

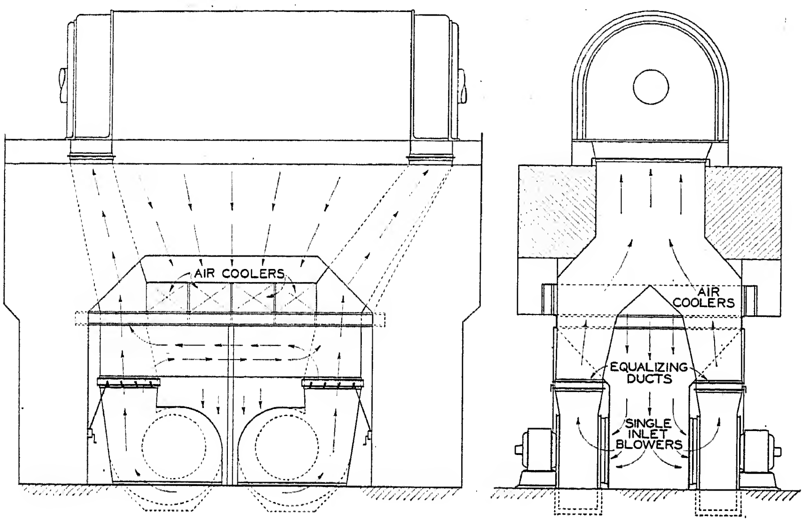


Fig. 11—General Arrangement of Motor-Driven Blowers

For cooling the air, condensate is circulated through a fin type radiator cooler mounted directly under the generator. The cooler is designed to handle 200,000 cu. ft. (5,670 cu. m.) of air per minute with a pressure drop not exceeding $1\frac{1}{2}$ in. (3.81 cm.) water. With cooling water temperature not exceeding 80 deg. fahr. (26.6 deg. cent.), it will absorb 6,985,000 B.t.u. per hour and deliver air to the generator inlets at a temperature not exceeding 104 deg. fahr. (40 deg. cent.).

Excitation

The rotor is wound for 250-volt excitation and current is supplied by a 350-kw. shunt wound exciter coupled to the turbo-alternator. The main exciter is a two-bearing machine with bearings lubricated from the oil circulating system of the main unit. A pilot exciter of $7\frac{1}{2}$ kw. output is overhung on the main exciter. This 250-volt machine provides excitation for the fields of the main exciter.

Characteristics

Fig. 12 shows the open-circuit saturation and short-circuit curves of the generator as obtained from shop test. At the time of preparing this paper, complete tests for efficiency, heating, etc., had not been made, but regular operation on the station load indicates that the temperature rise of the generator at full load is well within the guarantees of 60 deg. cent. rise by detector on the stator, and 85 deg. cent. rise by resistance on the rotor. The estimated efficiency of the generator at rated load, including power required to drive the separate fans is 98.1 per cent.

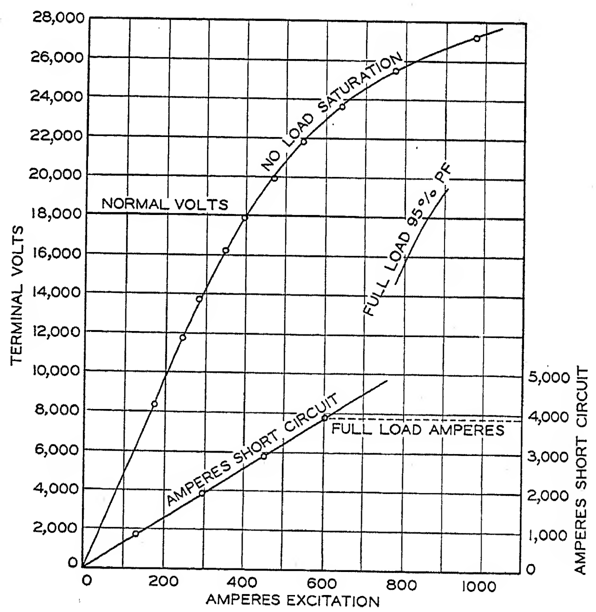


Fig. 12—Open-Circuit Saturation and Short-Circuit Characteristics

Discussion

S. H. Mortensen: If in the design of a machine of the size described in the paper the effect of vibrations, stray fields, heating, etc., could be forgotten it should be possible to determine its most economical all around design from a careful analysis of the relative cost of labor and material involved, combined with the operating cost per kilowatt after installation. From such an analysis the corresponding generator efficiency could then be derived to serve as a guide for the design.

However, vibrations, stray fields, heating, etc., if forgotten are apt to become exceedingly real and furthermore as present day tendencies combined with competition have made it necessary

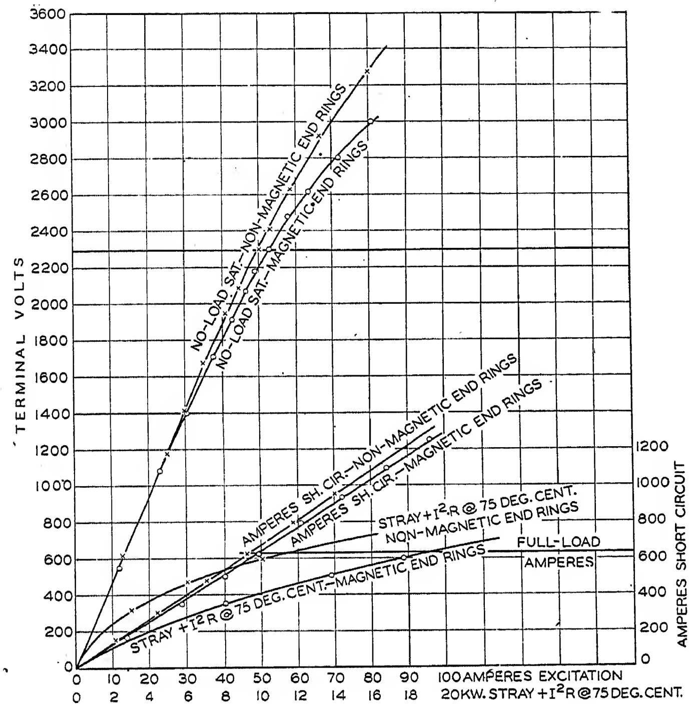


FIG. 1—NO-LOAD SATURATION AND SYNCHRONOUS IMPEDANCE CURVES AND STRAY LOSS

On a 2,500-kva. turbo-generator tested with magnetic and also non-magnetic coil support rings

to give efficiency guarantees which approach those of transformers, it becomes incumbent on the designer to combine his best skill with that of the best material available for the generator construction. For the machine in question this has resulted in the use of silicon core steel and non-magnetic material wherever practicable. The use of silicon steel which is brittle when cold, makes it necessary to omit sharp corners from slots and dovetails and to provide core clamping devices which eliminate the danger of vibrations and loose core iron. Fig. 2 of the paper shows that this machine has sectionalized core clamping plates. As each section only covers 1/14 of the circumference they can be pulled up independently and compensate for variations in core length. To reduce stray losses the clamping plates and the spacers on the teeth of the center ventilating segments are made from non-magnetic material as are also the stator end covers. To further reduce stray fields and losses the material in the stator coil support rings is steel rendered non-magnetic by alloying it with manganese, nickel and chromium. On physical tests, this material had an ultimate strength of 135,000 lb. per sq. inch, a yield point of 92,000 lb. with an elastic limit of 56,000 lb. Its reduction in area was 42 per cent and the elongation 36 per cent. The effect of substituting non-magnetic for magnetic coil support rings on the leakage fields and stray losses of a 2,500-kva., 3,600-r.p.m. generator is depicted in Fig. 1. The respective curves show that fewer field ampere turns are required to generate a given voltage or short-circuit current with non-magnetic rings, which, interpreted in kva. means a larger output for a given rotor heating. In addition, the short-circuit losses with non-magnetic rings were only 58 per cent of those corresponding to magnetic rings. This, of course, means a decrease in the operating cost which, under circumstances may justify the increased first cost inherent to the use of the more expensive non-magnetic material in the support rings.

In connection with the operating voltage of the 115,000-kw. machine it is of interest to note that it was chosen so 1-turn coils could be used in its armature windings. It would seem that in installations where generator and step-up transformers can be considered as a unit the value of the generator voltage should be chosen to give the most favorable generator design, irrespective of voltage standards.

Prior to adopting the rotor ventilating scheme proposed by Mr. Williamson and shown in Fig. 9, a number of tests was made on models and also on a 4,500-kva. machine without and with the impact ventilating holes. The respective tests showed the new scheme to give better cooling than obtained with any of the schemes previously used, such as ventilating ducts in the rotor body, axial ducts under the rotor coils, axial slots in the pole center, etc., combined with the grooving of the rotor surface which Mr. Williamson introduced prior to 1911, to increase heat radiation from turbo-rotors. Curve *A*, Fig. 2 shows the rotor heating before, and curve *B* the results obtained after the surface on the 4,500-kva. rotor had been finished along the lines shown in Fig. 9. The respective curves in Fig. 2 show that for 85 deg. cent. rise the new method of ventilation increases the rotor ampere turns 8 per cent. It also increases the volume of generator ventilating air and the rotor windage loss. The effectiveness of the impact method of ventilation increases with the axial rotor length and a preliminary test on the 115,000-kw. machine indicates its radiating capacity to be 15 to 20 per cent higher than the best results previously obtained on shorter rotors.

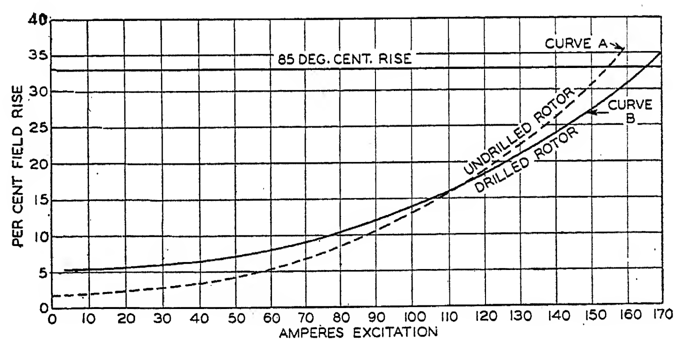


FIG. 2—ROTOR HEATING ON A 4,500-KVA. TURBO-GENERATOR BEFORE AND AFTER ITS SURFACE WAS DRILLED

The short-circuit ratio of the 115,000-kw. machine is less than that of the smaller machines previously installed in the Waukegan station. The short-circuit ratio of a machine does not materially alter its efficiency, but if no other restrictions are involved, it does affect its size and cost and should for that reason be kept as low as experience and careful analysis of system characteristics will permit. For generators operated with full field excitation, a short-circuit ratio of 0.5, as has been used in Europe, or even less would probably give ample stability on metropolitan systems. Further for equal stability a double winding generator could have a lower short-circuit ratio than a single winding machine as it is less affected by disturbances.

The heavy weight and large diameter of the forging required for the rotor of the 115,000-kw. machine taxed the art of steel

making to the limit. The loss of such a forging due to a blemish would be a serious matter and increase the cost of the machine. Until further progress is made in steel making it seems economically necessary for still larger rotors, to develop a construction reducing weights and diameters of the forgings for the component rotor parts.

Carl J. Fechheimer: When considering the construction of a machine of the large output and high speed described in this paper, it is well to regard the limitations imposed. It is well known that the limits are in the rotor, and they are imposed by mechanical stresses, critical speed, and temperature rise. The use of the better grades of steel, annealed or heat treated, with greater assurance of homogeneity, has warranted the adoption of higher stresses. Annealing or other heat treatment of the steel insures having material of ample ductility so as to avoid failure, if the elastic limit is exceeded at parts where there is still liable to be stress concentration, and plastic flow then secures more uniform stress distribution. Although the author does not so state, in all probability the running speed is above the first critical speed, and as this practise has been common for two-pole, 60-cycle machines of the larger outputs, there is no reason why the same practise should not be followed for four-pole machines. It is important to consider that the influence of the foundation may be to lower the critical speed, and there is danger of encountering a higher critical speed near the running speed.

The scheme adopted for rotor ventilation is ingenious. Determination of the air volumes that flow through the ventilating holes by means of models is in accordance with the desirable methods in use when such a complicated phenomenon as air flow is to be studied. Model tests are, however, not always easy to make in such a way that the conditions in the machine are completely simulated, and measurements are ofttimes difficult and may lead to false conclusions. As the author has not described the model and methods of measurement, it is difficult to comment at length. In this case it is well to consider, in addition to the flow in the rotor ducts as such, the effect of the variable back pressure in the air gap, and the loss of head at emerging, which is greater than the velocity head of the discharged air streams. If they have not been given due attention, the velocity may be lower than the values given of 6,000 to 8,000 ft. per minute.

It is believed that if consideration is further given to the flow of heat in the steel and copper, and to the rates at which heat can be transferred from the ventilating holes and external surfaces to the moving air streams, the temperature rise of the embedded copper should admit of a fair degree of accuracy in calculation. The engineer should also be in position to decide upon the best number and proportions of holes for economical design.

It is well to consider also the importance of ventilating the end windings of the rotor adequately. Cases have arisen in which the length of the outer coils is sufficiently great to cause large temperature differences when the ends were not ventilated. It is of interest to note that lower temperatures can be secured by reducing the sections, thereby raising the air velocities, even though the air volumes are reduced somewhat thereby.

As the rotor temperature rise depends upon the losses, it is interesting to note that the short-circuit ratio has been reduced about 0.67. This is in accordance with present tendencies in design. It is doubtful, however, whether a further reduction is warranted, as there would be but little reduction in excitation, and other effects, such as stability, must be given adequate consideration.

Philip Sporn: Was the use of hydrogen given any consideration in connection with the design of the machine in question? It seems to me that in a machine of this size there was a considerable margin to work on in order to make hydrogen justifiable. Besides the undoubted improved performance of an 18,000-volt winding due to the use of hydrogen, it would seem as if there was a possibility in this case of raising the efficiency to possibly as high as 99 per cent. If this was the case, it would have increased the rating of the machine by well over 1,000 kw., and an additional 1,000 kw. would obviously justify considerable extra expenditure. If hydrogen was considered, it would be interesting to obtain the estimated increase in cost and also the estimated increase in efficiency.

R. B. Williamson: Mr. Mortensen's discussion brings out some of the experiments and investigations upon which the design of the 115,000-kw. generator was based, and serves, for that reason, as an amplification of the paper. His remarks pertaining to the short-circuit ratio of turbo-generators are of interest and warrant consideration.

In answering Mr. Sporn's question with regard to hydrogen cooling, it can be said that the advantages of this type of ventilation were not considered of sufficient importance to warrant its use in this installation.

Mr. Fechheimer is correct in his assumption that the rotor design of the 115,000-kw. generator is such that its first critical speed is below its operating speed. This practise has been followed for the last 20 years by the Allis-Chalmers Manufacturing Company, not only on the 3,600-r.p.m., but also on the 1,800- and 1,500-r.p.m. generators and has given good results. In no case have operating difficulties been experienced on that account. With further increases in rotor length and weight, the danger that the second critical speed of the combined rotor, rotor bearings, and generator foundation may coincide with the running speed of the rotor, makes a most careful study necessary of the design and proportioning not only of the generator rotor but also of its bearings and foundation.

We agree with Mr. Fechheimer that it is difficult to make a model test for rotor ventilation which anywhere nearly represents actual conditions. Although the model mentioned in the paper was no exception to this, nevertheless the air velocities and general results obtained seemed sufficiently encouraging to warrant the application of the rotor ventilating system, described in the paper, to the rotor of the 4,500-kva. machine mentioned in Mr. Mortensen's discussion. The test data obtained on the 4,500-kva. machine were used as a basis for the calculations of the rotor heating on the 115,000-kw. machine. Subsequent tests bear out that the advantages of this method of rotor ventilation increase with the axial rotor length. The results thus far obtained, indicate that this type of ventilation is adequate to cool rotors of considerably greater axial length than the rotor for the 115,000-kw. machine.

The Mercury Arc Rectifier Applied to A-C. Railway Electrification

BY OTHMAR K. MARTI*
Member, A.I.E.E.

Synopsis.—A new commutatorless motor with series characteristics has been developed and is briefly reviewed. The motor is particularly suitable for traction purposes and can be supplied from an overhead trolley at the usual a-c. electrification voltages and any commercial frequency.

It is shown how it is possible, by the use of a grid-controlled rectifier, to eliminate not merely the commutator of the motor, but practically all of the expensive control, switching, and reversing equipment commonly used on a-c. and d-c. locomotives. Regenerative braking is also made inexpensive and practicable.

The first part of the paper explains the general theory of the grid-controlled rectifier which is necessary for an understanding of the commutatorless motor.

The data and layout of a 1,000-hp., 50-cycle, 15,000-volt, single-phase locomotive, now under construction, are given.

* * * * *

Introduction

THE extensive research work carried on for improving the mercury arc rectifier has led not only to improvements in its operation but also to entirely new applications of the device, and to new applications of the valve action on which its rectifying property is based.

In recent years considerable experimental and development work has been carried on with rectifiers provided with control grids, utilizing the grids for interrupting short circuits and backfires, for d-c. voltage control, for the conversion of direct current to alternating current, etc. In this paper is described a new application of grid-controlled mercury arc rectifiers in connection with a new single-phase motor for a-c. railway service. By the use of the rectifier it was found possible to eliminate the commutator of the motor and practically all the expensive switching and control equipment commonly used on a-c. locomotives, which will result in considerable simplification of the control, and a reduction in cost.

The experiments carried out on a locomotive equipment with this type of motor have demonstrated the practicability of this scheme. Besides the advantages from the point of view of locomotive operation, this scheme makes it possible to use a power supply of any available frequency, whereas at the present time frequencies of 25 cycles or less are generally required. While it is too early to predict to what this development may lead, the marked advantages of a locomotive equipped with motors of this type, which are already apparent, may revolutionize the practise of railway electrification.

The Principle of Action of Energized Grids

The phenomena and characteristics of the mercury vapor arc in vacua have been treated in great detail in various scientific journals. Briefly stated, in a tank containing highly rarefied ionized gases only a small potential in a given direction is required to cause current to flow between two electrodes of certain types, while a much larger potential must be applied to the electrodes in the opposite direction before an appreciable current flows.

Fig. 1 shows an evacuated vessel with two electrodes: an electron-emitting electrode, c, called the cathode, and an electrode which does not emit electrons, a, called the anode. The cathode can be made to emit electrons by raising its temperature, by imposing a sufficiently high voltage between the anode and the cathode, or by striking an arc by contact. By using a highly evacuated tank, and by using mercury for the cathode material, a mercury arc power valve is obtained, of a high current-carrying capacity.

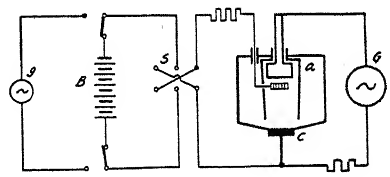


Fig. 1—Elementary Diagram of a Grid-Controlled Mercury Arc Valve

This mercury arc valve can be made useful for many purposes if equipped with a control grid which is introduced into the path of the arc. This grid is insulated from the anode a, and may be energized from an outside source of potential, such as a battery, auxiliary transformer, generator, or a combination of these sources. Non-energized grids or screens are also used to a large extent in mercury arc rectifiers for a number of purposes.*

In all considerations stated below it will be assumed that the cathode is brought into a condition such that it emits electrons. If the battery B is so connected to the grid as to make it positive with respect to the cathode, then current from the generator G will flow from the

*Engr.-in-Charge, Rectifiers and Rly. Equipment, Allis-Chalmers Mfg. Co., Milwaukee, Wis.

Presented at the Great Lakes District Meeting of the A.I.E.E., Milwaukee, Wisconsin, March 14-16, 1932.

*See Marti and Winograd, "Mercury Arc Power Rectifiers," (McGraw-Hill), pp. 42, 227, 232, 397.

anode a to the cathode c. Reversing the polarity of this applied potential, so that the grid is negative with respect to the cathode, will prevent any current from flowing from the anode to the cathode, provided the current is not already flowing when the negative grid potential is applied.

If the sequence of the changing of the grid potential is according to curve 1-2-3-4-5-6-7-8 in the lower part of Fig. 2, and the voltage e_g of the generator G of

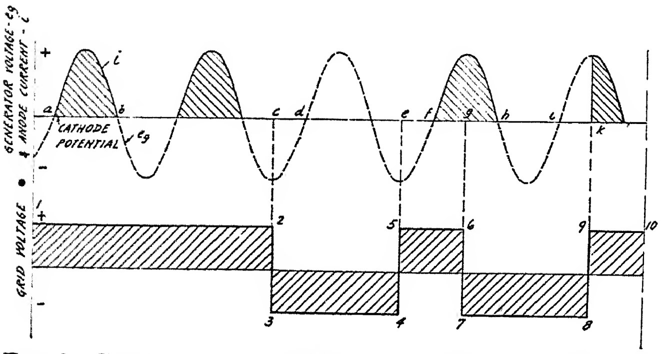


Fig. 2—Control of Arc by Means of Positive and Negative Grid Potentials

Fig. 1 is as shown in the upper part of Fig. 2, then the operation of the valve is as follows:

Current will pass during the interval 1 to 2 every time the impressed generator voltage is positive. Since the grid is negatively charged from 3 to 5, no current will flow when the generator voltage becomes positive at d. However, current will start to flow at f, as the grid voltage is positive between 5 and 6, and will continue to flow although the grid voltage is made negative at g; in other words, current will continue to flow as long as the anode remains positive, irrespective of the potential

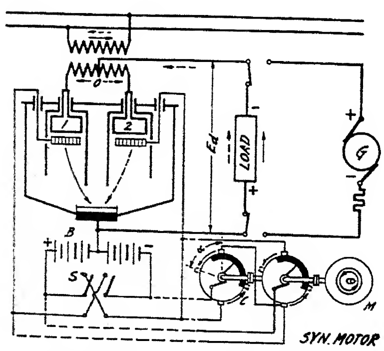


Fig. 3—Controlled-Grid Valve That Can Be Used Either as a Rectifier or a D-C. to A-C. Converter

of the grid. Due to the fact that the grid voltage is negative from 7 to 8, current can start to flow only at k, and will continue as long as the anode is positive; the anode therefore passes current only during one-half of the positive alternation.

Applying the above considerations to a simplified two-anode rectifier, as shown in Fig. 3, the working of energized grids can readily be followed, and the many possibilities of such a controlled rectifier become evident. Let the primary side of the transformer be impressed with a sinusoidal a-c. voltage and let the grids be non-energized. Then the anodes will be positive during alternate half-cycles. During the half-cycle that anode 1 is positive, current will flow from anode to cathode through the load circuit, as indicated by the solid arrows. When anode 2 is positive, current will flow from anode to cathode through the load circuit, as shown by the dotted arrows. The current therefore flows through the load circuit in the same direction during both halves of the cycle. The same rectifying action will take place if a positive potential is applied to the grids during the intervals when the voltages of the corresponding anodes are positive. This could be accomplished by throwing the switch S (Fig. 3) from one position to the other after each half-cycle, making the grid of anode 1 positive during the half-cycle when anode 1 is positive, etc., or by rotating the commutator C at synchronous speed by means of the motor M.

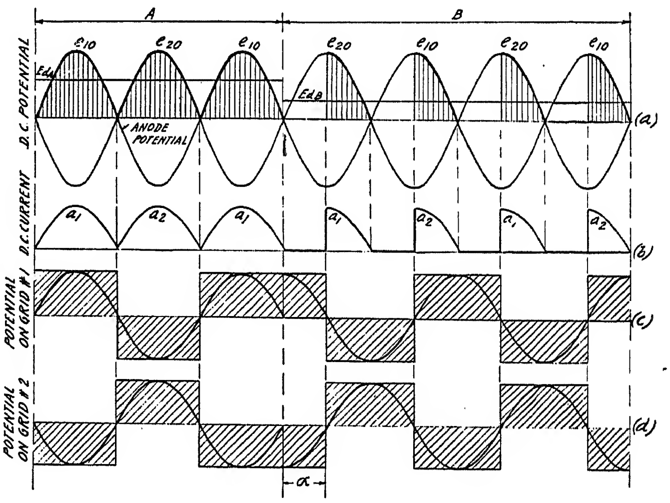


Fig. 4—Voltage Regulation and Control by Means of Energized Grids

This is illustrated in Fig. 4. Section A shows the anode voltages e_{10}, e_{20}. The shaded portions represent the d-c. voltage wave (neglecting the arc drop). Section B represents the d-c. wave for a resistance load. Sections marked c and d indicate the potentials applied to the grids; the shaded blocks represent the potentials if direct current is used, with a reversing or rotating switch, as shown in Fig. 3, and the sine waves represent the potentials if alternating current is used.

In A of Fig. 4 is shown the condition when the grids are made positive at the instant when the corresponding anodes become positive. Each anode then operates during half a cycle, and the average d-c. voltage is shown by line E_{dA}. In B of Fig. 4 is shown the condition obtained by shifting the brushes of the rotating switch C, Fig. 3, through the angle α. The grid potential then becomes positive α electrical degrees after the anode potential, and the point at which the

anodes start firing is delayed by the angle α, which reduces the d-c. voltage to the average value E_{dB}.

It can therefore readily be seen that by shifting the brushes of the commutator C, that is, by changing the timing with respect to the voltage of the anodes, the rectifier can be stopped from passing current, without interrupting the primary supply (see Fig. 2, interval 3 to 4), or the voltage can be changed from the maximum value E_{dA}, to a value E_{dB}, or as low as zero. The source of potential for energizing the grids, the battery B, can be replaced by an a-c. source, which could be an auxiliary transformer, a small generator, or the like.

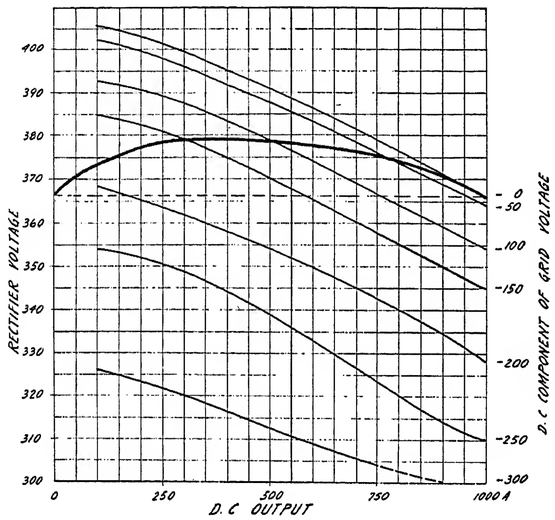


FIG. 5—GRID-CONTROLLED SIX-PHASE RECTIFIER (WITHOUT INTER-PHASE TRANSFORMER)

• Six-phase voltage superimposed on a d-c. grid potential, lagging anode voltage by 60 deg.

For controlling the grids of power rectifiers only a fraction of an ampere per anode is required. It therefore follows that the arrangement for controlling large units makes use of very simple and inexpensive equipment, doing away with much costly control, regulation, and protective apparatus. These advantages will become particularly evident in the description of the new motor and locomotive.

VOLTAGE REGULATION AND COMPOUNDING OF COMMERCIAL RECTIFIERS

Fig. 5 shows some experimental curves obtained by imposing a voltage on the grids of a six-phase rectifier. The diagram of connections is shown in Fig. 6. The grid voltage consisted of a negative d-c. voltage, on which was superimposed a six-phase a-c. voltage lagging the respective anode voltages by 60 degrees. The rectifier transformer was connected in six-phase star, without an interphase transformer. The light lines represent the regulation curves of the rectifier with the d-c. component of the grid potential varying from zero to 300 volts. The steep regulation curves are characteristic of this type of rectifier transformer connection.

The heavy line has been constructed from calculations based on the experimental regulation curves, and shows the regulation curve that would be obtained if the d-c. component of the grid potential were decreased uniformly as the load on the rectifier increased. As can be seen from this curve, such a control of the grid potential would give a flat-compounded rectifier, with a regulation curve very similar to the type obtained by flat-compounded generators. The high peak on this curve is of course due to the high regulation obtained with this particular transformer connection.

It is quite obvious that other combinations of d-c. and a-c. potentials can be applied to the grids, so as to obtain automatically almost any form of regulation desired. The broken horizontal line shows a regulation curve that could be obtained by the proper control of the grid potential. Such a curve could be obtained by varying the d-c. potential, the a-c. potential, or the phase angle of the a-c. potential on the grids with respect to the anode voltage. The d-c. voltage of the rectifier can be controlled as a function of the d-c. load, or could even be maintained constant, irrespective of any fluctuations of the a-c. voltage.

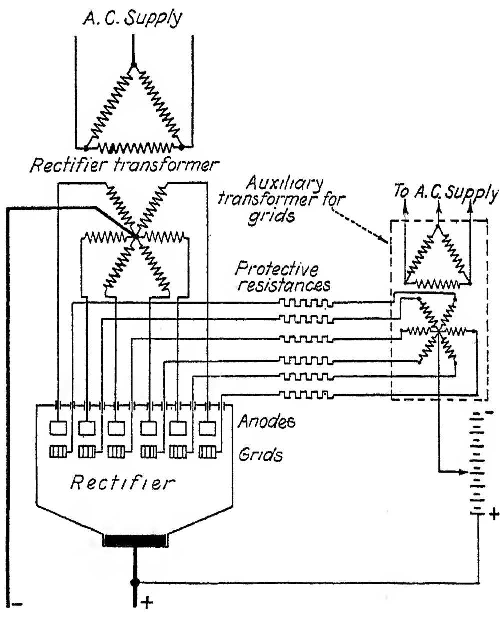


FIG. 6—CONNECTIONS USED IN OBTAINING CURVES SHOWN IN FIG. 5

Fig. 7 shows a 3,000-kw. mercury arc rectifier of standard design with bushings and connections for grid control, at the West Allis Works of the Allis-Chalmers Manufacturing Co.

INVERTED RECTIFIER

In the arrangement shown in Fig. 3, let us replace the load by a d-c. generator. If a positive voltage is applied to the grid of anode 1, current will flow through the left-hand portion of the secondary winding of the transformer, inducing a voltage in the primary winding.

After a certain time interval let the grid of anode 2 become positive, and current will start to flow through the right-hand portion of the secondary winding of the transformer. Let us assume that at the same time the current in anode 1 can be reduced to zero by some means or other, then the rectifier and associated equipment would be nothing else than a converter feeding power from the d-c. into the a-c. network. Although the flow of current from generator G through the anodes can be

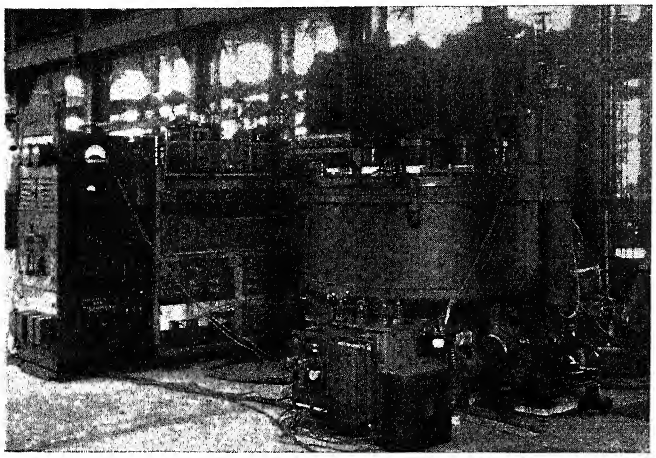


Fig. 7—3,000-Kw. Mercury Arc Power Rectifier for Railway Service on Test at Allis-Chalmers Plant

controlled by energized grids only as to the time of starting, other methods have been developed for reducing the anode current to zero in the proper sequence to obtain inverted operation of a conventional rectifier so as to convert direct current into alternating current.*

This feature opens up great possibilities for the now fully developed mercury arc rectifier, and many new applications for the mercury arc valve. One of the most important of these applications to become practicable will be described in the next section. It will be seen how the controlled valve action and voltage control have been applied to the design of a commutatorless motor with series characteristics, and how the inversion of the rectification process has been applied to make possible regenerative braking of a locomotive equipped with such motors.

Commutatorless Single-Phase Railway Motor for any Characteristic and any Frequency

For some time portable substations equipped with mercury arc rectifiers have been built abroad and in this country. As far back as 1914 a 250-hp. motor car, equipped with a mercury arc rectifier, was in service in this country, on the New York, New Haven and Hartford R. R. As some experience is therefore already at hand, it will not be a novelty to put a rectifier on a locomotive and use it as a mobile electric power converter.

During the last few years the mercury arc rectifier has influenced the electrification of railroads to a very great extent, because of its ability to convert a-c. to d-c. power at high voltages and with a high degree of economy and reliability. Again, it is the rectifier which may give another impetus to railroad electrification, for, as indicated in the subtitle to this section, it is now possible, by the use of a commercial mercury arc rectifier equipped with the grid control described above, to build a commutatorless motor which does not require the elaborate and costly switching, reversing, and control equipment used at present. Furthermore, such a motor can be used with current of any frequency, whereas up to the present time it has been necessary to supply electrified railroads using a-c. power with current at a frequency of 25 cycles or less. This fact may be of particular interest to central station companies, as they would be able to supply power to railroads from their 60-cycle power circuits, without the costly frequency changer and 25-cycle transformer equipment.

Single-phase a-c. motors for traction purposes can be divided into two main classes. The first class comprises motors whose speed is dependent on the frequency of the current supply. These motors lend themselves readily to high voltages but they cannot be given a continuous speed regulation. The second class includes commutator motors whose speed can be regulated, but which are limited to comparatively low voltages and frequencies, due to the presence of the commutator. The commutator also makes the construction of the motor and its control very expensive, and involves the troublesome difficulties inherent in commutation.

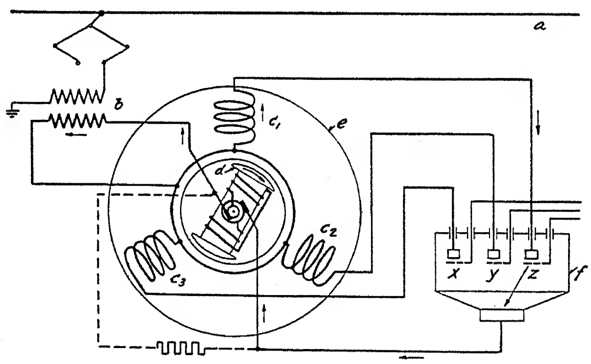


Fig. 8—Elementary Commutatorless Motor

In a motor of the second class, it is possible to replace the commutator by a series of electric valves, *viz.*, a mercury arc rectifier, by the use of which the above-mentioned disadvantages are obviated. As will be seen below, this scheme can be applied to single-phase motors so that relatively high voltages can be used, thus reducing the size and weight of the copper used in the connections. At the same time, a very desirable speed-tractive effort characteristic can be obtained. Further-

*See Kern, *Bulletin Schweizerischer Elektrotechnischer Verein*, 1931, p. 538.

more, all expensive control, switching, and reversing equipment is dispensed with, while only a very simple and cheap grid control is added. As explained above, only a very small current is needed for the grid control, and the cost of the equipment is naturally low.

In Fig. 8, a designates a single-phase a-c. supply line, and b a single-phase transformer. The transformer is connected to the stator winding c and rotor d of the motor e, both of which are connected in series, using two slip rings. The rectifier f should be located as near as practicable to the motor. It is equipped with grids and control, as shown in Fig. 3. The motor is shown as a salient pole machine, but does not have to be such. In the following description it will be assumed that the field of the motor is rotating, and that the armature is stationary. It is evident, however, that this could be reversed. Each armature winding is connected to one anode of the rectifier, each anode being provided with control grids for regulating the flow of current.

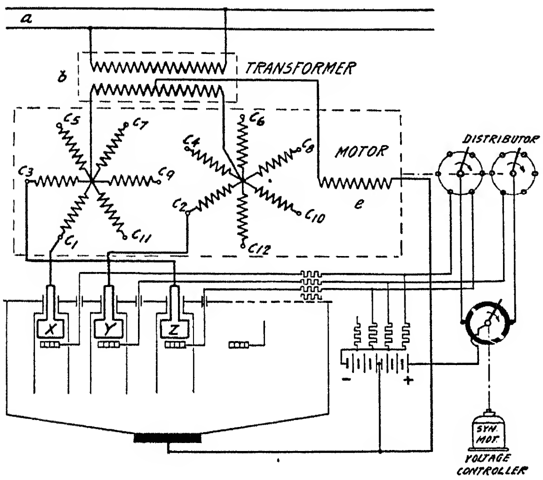


FIG. 9—COMMUTATORLESS MOTOR UTILIZING BOTH HALVES OF VOLTAGE WAVE

In operation, let it be assumed, first, that the rectifier equipment is in the position shown, and that one grid (of anode z) is positively energized through the distributor. This grid accordingly permits current to flow through the anode which it controls. All other grids will be negatively energized thus preventing current from flowing through their anodes. As the primary winding of transformer b is energized from the line, a half-wave current will flow through the winding of the rotor d and through c_1, as indicated by the arrows. As can readily be seen, the field produced in the windings c_1 and d will produce a torque which causes the field d to rotate and reach, with respect to winding c_2, the position which it previously had with respect to winding c_1. At the same time, the shaft of the motor e brings the brushes of the distributor to such a position that the grids of anodes z and x are now negatively energized, while the grid of anode y is positively energized, permitting current to flow from the anode y into the winding d, and through c_2. The above process is repeated in sequence for all windings of the stator, thereby producing a continuously rotating motion of the rotor.

It can be seen from the above that the motor simply constitutes a load for a conventional rectifier f, which may therefore be provided with all improvements known to the rectifier art. In particular, the voltage impressed on the stator windings may be regulated by any of the various means of voltage regulation used with rectifiers, and which have been described above. This voltage regulation can be used here, the voltage being reduced to a low value when the motor is started, and later on being adjusted for purposes of speed regulation. There are also other means for changing the speed or the characteristics of the motor, by a control of the anode grids.

As stated above, this arrangement utilizes one-half of the applied voltage wave. The arrangement shown in Fig. 9 will permit the utilization of both halves. The regulation of this motor is also effected by means of energized grids, and the speed can be varied and the characteristics altered by the same means.

The two motors mentioned above both have series characteristics which are desirable for traction purposes. In the first motor, a resistance could be connected across the stator winding, so that the motor can be given the characteristics of a repulsion motor. It is also possible to connect only the rotor to the source of power, in which case the stator circuits are closed only through the rectifier, and the stator currents are obtained by induction through the rotor. The stator, being then no longer connected to the circuit, can be wound for whatever voltage is found most desirable.*

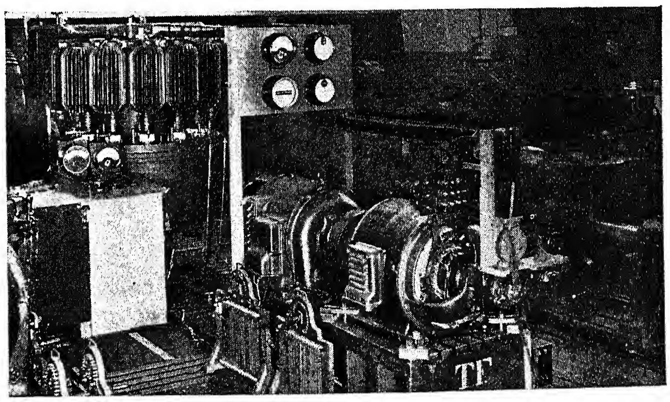


FIG. 10—COMMUTATORLESS RECTIFIER MOTOR ON TEST

The operation of the transformer, rectifier, and motor, does not require a source of any definite frequency. This makes it possible to operate the unit with power from either 25-cycle or 60-cycle commercial lighting and power circuits. Moreover, if properly designed, a single motor can be operated on lines of different frequencies. It may even be possible to omit the transformer, and feed the motor directly from the line, through the recti-

*See Kern, *Elektrische Bahnen*, Nov. 1931.

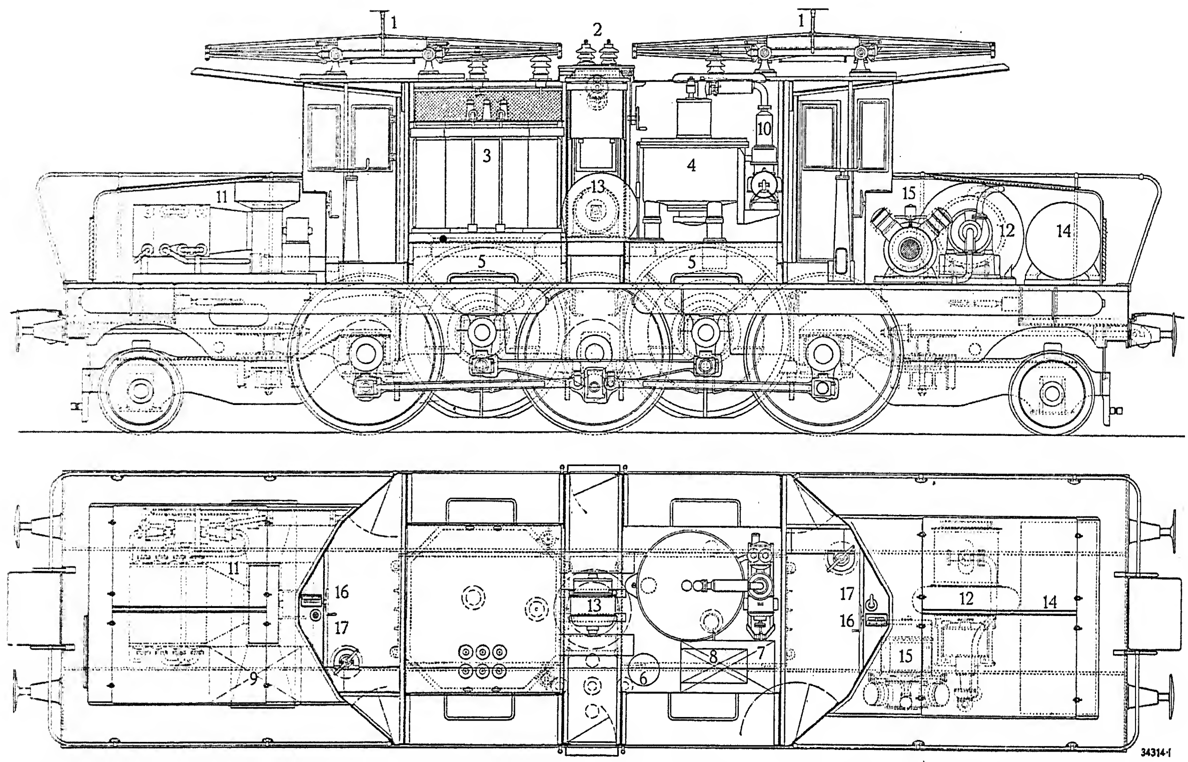


Fig. 11—General Layout of Single-Phase Test Locomotive with Rectifier Grid-Controlled Motors

1. Current collector
2. Main switch
3. Transformer
4. Rectifier
5. Driving motors
6. Control button
7. Central controller
8. Contactor
9. Motor switch
10. Vacuum pump for control valve
11. Cooling set for control valve
12. Cooling set for transformer
13. Fan for driving-motors
14. Converter for auxiliary services
15. Air compressor
16. Driving controller
17. Reversing switch for running and recuperation

fier. This would result in an arrangement particularly desirable for suburban lines using multiple-unit cars. Only one rectifier is required to control all the motors of one locomotive or motor car. We therefore have a very simple arrangement with the greatest possible range of control, namely, from zero to maximum, using a very light and inexpensive grid-control equipment, and without the least degree of energy loss.

With a suitable arrangement of apparatus, this type of motor can also be used for regenerative braking while the locomotive or car is proceeding down grade.

Fig. 10 shows the first experimental model of such a commutatorless motor directly coupled to a generator as load, and with a commercial rectifier of very large capacity. The control hand-wheel is visible on the right.

In this connection it is of interest to note that Brown, Boveri & Co., Ltd., of Baden, Switzerland, is at present building a single-phase locomotive to be operated directly with single-phase power of 50 cycles at 15,000 volts. The locomotive is of standard gage, and is equipped with two motors of 500 hp. one-hour rating, for a maximum speed of 90 km. per hr. (56 mi. p. hr.). (See Fig. 11.) It is designed for regenerative braking. This locomotive has no commutators, tap switches, starting resistances, nor reversing switches, in the motor circuit. All such equipment is replaced by a high-voltage mercury arc rectifier with control grids. The tests with this locomotive will be highly significant in view of the possible utilization of alternating currents of commercial frequencies for the electrification of main-line railroads.

Discussion

L. R. Ludwig: For a great many years engineers have appreciated the advantages which may be gained by introducing grids into mercury arc rectifiers for the purpose of controlling them. A large number of patents has demonstrated the realization of the possibilities inherent in this type of device; in particular, the early fundamental patent on inverters (L. W. Chubb, Re. No. 17693) and later patents by Slepian have helped lead the way in these developments. Today the average design engineer accepts the possibility of a grid-controlled rectifier, perhaps without enough reservation as to the difficulties which the grid carries with it. Inherently, a grid-controlled rectifier is similar to a uni-directional switch, provided suitable extinguishing means are supplied if a d-c. circuit is to be interrupted. However, the grid-controlled rectifier suffers from the very serious disadvantage that the grids frequently lose control without apparent cause. In this respect the grid-controlled rectifier is

much inferior to a switch because, as pointed out in a paper by Slepian and Ludwig,* the loss of grid control, which is most often in the nature of a backfire, does not occur above certain definite limits but may occur at random throughout any operating range of current or voltage. In other words, the loss of control of the grids constitutes a major problem, the solution of which today is more incomplete than that of the backfire problem in general. Another serious cause of loss of control of the grids when such devices are used as inverters is the fact that a relatively long deionizing time is necessary for a current zero in order for the grid to gain control. Careful measurements of this time made by Slepian and Crago in 1923 indicate hundreds of microseconds. In the inverter circuit the necessity for this deionizing time requires the use of special commutating means such as condensers, which must be comparatively large.

From these brief comments, it can be seen that the requirements of an ideal controlled rectifier are freedom from backfires and freedom from a long time lag necessary before the grid can gain control of the discharge to its respective anode. The large amount of research work which has been done by electrical manufacturers toward the perfection of the mercury arc rectifier and its grid-controlled brother will probably soon make the use of such devices technically possible. However, if one considers what is really required of the rectifier or inverter in an electrical circuit, it will readily be seen that the large and rather cumbersome devices which are on the market today really represent a rather crude development as compared with the device which may be imagined as being sufficient to accomplish the purpose which is really accomplished by the rectifier or inverter. The present size and complexity of this type of apparatus are no doubt due to the as yet imperfect understanding of the nature of backfire. For example, the ideal rectifier may be quite small, having very low loss and free from the complexity of the various grids and shields as employed at present. The ideal converter might well be imagined as a similar device in which satisfactory control is obtained perhaps with means other than grids. Consequently, there may be some question as to whether the present rectifier and converter must not be very materially improved before startling changes in the transmission of electrical power can be expected from their use.

It has also been recognized for some time that rectifiers may be used instead of commutators in conjunction with rotating electrical machinery. However, it would be no more correct to state that the rectifier can be used to replace the commutator than it would be to state that a gasoline engine can be used to replace the horse. Both gasoline engine and horse may be used as a source of power for locomotion, but certainly no one would expect to hitch a gasoline engine to the shaft of a wagon for the purpose of pulling it. Similarly, the rectifier is not the exact counterpart of the commutator. Two differences exist: (1) the rectifier is a uni-directional switch, whereas the commutator and brush will pass current in either direction, and, (2) the rectifier for practical reasons must have a comparatively small number of anodes or segments, whereas the commutator usually has a large number. The result is that when a rectifier is used in place of a commutator, as suggested by Mr. Marti, the utilization of the windings on the motor is not nearly so good as the utilization if a commutator is used. For this reason, one may question whether it would not be better practise to use a controlled rectifier merely as a frequency-changer without direct regard to the motor, and to use with it some type of motor in which the utilization of copper is considerably better. Certainly with this poor utilization, the requirements of the controlled rectifier as to size, cost, and reliability seem to be much greater than can be met at the present time.

R. E. Hellmund: The possibility of using rectifiers on locomotives running on an a-c. system has been rather intriguing to engineers ever since the conception of the mercury arc rectifier. This is evidenced by the early operation of a rectifier car by the Westinghouse Company on the New York, New Haven and Hartford system in 1914. The possibility of commutatorless motors operated by means of rectifiers was conceived very soon after the advent of the rectifier, as is evidenced by the early work of Chubb, Slepian, Hazeltine, Alexanderson, and others. The failure to introduce such arrangements for commercial operation was primarily due to the fact that they could not economically compete with arrangements using the ordinary single-phase commutator motor, and in the case of the schemes of operating commutatorless motors the development of suitable rectifiers in the early days was perhaps not far enough advanced to give sufficiently reliable operation. In the meantime, considerable progress has of course been made in the development of rectifiers and similar electronic devices, but probably even greater advances have been made in the design of single-phase motors by those engineers who realized the possibility of this type of motor for railway work. As a result of such progress in both lines of activity, their relative position today is about the same as in the early days, namely, the present rectifier arrangements cannot compete economically with single-phase commutator motors.

The fact that the single-phase commutator motor presents certain engineering problems in connection with commutation and that as a matter of course such problems have been discussed a great deal by the interested engineers, has led to the impression that if only a commutatorless motor could be devised for an a-c. railway the millennium would have been reached. In other words, the fact that the single-phase motor offers one major problem in its design has distracted attention from the fact that it offers some very marked advantages over any other type of motor for conditions prevailing in railway work. The very feature which has been introduced in the single-phase motor design to take care of the commutation problem, namely, the use of a large number of poles, brings with it the possibility of obtaining a higher rating within the definitely limited space available and at a lower weight than is possible with other motors.

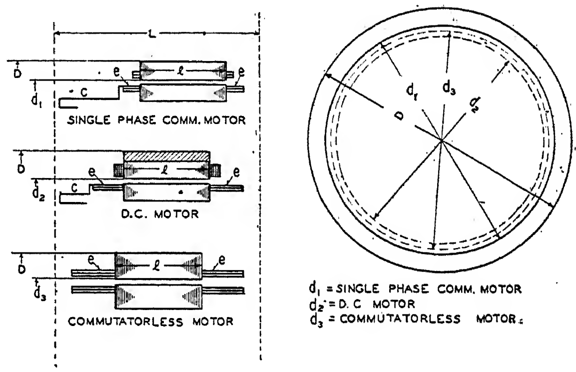


Fig. 1

This can best be pointed out by reference to Fig. 1. With an available outside diameter D and a large number of poles, the core naturally will be shallow, giving an air-gap diameter d_1 in the case of a single-phase motor. In a d-c. motor on a locomotive, at least an appreciable part of the total line voltage has to be applied to each motor, which necessitates large distances between brush-holders and, as a consequence, a small number of poles. This in turn requires appreciable space for the yoke section and the field coils, so that the air-gap diameter d_2 will have to be very much smaller. In most commutatorless motors the number of poles must also be made smaller because such motors, unlike the single-phase commutator motor, are unable to operate above synchronous speed. In the case of induction and synchronous motors this is of course evident, but it is also true with the motor described in the paper unless additional complicated

*Backfires in Mercury Arc Rectifiers, A.I.E.E. Trans., March 1932, p. 92.

means are introduced. While with the arrangement given, a number of cycles can be used in succession for each of the windings, thus making it possible to operate at low speeds; there seems at present to be no simple and inexpensive way of stopping the current flow in any one winding before the end of the half cycle. This means that with a given number of poles the speed cannot be increased indefinitely, which in turn means that in order to obtain high speeds a small number of poles has to be chosen. As a consequence, the depth of the stator core will be greater than with the commutator motor and therefore the air-gap diameter d_3 will be smaller. The same figure gives a comparison of the three types of motors with regard to filling the limited width available in railway work. The latter is usually taken up by some clearances, by the core width, the length of the coil-end connections, and the commutator where there is any. In the upper figure showing the single-phase motor, the commutator is long, but on account of the large number of poles the coil ends are short. In the second figure illustrating a d-c. motor, the commutator is shorter but the end connections are longer. In the third figure, representing the commutatorless motor, there is of course no commutator but the coil ends are long. If slip-rings are used inside of the bearings, as usually will be the case, further space may be taken by the slip-rings and the brush-holders, unless it is possible to locate them under the winding. In other words, the length of core available for the three types of motors probably does not vary to any great extent. This then means that, owing to the fact that the single-phase commutator motor permits larger air-gap diameters, its space and weight efficiency is indeed very favorable. It is further favored by the fact that because of its shallow cores and field coils there is good heat dissipation and a low temperature gradient between the hot spot and the surface. The motor arrangement described in the paper is, on the other hand, considerably handicapped by the fact that each of the stator windings is used only for part of the time, similar to the transformer windings used in connection with rectifiers. Again, if a high-voltage winding is used as proposed, it will necessarily prove a further handicap. All in all, it seems very unlikely that under the conditions described the proposed type of motor will within a given space equal the rating obtainable with the single-phase commutator motor.

With this picture of the motors before us, it is not difficult to see that, with no gain in the motor situation, the addition of a rectifier to the locomotive in the present state of the art is not likely to lead to an all-round economic solution from either the first-cost or the maintenance point of view. It is true that commutators as well as switches require some maintenance, but this is also true to a greater extent with the rectifier. Further consideration must be given to the fact that rectifier arrangements will be handicapped by the necessity for filtering equipments on the locomotive if telephone disturbances are to be avoided.

It is suggested in the paper that the proposed scheme might be used for 60-cycle electrification, thereby obviating the necessity for 25-cycle systems. Here again it so happens that although the considerations of the motor commutating problem have brought about the use of 25 cycles for railway work, such a system is advantageous from various points of view. The early claims that the 60-cycle system would eliminate rotating conversion apparatus have not materialized because it has been found that phase-converting apparatus would be necessary anyway. Furthermore, the use of 25 cycles under railway conditions, with the heavy loads of changing location, has shown quite some superiority from a voltage regulation point of view and also as far as telephone interference is concerned.

This discussion of the economic conditions is not intended to convey the idea that studies like that presented in Mr. Marti's paper are futile. However, the scheme presented, like many others previously suggested for the utilization of electronic devices both for traction and other purposes, will be dependent to a great degree upon further developments of electronic devices. Many of them will not find commercial utilization until the electronic devices have been developed to the point where they are considerably less expensive and where a greater reliability of various grid control schemes has been obtained.

E. F. W. Alexanderson: The vacuum tube technique has many ramifications and promises to become an important factor in most branches where electric power is used. One of those possibilities is to substitute grid control rectifiers or thyratrons, as we call them, for the copper and carbon brush commutator in the operation of adjustable speed motors. The largest field for the use of adjustable speed a-c. motors is undoubtedly railroad electrification and much thought and research have been given to this subject in the last ten years or more by those interested in the vacuum tube technique. Those who look at this development from outside may well ask: Why, if the problem has been clearly understood for so long, is the practical solution so slow to be realized? The answer to this is apparently that the vacuum tube technique is not a simple engineering problem but an evolution and like all evolutions it is necessarily slow. It involves the work of the physicists who are contributing new knowledge of electricity and matter. It involves the manufacture of the vacuum tubes themselves, and the evolution of a new technique for embodying those new principles in practical devices and finally, it involves the electrical engineers who must interpret the usefulness of these devices with relation to the more conventional types of electrical machinery.

The history of this evolution for the last ten years is that new principles have constantly been introduced in the manufacture of superior vacuum tube devices and these devices in their turn have made possible new and improved methods of engineering application. The commutatorless a-c. motor developed by the Brown-Boveri Company and described by Mr. Marti is one of the steps in this evolution. From personal experience I can say that motors of this type will operate with satisfactory characteristics and I do not doubt for a moment that a motor just as described can be designed to haul a locomotive.

Another question is: Will such a locomotive be economically competitive with other and existing types of electric locomotives? Serious efforts are being made to answer that question. Personally I am of the opinion that this development will lead to such a practical solution and my reason for this belief is that progress of the vacuum tube technique is rapidly leading to new and superior forms which in their turn make possible improved methods of correlating the power system, the vacuum tube, and the motor. From the point of view of engineering research, this subject is of intense interest. But no matter how great our enthusiasm, I think it is better and wiser to admit that from the point of view of practical railroad electrification it is not possible to discuss this matter intelligently until we have before us a locomotive with vacuum tube commutation which is not only operative but is economically competitive with the now existing types. If, and when, these hopes are realized it will mean a substantial boost to the spread of railroad electrification because locomotives of the new type may be operated side by side with locomotives of the old type on the same lines, each type filling its place according to its own merits. From this point of view, it is fortunate that in this country a power frequency of 25 cycles rather than the lower frequency of $16^2/_3$ cycles has been adopted for railroad electrification because 25 cycles is better adapted to various uses. The anticipation of new and improved types of locomotives is thus no reason for delaying practical progress in electrification, because the new methods when realized, will fit in with the old plans.

R. G. McCurdy: The developments described by Mr. Marti on rectifiers provided with control grids open up some very interesting possibilities in the way of new problems for the telephone companies as well as possible advantages to power and electric railway companies.

We have had, of course, no experience with the new applications described by Mr. Marti but must base our comments on our experiences with these devices as used for conversion from alternating to direct current.

Considerable information has been published in the technical press, including the TRANSACTIONS of the Institute, regarding the effect of mercury arc rectifiers on the wave shape of systems to which they are connected and the resulting effects on coordination with telephone lines. This has been more particularly in reference to the effects on the d-c. side. The joint studies which have been made of the problem by the electrical manufacturing and telephone companies have led to the development of filters for the d-c. side which reduce the harmonic voltages and currents to values which permit satisfactory coordination between the d-c. systems and nearby telephone systems. These have been employed in situations where it appeared that conditions of proximity with the telephone plant were such as to justify their use.

The a-c. side of the device presents a more difficult problem. As compared to the d-c. side the percentage harmonic voltages and currents are much greater in magnitude. Moreover, there is a larger number of harmonics present. Both of these factors

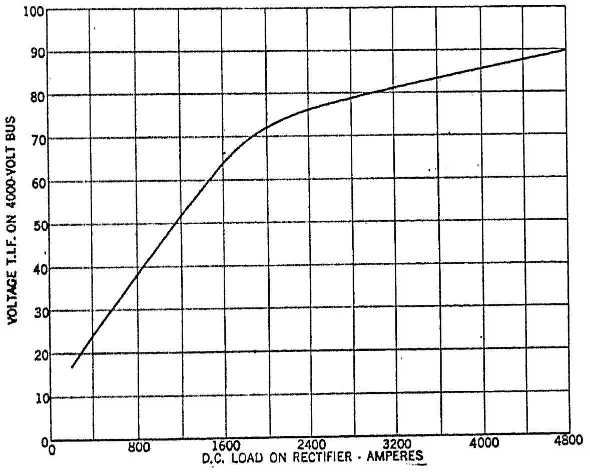


FIG. 2—VARIATION OF VOLTAGE TELEPHONE INTERFERENCE FACTORS ON A 4,000-VOLT BUS

With load on rectifier fed from same transmission system

considerably increase the cost and difficulty of filtering. Where the rectifier is close to a source of power of relatively large size and supplied over lines not involved in appreciable exposures to telephone plant, these relatively large harmonic currents are not of importance. The impedance of the system under such conditions is low enough so that no appreciable increase takes place in the harmonic voltages and the telephone interference factor to which they contribute. On the other hand when the rectifier load is a relatively large part of the total power rating of the system, or when lines and transformers of considerable impedance intervene, or if the line supplying the rectifier is involved in telephone exposures, the difficulties in coordination are greatly increased.

On balanced polyphase power systems mercury arc rectifiers make no direct contribution to the residual voltages and currents of the system. Thus the usual methods of transposing the lines are applicable. In many cases, however, a distribution system involving numerous single-phase branches involved in exposures with telephone circuits may be supplied from the same system as a mercury arc rectifier. Under these conditions the harmonic voltages introduced by the rectifier will be residual so that means other than power transpositions will need to be resorted to in order to reduce the influence.

In Fig. 2 are shown some values of telephone interference factors on a 4,000-volt bus supplying distribution circuits, as a function of the load on a large mercury arc rectifier supplied from the same high-voltage transmission circuit as this 4,000-volt bus. It will be noted that under maximum load conditions on the rectifier, the bus voltage telephone interference factors are increased by a factor of more than 5:1. It will be apparent that coordination measures which are adequate for the minimum telephone interference factors values may be entirely inadequate when the load on the rectifier is large.

It is well known that the magnitudes of the harmonic voltages and currents on both the a-c. and d-c. sides are dependent in a large measure on the number of phases involved. A 12-phase rectifier for example has materially lower values of certain harmonics than the 6-phase rectifier. The wave shape of the two devices may be made substantially equivalent on the d-c. sides by filtering, a more effective filter however being required with the 6-phase device. On the a-c. side the 12-phase rectifier involves somewhat smaller fifth, seventh, seventeenth, and nineteenth harmonics. The eleventh and thirteenth harmonics are about the same for the 12-phase and 6-phase devices. The telephone interference factors so far observed have been in the ratio of about 1:3 in favor of the 12-phase device.

This matter is being actively studied by the Project Committee on Wave Shape of the Joint Subcommittee on Development and Research of the N.E.L.A. and Bell Telephone System. It is hoped that technical information which will permit the making of quantitative estimates of the effects of rectifiers on the wave shape of supply systems will be soon forthcoming from this committee.

In examining the effect of the controlled grid rectifier as described by Mr. Marti, it seems evident that the load regulation by these means must result in further reaction in wave shape. This may be seen by examining the graphs shown in Fig. 4 of Mr. Marti's paper. The effect on the wave shape of the d-c. side may, of course, be compensated for by suitable modification of the d-c. filter.

While I have not made a detailed quantitative examination of the wave shape of the current of the commutatorless single-phase railway motor described by Mr. Marti, it seems apparent that harmonics will exist in the a-c. side comparable at least with those experienced with the polyphase rectifiers previously mentioned.

Noise frequency induction has not in general been a problem with single-phase railway electrifications due to the small wave-shape distortion experienced, induction at the fundamental frequency being a controlling factor. No doubt the exacting commutation requirements of the single-phase motor have been a controlling factor in this situation. The controlled grid mercury arc rectifier would apparently seem to solve the commutation problems, but at the expense of a considerably increased wave-shape distortion. This may be expected to complicate the problems of coordination between single-phase a-c. railways and telephone circuits. It is hoped that the joint development effort previously referred to in securing inductive coordination with the conventional type of mercury arc rectifiers will lead to methods which may be applicable in the a-c. railway case.

H. M. Trueblood: The possibility of direct utilization of 60-cycle energy on the contact systems of electrified railways has engaged the attention of engineers occasionally in the past, and has led to certain preliminary exploration of its probable reaction on inductive coordination at fundamental frequencies. Mr. Marti's paper brings this question up again in an interesting way. In line with American practise in electric railway and commercial power engineering, the following general observations are based upon a comparison of the use of single-phase 25-cycle and 60-cycle energy upon the contact wire of an electrified railway using the running rails for return. Several factors are involved in a rather complicated way, and simplifying assumptions underlie this preliminary comparison. These are: first, the power supply

is unidirectional *i. e.*, it is what is generally known as "stub-end feed;" second, the currents—load or short-circuit—are of the same magnitude in the two systems.

There are involved the following main factors:

Average earth current throughout the inductive exposure (it is chiefly this current, as distinguished from the current remaining in the rails, that causes fundamental frequency induction in communication circuits in the usual case). In addition to differences in track-earth circuit characteristics at the two frequencies, this point involves also the length of feed and the length of exposure.

Relative magnitudes of coefficients of induction at 25 and at 60 cycles. This depends upon the resistivity of the earth and upon the separation between the railroad and the communication line.

The efficacy of shielding conductors at the two frequencies. Such shielding conductors may exist, irrespective of any inductive coordination questions, in proximity to either the communication conductors (for example, cable sheaths) or the railway system (for example, structure ground-wires), or they may be installed primarily for purposes of coordination. Here again the earth resistivity is a factor, and, as exemplified particularly in the case of underground communication cables, the amount of metal in the shielding conductors and its distribution are important.

Estimates have been made of the effects of these factors, using reasonable figures for items of design, and, in the main, experimental data for physical characteristics such as rail impedance, track leakage, etc. It is not proposed here to attempt more than brief summaries. These estimates indicate:

1. That for a representative value of track leakance per track, the ratio of average earth current at 60 cycles to that at 25 cycles does not depart from unity by more than about 10 per cent for lengths of feed ranging from some 3 miles up to 20 miles or more.

2. That the ratio of the mutual impedance of two ground-return circuits at 60 cycles to the same quantity at 25 cycles ranges from 2.2 for 50 ft. separation and earth resistivity of 1,000 meter-ohms to about 1.2 for 1,500 ft. separation and earth resistivity of 10 meter-ohms.

3. That for optimum conditions (zero impedance ground connections at the ends of the exposure) the ratio of the reduction in induced voltage due to cable-sheath shielding at 60 cycles to the corresponding reduction at 25 cycles ranges from about 2.0 for one full-sized cable to about 1.5 for two full-sized cables in adjacent duct runs. (This is for an earth resistivity of 100 meter-ohms.)

4. That the efficacy of track feeders in reducing the amount of current in the earth, for long feeds, is some 5 to 10 per cent (depending on the number of feeders and the number of tracks) better for 60 cycles than it is for 25 cycles.

On the basis of the estimates that have just been summarized and under the above-stated simplifying assumptions, certain statements become possible with regard to the induced longitudinal voltages:

5. The ratio of the longitudinal induced voltage at 60 cycles to the longitudinal induced voltage at 25 cycles is greater when the track leakance, the length of feed, the separation, and the amount of shielding provided are small than when they are large.

6. For an earth resistivity of 100 meter-ohms this ratio ranges from about 2.4 for a track leakance of 0.5 mho per track mile, a 3-mile length of feed, a 50-ft. separation and no shielding, to about 0.8 for a track leakance of 1.0, a 10-mile length of feed, a separation of 1,500 ft., and shielding due to two full-sized cable sheaths in adjacent ducts under optimum (ideal) conditions. (These figures assume that the exposure embraces the length of feed.)

It may be said that in the general run of the more important situations the induced voltage with the 60-cycle trolley would be higher than with the 25-cycle trolley, by amounts varying from a small, probably a negligible, percentage, up to large percentages—of the order of 50 to 100 per cent. This ignores so-called "ground potentials," an important item in inductive coordination involving electrified railways. To a first approximation, we may say that ground potentials would probably be about the same for the two systems.

Insulator Sparkover

Factors Affecting the Sparkover Voltage of Insulators Used on High-Voltage Transmission Systems

BY W. L. LLOYD, JR.*
Member, A.I.E.E.

INTRODUCTION

WHILE the mechanical properties and strength of the various parts entering into the design of a modern transmission system are quite well known, the electrical characteristics of the various parts are often not so definitely understood. This is particularly true of the electrical properties of the insulating parts, including the line and station insulation and bushings used on the station equipment. It is the purpose of this paper to present data upon the sparkover strength of the various parts used for the insulation of modern high-voltage transmission systems.

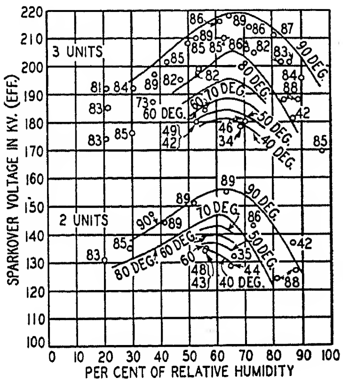


FIG. 1—EFFECT OF RELATIVE HUMIDITY AND DRY BULB TEMPERATURE ON THE 60-CYCLE SPARKOVER OF SUSPENSION INSULATORS

Number on points indicates dry bulb temperature at time of test. Air density = 1.00

Temperature and Barometric Pressure. The effect of temperature and barometric pressure upon the 60-cycle sparkover voltage was determined a number of years ago for sphere-gaps, point-gaps, bushings and various types of suspension and pin type insulators.[1] In this early work it was found that the sparkover voltage of gaps, varies with varying air density factor δ.

$$\delta = \frac{3.92\, b}{273 + t} \text{ in metric units, where}$$

b = barometric pressure in centimeters of mercury
t = temperature in degrees centigrade

At 25 deg. cent. and 76 cm., the air density factor is unity.

Or,

$$\delta = \frac{17.9\, b}{459 + t} \text{ in English units, where}$$

b = barometric pressure in inches of mercury
t = temperature in deg. fahr.

At 77 deg. fahr. and 29.9 inches, the air density factor is unity.

It is generally sufficiently accurate to assume that the change in sparkover voltage is directly proportional to the change in air density. Although the effect varies somewhat with the design of the insulator or bushing, the above rule will usually apply over the practical operating range. For example, the sparkover voltage of three and four unit insulator strings has been found to vary directly with the air density factor down to and including an air density 50 per cent of that occurring at standard temperature and barometric pressure (77 deg. fahr. and 29.9 inches).

Air density affects the lightning sparkover of gaps and insulators in the same general way as for 60 cycles.

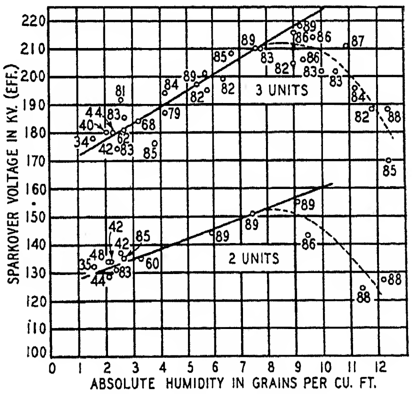


FIG. 2—EFFECT OF HUMIDITY ON THE 60-CYCLE SPARKOVER OF SUSPENSION INSULATORS

Note that plotting sparkover voltage against absolute rather than relative humidity results in a single curve for all dry bulb temperatures (as indicated by number on points). Air density = 1.00

However, the effect for lightning is usually somewhat less.

All sparkover voltages in this report have been corrected to an air density of unity by using the air density factor as given above.

Humidity. Only a small amount of data giving the effect of humidity has heretofore been published.[2,3] Tests made in different laboratories under identical conditions, in so far as A.I.E.E. specifications are concerned, often fail to produce identical results. Data included in this

*High Voltage Engineering Laboratory, General Elec. Co., Pittsfield, Mass.

1. For references see Bibliography.

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paper indicate that the larger part of these differences can usually be attributed to the effect of humidity or water vapor in the atmosphere in which the tests are made. A humidity of 6.5 grains per cu. ft. (65 per cent at 77 deg. fahr.) has been used by this laboratory as the standard condition of humidity in giving out sparkover data. This is equivalent to a vapor pressure of 0.608 in. (15.4 mm.) of mercury.

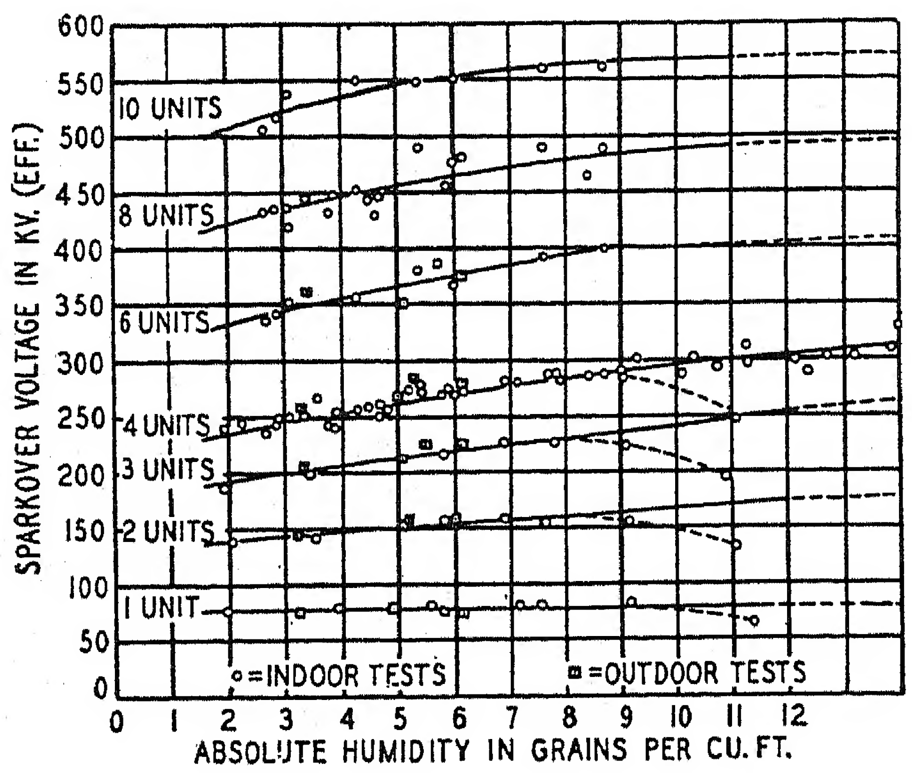

FIG. 3—EFFECT OF HUMIDITY ON THE 60-CYCLE SPARKOVER OF SUSPENSION INSULATORS

10-in. diameter, 5¾-in. spacing, standard duty. Air density = 1.00

Smoke, Steam, Dew, Fog, Rain, and Surface Dirt and Moisture. While the effects of moisture and dirt have been recognized for a much longer time, specific quantitative data have often been lacking. Some heretofore unpublished data which are believed to have a direct practical bearing on the insulation design of many lines of systems are included in this paper.

Lightning Sparkover. The effects of the preceding

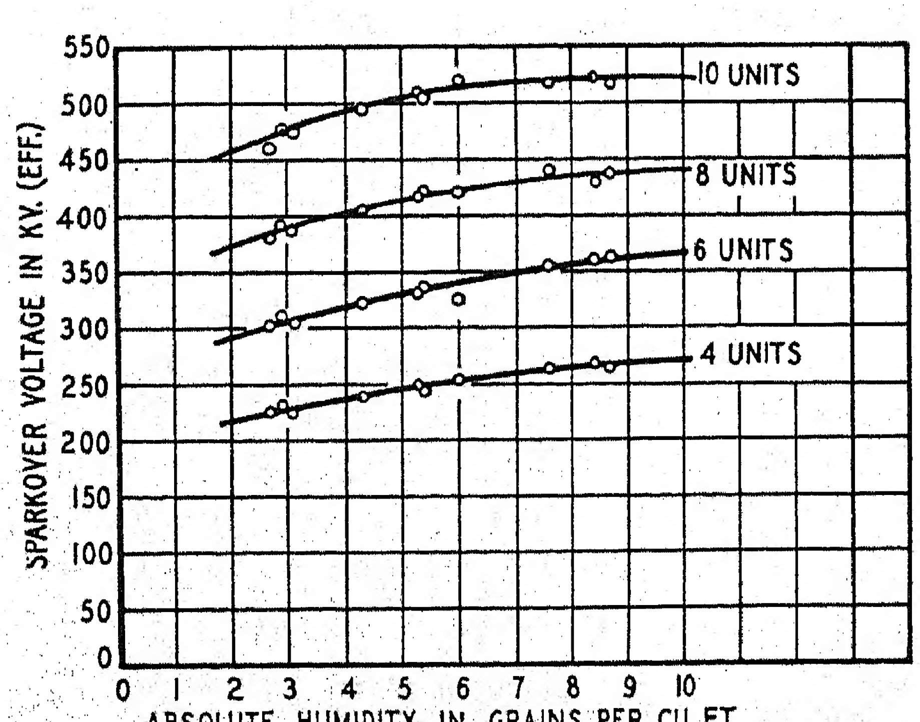

FIG. 4—EFFECT OF HUMIDITY ON THE 60-CYCLE SPARKOVER OF SUSPENSION INSULATORS

10-in. diameter, 4¾-in. spacing, standard duty. Air density = 1.00

factors on the lightning sparkover of various insulations are discussed and some data submitted.

HUMIDITY

Where the humidity for a given sparkover test has been specified it has been common to express the humidity in terms of relative humidity. No confusion would result by such a procedure if all sparkover tests were made at approximately the same temperature. The effect of humidity can then be indicated by plotting sparkover voltage as a function of relative humidity. Sparkover tests are, however, sometimes made over a wide range of temperature and results so plotted are of little value in comparing the humidity effect and in finding an agreement in the sparkover voltage at some given humidity. A single curve of sparkover voltage as a function of relative humidity for different temperatures cannot be drawn. (See Fig. 1.) The effect can only be shown by drawing in a group or family of curves, each individual curve pertaining to a particular temperature. Since humidity has generally been expressed in terms of relative values, it has been difficult to interpret its true effect and different laboratories have found it difficult to agree upon the sparkover voltage at any given humidity. Note that for a relative humidity of 65 per cent, the sparkover for two units varies

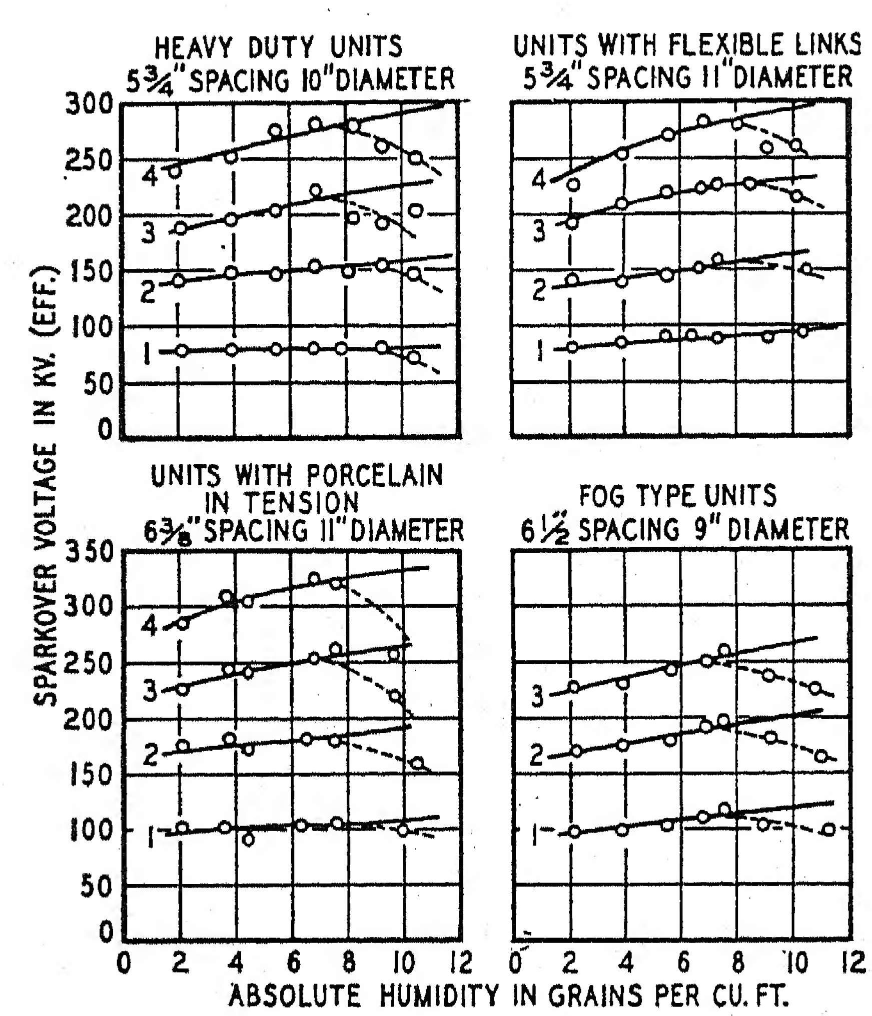

FIG. 5—EFFECT OF HUMIDITY ON THE SPARKOVER OF SUSPENSION INSULATORS, 60 CYCLES

Air density = 1.00

from 130 kv. to 155 kv., and for three units from 180 kv. to 220 kv. for a temperature variation of 40 deg. to 90 deg. fahr. or plus or minus approximately 10 per cent in voltage even after correction for temperature.

Fig. 2 shows the same data when plotted in terms of the absolute humidity. The family of curves reduces to a single curve indicating the true effect of humidity. The dip in the curve beyond about 9 grains per cubic ft. will be discussed below.

Fig. 3 presents the sparkover voltage for 10-inch diameter standard-duty suspension insulators having a spacing of 5¾ inches. These tests have been made with four different sets of testing equipment in four laboratories. Three of the laboratories employed artificial humidity control; the fourth was an outdoor laboratory in Pittsfield using natural humidity occurring from day to day and season to season. All these tests were conducted in accordance with A.I.E.E. Standards No. 41. The importance of recording absolute rather

than relative humidity is most apparent when the same relative humidity is measured in a spring or fall outdoor test as compared with a summer test. Although the relative humidity may be the same, the absolute

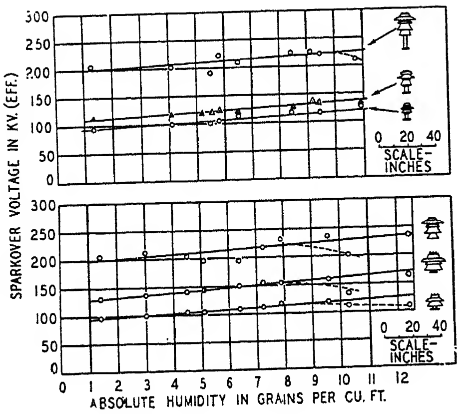

FIG. 6—EFFECT OF HUMIDITY ON THE 60-CYCLE SPARKOVER OF PIN AND PEDESTAL TYPE INSULATORS

Air density = 1.00

humidity because of a considerable change in temperature may be considerably different. Under such conditions, unless the absolute humidity is used, considerable variations are indicated.

The effect of humidity on the sparkover voltage of

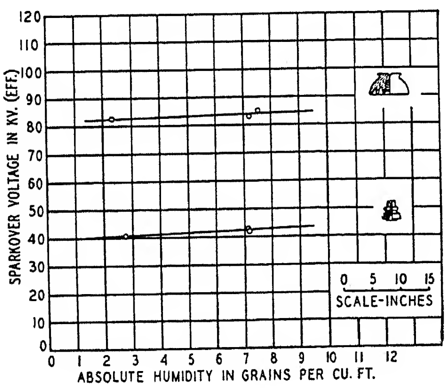

FIG. 7—EFFECT OF HUMIDITY ON THE 60-CYCLE SPARKOVER OF PIN TYPE INSULATORS

Air density = 1.00

10-inch diameter standard-duty suspension insulators having a spacing of 4¾ inches is shown in Fig. 4. The effect of humidity on the sparkover for various types of suspension insulators is shown in Fig. 5 while Fig. 6 presents the effect of humidity on certain sizes of pedestal insulators and a large pin type insulator. Fig. 7 presents similar data for certain sizes of small pin type insulators. The effect of humidity is not great on single suspension disk insulators or on these small pin type insulators.

Fig. 8 shows the effect of humidity on the sparkover of an oil-filled bushing.

Fig. 9 is a typical humidity curve for the sphere gap. The sphere gap appears to be practically independent of the humidity. Fig. 10 presents the effect of humidity on the 60-cycle sparkover voltage of the needle gap. The data in Figs. 9 and 10 are in agreement with previous data on the effect of humidity on the sparkover of spheres and needles.[3]

Fig. 11 is a plot of the data from Fig. 10 in sparkover voltage as a function of the spacing. The standard needle gap curve is also indicated.

It will be noted that several of the humidity sparkover curves for insulators indicate a lowering of the sparkover for absolute humidities above approximately 9 or 10 grains per cu. ft. That this is an effect of surface moisture or condensation of the water vapor from the atmosphere on the porcelain surfaces at these high absolute (and generally relative) humidities and not an effect in the air gap, is indicated by the absence

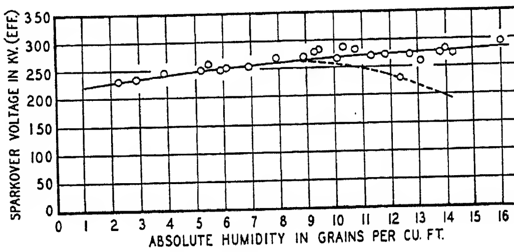

FIG. 8—EFFECT OF HUMIDITY ON THE 60-CYCLE SPARKOVER OF AN OIL-FILLED BUSHING

Air density = 1.00

of this effect in the case of the needle gap and sphere gap, particularly the former. This conclusion is also indicated by the data in Fig. 12 where tests were made on chilled and slightly heated insulators. It is also indicated by the difficulty commonly experienced when tests on insulators at high humidities show a constantly rising sparkover in repeated tests. The surface moisture

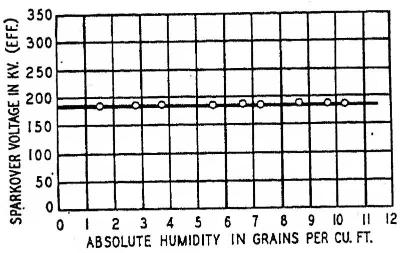

FIG. 9—EFFECT OF HUMIDITY ON THE 60-CYCLE SPARKOVER OF A 25-CM. SPHERE GAP SPACED FOR 187 KV.

Air density = 1.00

results in a low sparkover in the initial test. Repeated sparkovers remove this surface moisture and the insulator gradually acquires the higher sparkover resulting from the high humidity. Frequently this increase in sparkover continues for twenty-five or more tests before the sparkover voltage becomes constant at the upper value.

Smoke, Steam, Dew, Fog, Rain, and Surface Dirt and Moisture

The 60-cycle sparkover of clean 10-inch diameter standard-duty suspension insulators of 5¾-inch spacing for dry, light rain, and heavy rain conditions is shown in Fig. 13. The tests apply to insulators in a vertical

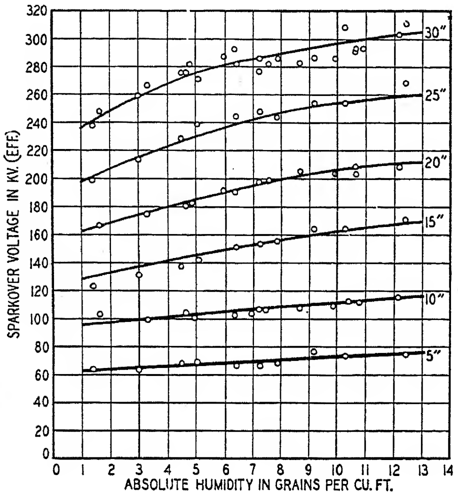

Fig. 10—Effect of Humidity on the 60-Cycle Sparkover of the Needle Gap

Air density = 1.00

position. Rain tests with the insulators in a horizontal position indicate no lowering of the sparkover by rain with the rain directed either at right angles to the string or from, or towards, the line end at an angle of 45 deg. to the string and always at 45 deg. from the vertical. On a horizontal string rain sufficiently

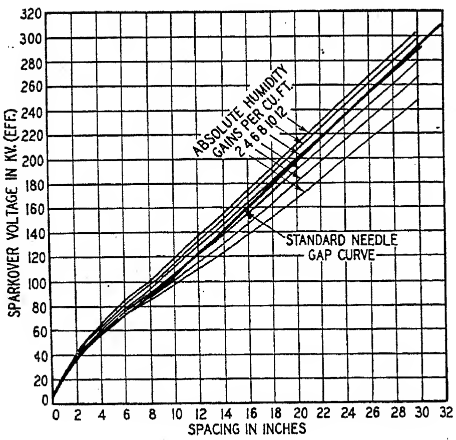

Fig. 11—Sparkover of the Needle Gap. Effect of Humidity on Sparkover. 60 Cycles

Air density = 1.00

improves the poor voltage distribution along the string so as to raise the rain sparkover to that of the dry. The drip along the vertical string overbalances this effect and actually lowers the sparkover.

Fig. 14 shows the effect upon the sparkover voltage of the resistivity of the water and the rate of precipitation used in making artificial rain tests on clean insulators and bushings.

That the dry 60-cycle sparkover of 10-inch diameter insulators strings having the same overall length is apparently independent of the spacing of the individual units making up those strings is indicated in Fig. 15.

Fig. 16 indicates the effect of surface dirt and moisture

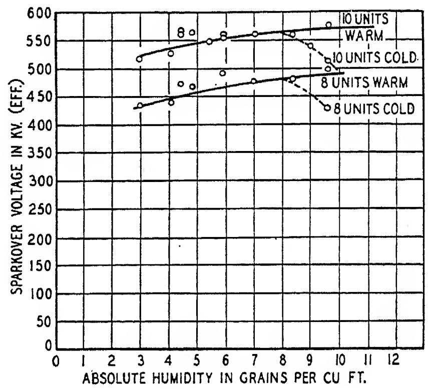

Fig. 12—Effect of Humidity on the 60-Cycle Sparkover of Suspension Insulators

10-in. diameter, 5¾-in. spacing, standard duty. Air density = 1.00

upon the 60-cycle sparkover of suspension insulators. The dirt used was that resulting from long exposure of the insulators to locomotive steam and smoke until the insulators were coated with a carbon deposit perhaps 1/32 to 1/16 inch in thickness. From Fig. 16 it will be observed:

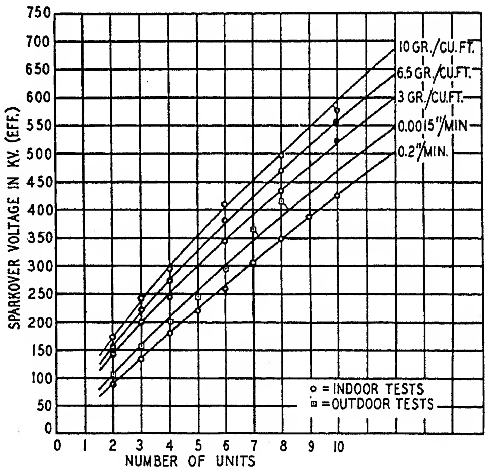

Fig. 13—Effect of Humidity on the 60-Cycle Sparkover of Suspension Insulators

10-in. diameter, 5¾-in. spacing, standard duty. Air density = 1.00

1. That this deposit of dirt when dry had practically no effect upon the insulator sparkover.
2. That the sparkover of the dirty insulators with top surface moistened was only slightly lower than the clean insulators with top and bottom surfaces moistened.
3. That the clean and dirty insulators had about the same sparkover under artificial rain conditions of 0.2

in. per minute at 45 deg. from the vertical, as specified by the A.I.E.E., presumably because some washing of the insulators took place.

4. That the dirty insulators with top and bottom surfaces moistened (by light spraying) had a very low sparkover voltage, and finally

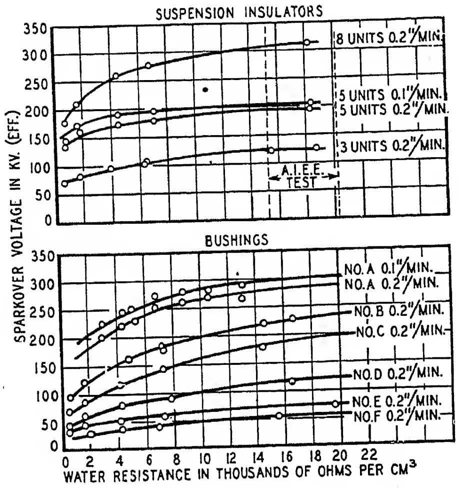

Fig. 14—Effect of Water Resistance on the Wet Sparkover of Suspension Insulators and Bushings. 60 Cycles

Air density = 1.00

5. That this latter condition combined with an atmosphere of heavy steam and locomotive smoke surrounding the string (see Fig. 17) resulted in the lowest sparkover of all. (See lowest curve in Fig. 16.)

It may be noted that under these conditions the sparkover of 14 disks was only 200 kv. effective, a reduction of 73 per cent.

Table I shows the effect of dew upon the sparkover voltage of clean and salt-coated suspension insulators.

The salted insulators in Table I were lightly coated with salt (sodium chloride) by hand-spraying with a saturated salt solution. The condensation of moisture on the porcelain surfaces was accomplished by exposure of the insulator string to the warm and somewhat humid atmosphere of the laboratory after cooling in an iced compartment. The duration of exposure in minutes is given in the table. Very little reduction in sparkover took place in five minutes. The maximum reduction was reached in about 10 or 15 minutes. The strings were observed to start to dry again after about 30 minutes of testing.

Surface moisture was found to have only a very slight effect upon the clean insulators. On the salt-coated strings a 30 to 40 per cent reduction in sparkover was common. In one case the reduction was nearly 50 per cent. The salt deposit was not excessive. Upon spraying with clean water, a salted string of nine disks would not hold 87 kv. This was a reduction of more than 85 per cent in sparkover. This reduction compares with a reduction of 50 per cent on nine disks and 60 per cent on fourteen disks for the locomotive soot-coated strings with top and bottom surfaces moistened.

Table II gives somewhat similar data on four small pin type insulators on which a deposit of mill dust had been accumulated as a result of a long time exposure in the vicinity of a Pittsburgh mill.

TABLE I—DEW TESTS ON CLEAN AND SALT-COATED INSULATORS
60 Cycles

String	No. of disks	Surface condition	Duration of exposure	Sparkover voltage	Sparkover reduction
No. 1	13	Clean and dry	0 min.	699 kv.	0 per cent
No. 1	13	Clean but moist	20 min.	692 kv.	1 per cent
No. 4	13	Salted and moist	6 min.	610 kv.	13 per cent
No. 4	13	Salted and moist	7 min.	531 kv.	24 per cent
No. 4	13	Salted and moist	8 min.	465 kv.	34 per cent
No. 4	13	Salted and moist	14 min.	425 kv.	39 per cent
No. 4	13	Salted and moist	24 min.	455 kv.	35 per cent
No. 4	13	Salted and moist	Held 127 kv. (eff.) for 7 minutes. No flashover.		
No. 4	13	String drying	31 min.	595 kv.	15 per cent
No. 4	13	String drying	44 min.	659 kv.	6 per cent
No. 5	13	Salted and moist	21 min.	466 kv.	33 per cent
No. 5	13	String drying	37 min.	594 kv.	15 per cent
No. 2	9	Clean and dry	0 min.	537 kv.	0 per cent (4)
No. 2	9	Clean but moist	6 min.	542 kv.	0 per cent
No. 3	9	Clean but moist	6 min.	300 kv. dried line unit in about 5 minutes	
No. 6	9	Salted and moist	7 min.	326 kv.	39 per cent
No. 6	9	Salted then sprayed with clean water		Less than 87 kv.	84 per cent
No. 7	5	Clean and dry	0 min.	345 kv.	0 per cent
No. 7	5	Salted and moist	6 min.	188 kv.	45 per cent
No. 7	5	Salted and moist	15 min.	175 kv.	49 per cent
No. 7	5	Salted and moist	20 min.	178 kv.	48 per cent
No. 7	5	Salted and moist	25 min.	201 kv.	42 per cent
No. 7	5	String drying	35 min.	301 kv.	13 per cent

From Table II, it may be seen that the reduction in sparkover voltage was about 66 per cent for the top and bottom moistened condition.

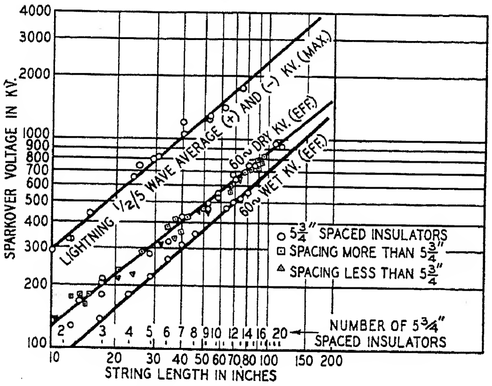

FIG. 15—SPARKOVER OF SUSPENSION INSULATORS
10-in. diameter. 5¾-in. spacing. Standard duty. Air density = 1.00

LIGHTNING SPARKOVER

The effect of humidity on the lightning sparkover of different insulators is indicated in Fig. 18. Similar tests have been made on various gaps and a corresponding

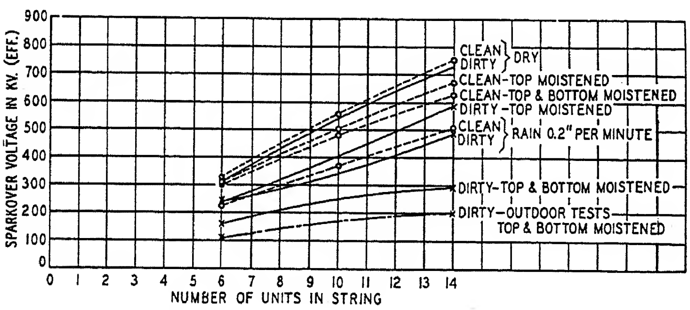

FIG. 16—EFFECT OF SURFACE MOISTURE AND DIRT UPON THE SPARKOVER VOLTAGE OF SUSPENSION INSULATORS
10-in. diameter, 5¾-in. spacing, 60 cycles. Air density = 1.00

effect noted. Taking 6.5 grains per cu. ft. as standard, the variation in sparkover between 3 grains per cu. ft. and 10 grains per cu. ft. was found to be approximately plus and minus 10 per cent for the sizes tested. This is of the same order as the effect with 60 cycles.

TABLE II—60-CYCLE SPARKOVER OF DIRTY PIN TYPE INSULATORS DRY, WET AND UNDER ARTIFICIAL RAIN

Sample No.	Dry	Top surface moistened	Top and bottom moistened	Rain at 45 deg. (0.2 in. per min.)
1	58.5	29.0	19.0	27.8
2-A	59.0	33.4	23.9	25.1
2-B	58.7	26.8	25.9	27.6
3-A	59.7	32.3	21.2	25.0

NOTE: On the foregoing insulators with top and bottom surfaces moistened corona was visible in a darkened room at the following voltages:
sample No. 1.........13.4 kv.
sample No. 2-A.......10.7 kv.
sample No. 2-B......less than 9 kv.
sample No. 3.........12.8 kv.
No corona was visible below 15 kv. on these insulators dry or with only the top surface moistened by spraying

That dew, fog, rain, and surface moisture have little effect upon the lightning sparkover of insulators is indicated by the data in Table III. In general, the effect of dew, fog, rain, and surface moisture decreases as the length of the wave decreases.

Tests on wood poles and crossarms with and without

TABLE III—LIGHTNING SPARKOVER OF VARIOUS INSULATORS DRY AND WET
Positive Polarity Lightning Wave

Sample	Condition	Approx. wave shape	Dry	Rain 0.2 in. per min.
(a) Pin Type Insulators (Same as in Table II)				
No. 1	Dirty	0.5/5	110 kv.	106 kv.
No. 2-A	Dirty	0.5/5	113 kv.	113 kv.
No. 2-B	Dirty	0.5/5	112 kv.	109 kv.
No. 3-A	Dirty	0.5/5	122 kv.	110 kv.
(b) Pedestal Insulators				
6 units	Clean	1.5/40	1,600 kv.	1,490 kv.
(c) Suspension Insulators (Various Types)				
8 disks	Clean	1.5/40	875 kv.	845 kv.
8 disks	Clean	1.5/40	890 kv.	880 kv.
9 disks	Clean	1.5/40	945 kv.	885 kv.
12 disks	Clean	1.5/40	1,300 kv.	1,280 kv.
14 disks	Clean	1.5/40	1,370 kv.	1,310 kv.
16 disks	Clean	1.5/40	1,740 kv.	1,670 kv.
16 disks	Clean	1.5/40	1,730 kv.	1,560 kv.
18 disks	Clean	1.5/40	1,800 kv.	1,740 kv.

porcelain insulators mounted on the crossarm give the same indication. Wood poles, crossarms and guy insulator sticks of different cross-sectional areas have a lightning sparkover of approximately 175-kv. per ft. of length whether the wood is dry or wet with clean water or saturated salt water and brine.[5] That water

TABLE IV—COMPARISON OF THE DISRUPTIVE STRENGTH OF WATER AND OF AIR

Gap in cm.	Impulse kv. water	Impulse kv. air	Ratio water to air
1-Inch Diameter Spheres			
0.1	49.5	3.5	14.1
0.2	86.0	7.0	12.3
0.3	105.5	11.0	9.6
0.5	126.0	17.5	7.2
0.7	137.0	24.0	5.7
1.0	149.0	33.0	4.5
1.3	159.0	42.0	3.8
1.5	165.0	46.0	3.6
60 Deg. Points on 1/8 In. Rods			
1.0	56.5	25.0	2.05
2.0	72.0	35.3	2.15
3.0	92.0	41.0	2.25
4.0	113.0	46.5	2.42
5.0	134.0	51.5	2.61
6.0	156.0	56.5	2.76

Tests were made inside a 6-in. diameter glass sphere. Resistance of water was measured at 1.0 cm. setting of gap:
R between spheres = 20,000 ohms
R between points = 40,000 ohms

has so little effect upon the lightning sparkover of porcelain insulators and insulations such as wood is probably because water is a fairly good insulator for impulse voltages as compared with air in which these materials are ordinarily tested. Table IV from "Dielectric Phenomena in High-Voltage Engineering," by F. W. Peek, Jr., Third Edition, p. 391, gives the relative

disruptive strength of water as compared with air for impulse voltages.

ACKNOWLEDGMENT

Some data included in this paper have been obtained from each of the following laboratories:

High Voltage Engineering Laboratory at the Pittsfield Works of the General Electric Company.

Pittsfield Works Laboratory of the General Electric Company at Pittsfield (Outdoor and Indoor Laboratories).

Insulator Testing Laboratory at the Baltimore Works of the Locke Insulator Corporation.

FIG. 17—SMOKE TEST ON SUSPENSION INSULATORS

Switching locomotive used to supply heavily laden atmosphere of steam and smoke

The valuable assistance of the various members of the staffs of the above laboratories in making the many tests included in this paper is hereby acknowledged. The author also wishes to acknowledge the valuable guidance of Mr. F. W. Peek, Jr. under whose direction the various investigations reported above were conducted.

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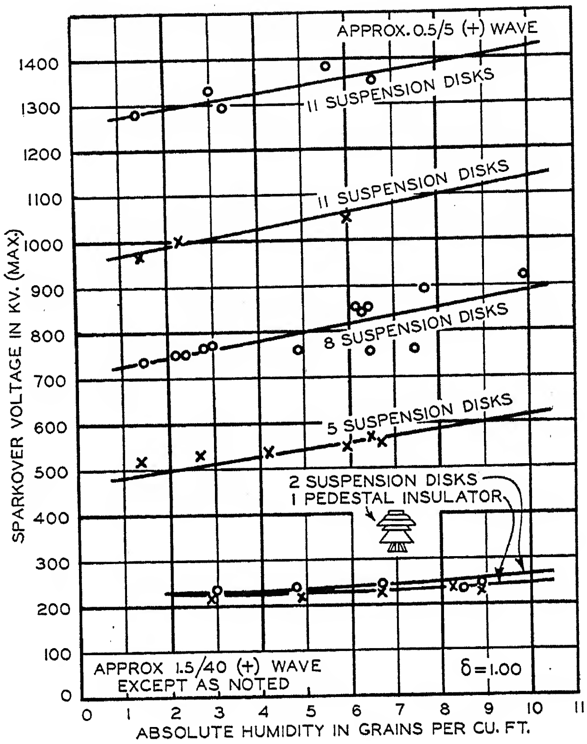

FIG. 18—EFFECT OF HUMIDITY UPON THE LIGHTNING SPARKOVER OF INSULATORS

Air density = 1.00. Approximately 1.5/40 (+) wave except as otherwise noted

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Discussion

For discussion of this paper see page 697.

An Improved Type of Limiting Gap
for Protecting Station Apparatus

BY A. O. AUSTIN*
Fellow, A.I.E.E.

Synopsis.—The wide range of transients to which station apparatus is subjected is pointed out and the desirability shown for having adequate protection. The different breakdown characteristics of the various insulating members of a station are illustrated and the importance emphasized for considering this in choosing a voltage limiting device. The need is also shown for having protective gap discharges reduced to a minimum in order to limit the number of service interruptions.

The characteristics and limitations of several forms of gaps are shown and discussed. A new form of limiting gap is described in which it is possible to change the sparkover characteristic over a considerable range to allow for the differences in time lag and polarity characteristics of station insulation. The advantage of using a limiting gap in multiple with a lightning arrester to provide maximum protection and reduce interruptions to a minimum is discussed.

* * * * *

Introduction

In the early transmission lines, comparatively little was known about the essential characteristics required in the insulation of transformers and other equipment attached to the line. In addition to this, there was no information as to the nature of transients and the number to which the station equipment would be subjected. It was clearly recognized, however, that lightning could impose stresses for which it would be impossible to provide sufficient insulation to prevent breakdown unless a safety valve were present.

It was only natural that an attempt was made to limit the voltage by the use of a discharge gap or lightning arrester. In many cases there was little difference between the protecting gap and the lightning arrester as to the ability to limit the overvoltage. It was recognized, however, that the operation of a protective gap generally caused an interruption due to the flow of normal frequency current following the high-voltage discharge. The desire to prevent the power outage following a discharge of the gap has led to many schemes for limiting the flow of current in the gap or for interrupting it.

It was recognized that the ideal lightning arrester should have a high discharge rate, small time lag, and should prevent the flow of normal frequency current, thereby giving it a great advantage over a limiting gap which would cause an interruption or material drop in voltage.

Records, as well as operating experience, show that stations may be subjected to a wide variety of transients. Although in practise it is necessary to limit the magnitude of the transients which may cause damage to the equipment, it is very desirable that any limiting device does not function unnecessarily and cause an interruption. This is particularly true where a protecting gap is used which does not have the ability to clear the line or prevent the flow of normal frequency current following discharge. This leads to the use of a limiting gap of relatively small time lag for transients of very high magnitude and steep front, in parallel with an arrester of greater time lag which will function for the many transients of lower magnitude without interrupting service.

Factors in the Protection of Station Insulation

It is exceedingly difficult to coordinate station insulation for the wide range of transients likely to be imposed, as the flashover characteristics may be quite different as to time lag and polarity, even though in close agreement at normal frequency. This is evident by reference to Fig. 1.

Owing to difference in time lag, a very high overpotential suddenly applied may cause one type of insulator to flash over, whereas a transient having a much lower crest value may cause another insulator to arc over. Furthermore, the difference in breakdown voltage for a negative and a positive transient may be quite large for bushings, bus insulators and the internal insulation of transformers and circuit breakers. A bus insulator or a disconnecting switch may have a high effective flashover under a negative impulse and a much lower flashover under a positive impulse. However, for some other piece of equipment, the reverse may be true. In station insulation it is therefore not only necessary to take into account the time lag in flashover, but the effect of polarity and oscillations as well.

In operation one station may be subject to very severe overpotentials, whereas the transients affecting another station may be made up largely of disturbances of lower magnitude. The transients imposed upon a given station will necessarily depend upon the storms affecting the station or the connecting lines, the number and magnitude varying greatly with the location and the line insulation.

A lightning arrester or a limiting gap which will protect station insulation against overvoltage transients has long been regarded of great economic importance, for it is evident that if a gap or lightning arrester may be depended upon to protect the insulation similar to the safety valve on a boiler, it is possible to lower the level of insulation and thereby effect a very material saving. To be fully effective, however, the limiting device must

*Chief Engr., Ohio Insulator Co., Barberton, Ohio, and Consulting Engr., Ohio Brass Co., Mansfield, Ohio.

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interrupt the discharge when the overvoltage is relieved without interfering with operation.

Characteristics of Several Forms of Limiting Gaps

Fig. 2 shows the impulse flashover values for a simple type of gap formed by two 5/8 in. rods. Fig. 3 shows the flashover characteristics for a ring gap. Fig. 4 shows the time lag characteristics for a 22-in. horn gap.

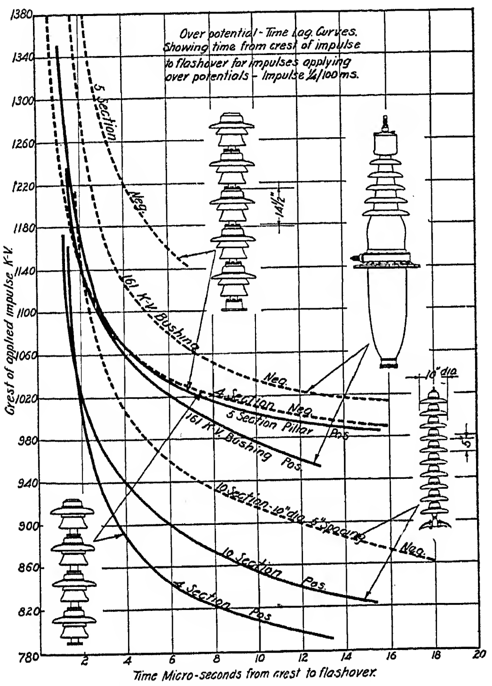

Fig. 1—Overpotential—Time Lag Curves

In considering the several types of gaps shown above, it will be noted that the flashover voltage tends to increase very rapidly for short waves or for short time intervals. Since equipment needs protection most for very severe direct strokes at or in the immediate vicinity of the station, it is evident that many gaps may not afford the desired protection, although having an arcing voltage comparatively low for less severe impulses or those of longer duration. It is also evident that the characteristics of some gaps may be widely different for a positive and a negative impulse. If the gap is to furnish efficient protection it is therefore essential that it have characteristics corresponding to those of the insulation in parallel with it, both with respect to polarity and time lag.

Fig. 5 shows the impulse time lag characteristics for a bushing and several types of limiting gaps. Typical time lag curves for only one polarity are shown in order to avoid confusion.

In order to provide protection for the bushing having the characteristics shown in curve A, it will be assumed that the voltage limiting gap should flashover at not more than 90 per cent of A for all time lags as indicated by curve B. The characteristic curve of the horn gap shown as C has approximately the same general shape as A for negative impulses but tends to depart somewhat for the shorter time lags. Reference to Fig. 4, however, shows that the difference between positive and negative sparkover voltages is likely to be very large for that type of gap.

By reference to Fig. 5, it will be noted that curve D for the ring gap intersects curve A at approximately 585 kv. For transients having a crest below 585 kv., the ring gap sparks over first. For transients having crest values above 585 kv., however, the ring gap does not afford protection as the bushing will arc over first. It is therefore evident that where the ring gap is set as high as possible so as to minimize unnecessary interruption for transients of lower crest and longer duration, it will afford little or no protection for severe overvoltages when most needed. To provide protection, the ring spacing and flashover voltage must be materially reduced thereby causing unnecessary interruptions. For example, the minimum flashover voltage shown in Fig. 5 for the ring is approximately 440 kv., as against 520 kv. for the bushing and a further reduction of the flashover voltage will be necessary in order to provide protection for high overpotentials.

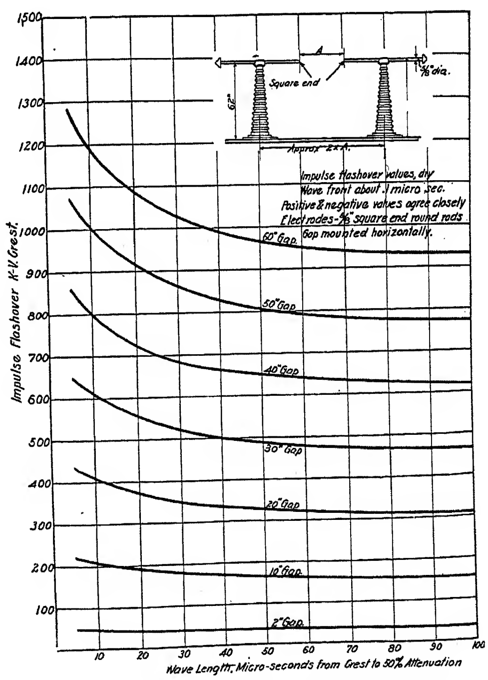

Fig. 2—Impulse Flashover Values, Dry

Curve E shows the characteristics of a sphere gap of negligible time lag having the same minimum flashover voltage as the gap of curve B. As the time of voltage application is decreased, however, insulators and most equipment will withstand much higher impulses. It is therefore desirable that the impulse flashover of the limiting gap be higher for short time lags. If a limiting gap having the rather uniform flashover characteristics of a sphere gap as shown in curve E is used, unnecessary

flashovers will result owing to the sparkover of the gap for the shorter time lags where the insulation can well withstand higher voltages. Of course, it is possible to modify materially the characteristic of the sphere gap by changing the relative sizes of the spheres with respect to each other and with respect to the gap spacing. As will be discussed later, this principle is made use of with the control type of gap in order to obtain the characteristic of curve A', Fig. 5, closely approximating the ideal characteristic curve B.

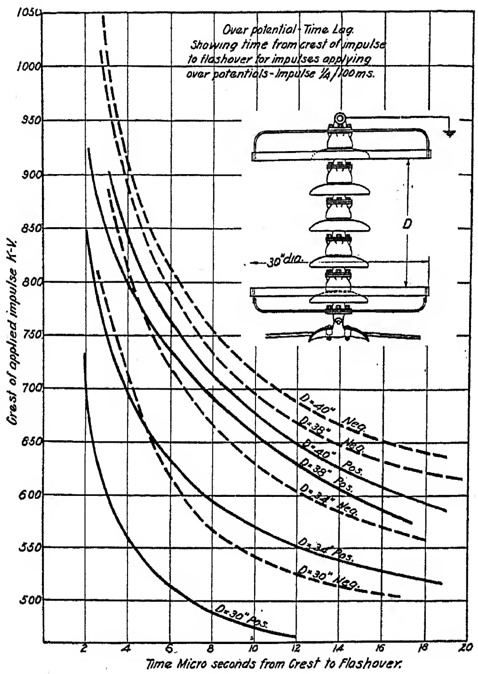

FIG. 3—OVERPOTENTIAL—TIME LAG CURVES

CONTROL TYPE OF LIMITING GAP

A protecting gap will approach the ideal in so far as the following can be carried out:

1. It should be possible to control the time lag characteristics such that the gap will afford protection over the range of transients likely to be imposed.

2. The negative and positive impulse breakdown characteristics of the protecting gap should correspond to that of the apparatus to be protected.

3. The flashover voltage of the gap should not be seriously lowered by rain or surface contaminations.

4. Means should be provided for changing the relative negative and positive flashover voltages where it is desired to work within close limits or to protect apparatus or insulation whose properties are not fully known.

5. It should be possible to change the time lag characteristics without materially affecting the minimum flashover voltage.

6. Where the discharge of the gap will cause a serious service interruption, a fuse or other device should be used for clearing the normal frequency current following discharge.

7. The gap should not discharge under high frequency wave transients of comparatively low magnitude due to switching or arcing grounds.

A gap conforming to the above requirements should have improved characteristics over types in general use in the past and a description is given below of the development of such a gap.

A protecting gap using spheres of proper size in relation to the gap spacing and of unequal diameters to provide for the difference in negative and positive characteristics of the equipment protected, could be used to comply with conditions Nos. 1 and 2 above. However, the effect of water or other surface contaminations upon the flashover voltage of spheres would make it difficult to conform to condition No. 3. Means for absorbing or removing the water from the face of the spheres have been used to improve this type of gap for condition No. 3. However, the characteristics of the gap made in this way could not be easily changed and therefore would not meet the requirements Nos. 4 and 5.

As the spacing of a protecting sphere gap is increased to lengths well beyond the diameter of a sphere, the flashover voltage under positive impulse will tend to be

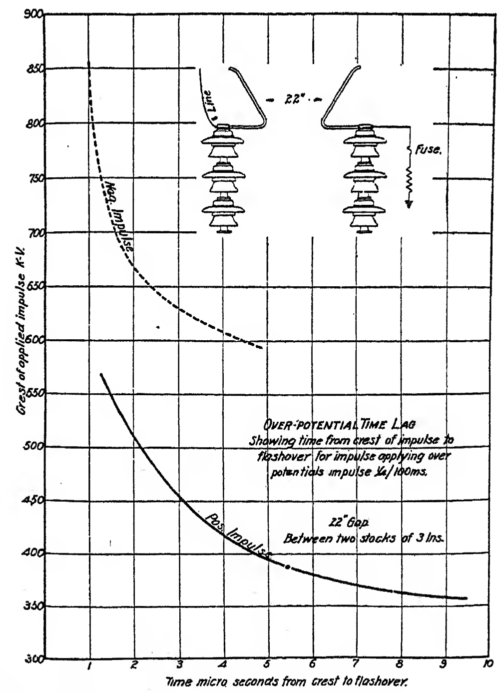

FIG. 4—OVERPOTENTIAL—TIME LAG CURVES

lower than under negative impulse. This can be offset by using a larger sphere for the live terminal and a smaller one for the ground terminal. Where the electrostatic fields of the two terminals of the gap are of equal intensity, the flashover will start from the positive terminal as it takes a lower density to start discharge with a positively charged electrode. It is therefore apparent that by controlling the flux density at the

surface of a gap terminal, it is possible to vary the discharge voltage.

This last principle is made use of in the newly developed limiting gap, one form of which is shown on Fig. 5. The control shields shown in the vicinity of the gap terminals can be moved forward or backward to change the field density and regulate not only the time lag but the difference in negative and positive sparkover characteristics.

The results of tests in Fig. 6 show that the relative negative and positive flashover voltage of the protecting gap may be readily controlled by changing the flux density of the arcing tip which is positive. It is evident that any screen or shield may be used to accomplish this. It is however, important that the shield does not cause erratic performance by becoming one of the electrodes so that an arc will be struck between shields or between one of the tips and a shield.

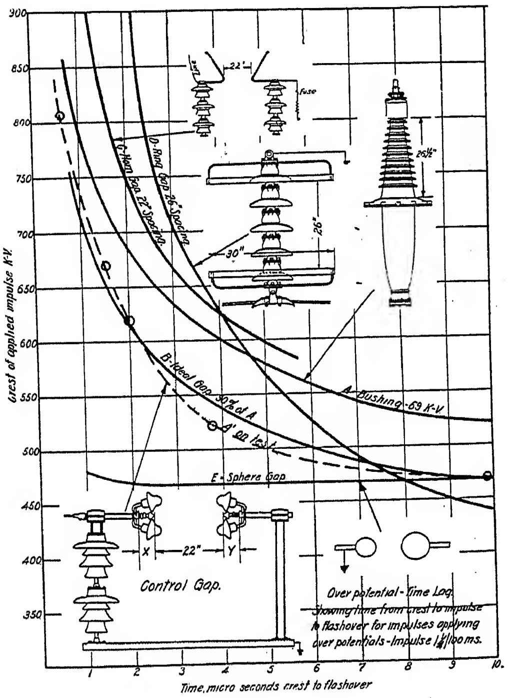

Fig. 5—Overpotential—Time Lag Curves

By using the insulated type of shield, this difficulty is prevented, even though the shields may be nearer together than the discharge points. This is particularly important under wet conditions or where the gap may be subjected to an oscillation, for under these conditions a gap having bare metal shields generally has a very low flashover voltage. Reference to Fig. 6 shows that the control of the flux density of the discharge points by means of the insulated shields is very effective in changing the relative arcing voltage for negative and positive impulse. In addition, it is seen that the arcing voltage under a damped wave or oscillation compares favorably with the arcing voltage under impulse thereby reducing unnecessary discharges.

In any discharge gap in which the electrodes are similar, the electrode or terminal on the live side generally has a much higher voltage gradient than that on the grounded side of the gap. It has been pointed out that increasing the size of this electrode would make it possible to reduce the gradient. Increasing the effective size of one electrode will also tend to increase the stress or lines of force on the electrode connected to the opposite side.

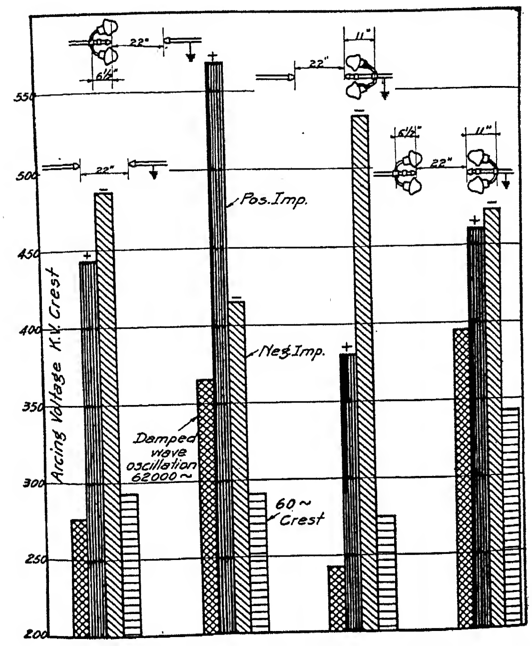

Fig. 6—Effect of Insulated Control Screens Upon Arcing Voltages of 22-In. Gap

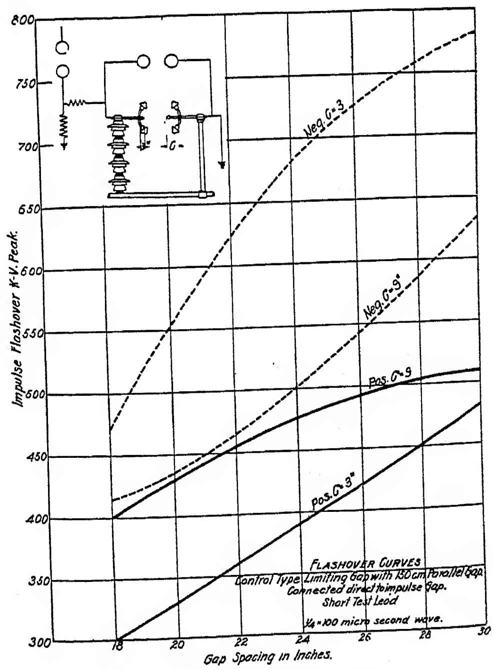

Fig. 7—Flashover Curves

Fig. 7 as well as Fig. 6 shows the change in relative

negative and positive impulse sparkover values which may be effected by changing the flux density on the electrodes with insulated control shields. Reference to Fig. 7 shows that the arcing voltage for negative impulse is approximately 75 per cent greater than for positive impulse where the controlled screens have the same relation to the arcing tips and where the control screens are well forward (C = 3 in.). Moving the control shield on the ground electrode back from the point of the gap, reduces the screening on this electrode so that the discharge will start at a lower voltage for a given distance between terminals when the opposite terminal is negative. Moving the shield back on the ground side also reduces the flux on the live terminal so that the arcing voltage will be raised under the positive impulse. For example, by increasing C to 9 in. and

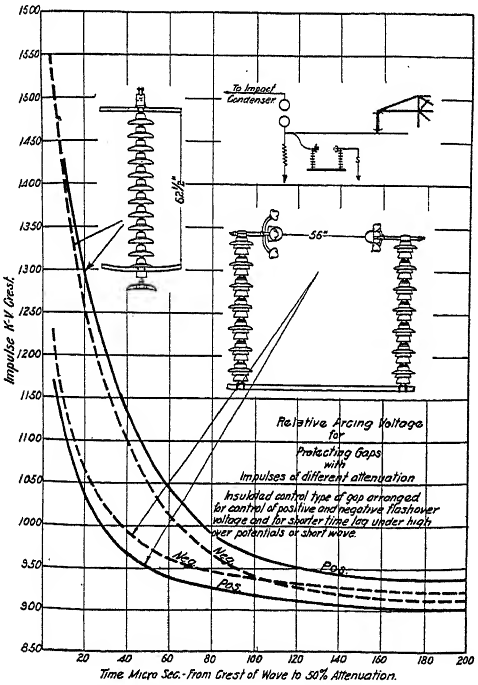

FIG. 8—RELATIVE ARCING VOLTAGE FOR PROTECTING GAPS WITH IMPULSES OF DIFFERENT ATTENUATION

reducing the screening effect of the control shield on the ground electrode, the positive and negative impulse values are brought in close agreement. Advancing the control screen on the live side toward the gap opening, has the same general polarity effect as moving the control screen on the ground side away from the gap opening.

The control gap allows one to make use of the effect of polarity and gradient upon arcing voltage similar to that with the point-to-plate discharge. It is evident that the arcing voltage will be increased for a given gap spacing where the stress upon the arcing tips is reduced by advancing the control shields. Therefore, by using the control shields, it is possible to reduce the gap spacing and the time lag for a given flashover voltage. It necessarily follows that the characteristics of the gap may be changed materially, both as to time lag and the effect of polarity upon discharge voltage by simply adjusting the control shields. As the projecting insulated control screen is affected but little by high fre-

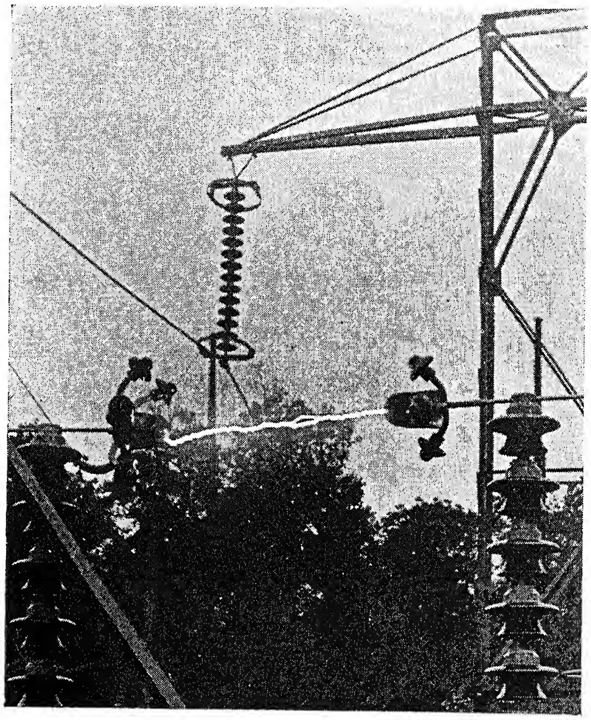

FIG. 9—POSITIVE IMPULSE 950-KV. CREST

Arcing-control gap in multiple with ring-equipped string. Wave ¼ x 100 microseconds. Control gap in artificial rain

quency transients, it is possible to reduce the time lag without lowering the flashover voltage for waves of lower crest and long duration. In Fig. 8, the impulse flashover values for waves of different attenuation are

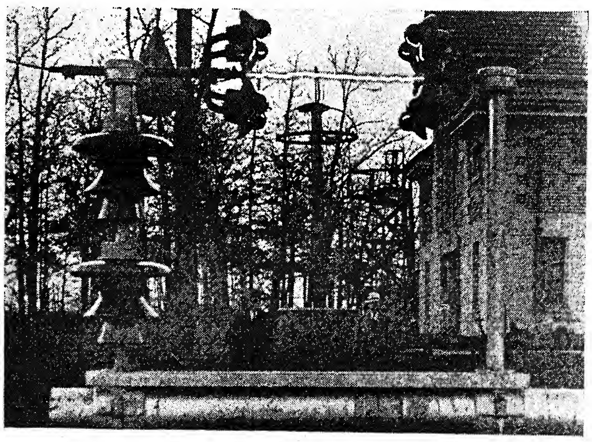

FIG. 10—SMALLER TYPE OF CONTROL GAP

given for a control type of gap and a ring equipped suspension string.

The control type of gap may be made in a number of different forms with plain shields or rings used in place

of the insulated control type of screen. The latter has the advantage in that the gap does not have a low flash-over value for oscillations and is affected but little by rain or conditions which may cause a gap having bare metal shield to be very erratic. Other forms of the gaps embodying the above principles may, in some cases, work out to much better advantage.

Fig. 9 shows the gap of Fig. 8 under test. The insulated support of the ground terminal may be replaced by

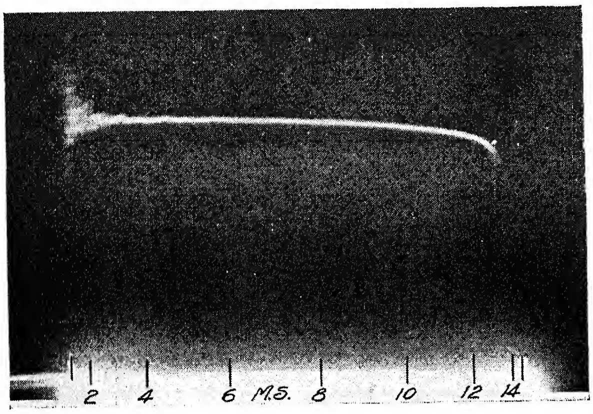

Fig. 11—Negative Impulse 460-Kv. $1\frac{1}{4} \times 100$ Wave Arcing 22-In. Control Type Protecting Gap

a metal support where the change in field will not disturb the gap or where no fuse or resistance is used between gap and ground.

An illustration of a smaller type of control gap is shown in Fig. 10. This gap was used in obtaining the data in Fig. 5. The oscillogram, Fig. 11, shows that the arcing voltage of the control gap is not unnecessarily low even for longer waves which would ordinarily arc a horn or ring gap.

Where a station is subjected to few high-voltage transients so that the gap discharges infrequently, the clearances and setting of the gap may be of little importance, provided satisfactory protection is obtained. However, where the station must withstand many transients, it may be desirable to adjust the gap to take all discharges and, thereby, protect the equipment fully. This can only be accomplished by trial in many cases. It is therefore necessary to be able to change the spark-over characteristic of the gap.

Where the limiting gap must operate frequently some form of clearing device may be added, such as a magazine fuse or circuit breaker. In general attention should be given to the use of a lightning arrester for limiting transients without causing an interruption, using the limiting gap only where necessary to afford absolute protection or where the expense of an arrester is not warranted.

Conclusion

1. An ideal voltage limiting gap for station use is one wherein both polarity and time lag characteristics are capable of adjustment over a sufficient range to allow protection of all apparatus in parallel with it for any transient wave form imposed upon the station.

2. A form of limiting gap has been developed making use of adjustable electrostatic field controls for the gap electrodes. This allows the breakdown characteristics of the gap, both as to time lag and arcing voltage for positive and negative impressed waves, to be varied over a wide range so as to conform to the characteristics of the station insulation.

3. The use of a control type of gap for limiting the most severe transients only and a lightning arrester for the remainder will, undoubtedly, provide the best station protection and operation available at present.

Discussion

For discussion of this paper see page 697.

Suspension Insulator Assemblies

Their Design and Economic Selection

BY J. J. TOROK*
Associate, A.I.E.E.

and

C. G. ARCHIBALD†
Non-member

***Synopsis.**—This paper presents a summary of data from extensive impulse and 60-cycle laboratory flashover tests upon suspension insulator strings relative to unit shell diameter and unit spacing. The insulating qualities and the economic advantages of the various sizes of units are discussed with due regard to the most recent lightning protection theories. Both steel tower and wood pole high-voltage transmission line insulation are given consideration from the electrotechnical and economic standpoint.*

Introduction

The pioneer high-voltage transmission lines were insulated to the extent deemed sufficient for the normal operating voltage. The designers based their selections upon the experience gained from operating lower voltage lines. The performance of these lines is responsible to a large extent for the arbitrary quantities of insulation provided for various voltage classes. However, the average operating record of these higher voltage lines was not, in general, satisfactory. A cursory analysis of the faults and their causes showed that the majority were caused by lightning.

The formation of the majority of the lightning surges was explained at that time by the "induced voltage theory." The possibility of strokes terminating upon conductors was admitted but it was believed that these were infrequent and impossible to protect against. Several methods were tried in an attempt to reduce the lightning disturbances. At first increasing the number of suspension insulators, or "over-insulation" was tried. The resulting improvement in performance was inappreciable and not commensurate with the increase in insulation. At the same time ground wires were installed as a possible remedy. In many of these latter cases material improvement in reduction of line outages was noticed but in others no great improvement resulted. A satisfactory explanation of these apparent discrepancies did not appear until the "direct stroke" theory was proposed.[1] Following this, extensive field tests[2] were made, in which the effectiveness of ground wires and tower footing resistances were determined. Subsequent analytical studies[3,4] have disclosed the wave form and magnitude of lightning voltages which may appear across the line insulation.

With the extensive and detailed knowledge of the voltage for which a high-voltage transmission line must be insulated, the designer is now able to make a more logical choice of insulation rather than rely upon tradition. However, there are economic influences as well as electrical characteristics to be considered in the design of high-voltage lines.

*Trans. Engr., Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

†Insulator Engr., Westinghouse Elec. & Mfg. Co., Derry, Pa.

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Presented at the Great Lakes District Meeting of the A.I.E.E., Milwaukee, Wisconsin, March 14-16, 1932.

Standards for Comparisons

Past and present papers dealing with studies of lightning phenomena have brought us to a point where we can make a logical approach to the selection of sizes of units and their spacings in suspension insulator assemblies. In a paper presented by Messrs. Fortescue and Conwell[3] and a discussion[4] on this paper, it was shown that on a transmission line equipped with ground wires, the reflections from tower footings altered the wave form so that the resultant voltage across the insulators had the approximate shape of the wave shown in Fig. 1A. Field tests have substantiated these analytical results[4] as can be seen by comparing Figs. 1A and 1B. If the insulators are to be subjected chiefly to this type of wave it is essential to determine how the insulators will perform under these types of waves.

During the early search for the proper amount of insulation for high-voltage transmission lines, various sizes of suspension insulator units, as well as different numbers of units per string, were used over a period of a decade or more. At the present time, operating and construction companies continue to insulate their new lines with the same size units previously employed, partly because of the advantages of interchangeability and partly due to acquiescence to a standard established some years ago before the requirements for the insulation of lines against lightning were understood. In the past the actual advantage of one size over another had not been definitely established despite the multitude of sizes operating more or less satisfactorily under a wide range of conditions. Consequently, an extensive group of laboratory tests was arranged to study the effects shell diameter and unit spacing have upon the electrical and economic characteristics of suspension insulator string assemblies applicable to high-voltage service.

To make an investigation of the characteristics of insulators comprehensive, all tests should be made on a volt-time basis, that is, waves of long duration should be applied to the insulators and the magnitude varied so that flashover will take place in time lags ranging from 16 or more microseconds down to one or two. The performance of the insulation should then be judged according to its application. If it is to be used on ground wire installations the performance of the insulation should be judged by the flashover voltage at short periods of time. If it is to be used on lines not equipped

with ground wires its performance should be judged by long time lags as well as short. However, the majority of the transmission lines now being planned and constructed are to be equipped with ground wires. Therefore, the emphasis of this analysis will be placed on short time flashovers.

Laboratory Tests

Tests Establishing Comparison Basis. Laboratory tests were made upon suspension insulator assemblies

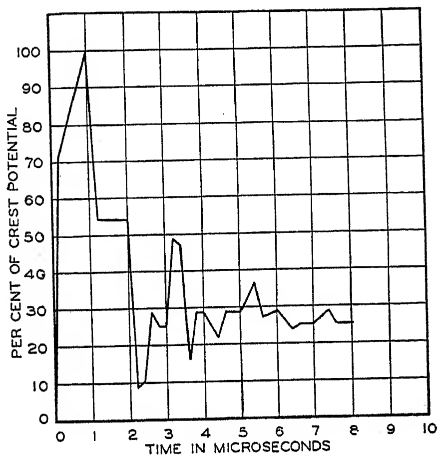

Fig. 1A—Calculated Wave Shape

and bushings with the voltage wave shown by the oscillogram of Fig. 2. This voltage wave very closely resembles the calculated curve shown in Fig. 1A. These tests were conducted to determine the relation between flashovers under this type of wave and those given by standard volt-time curves.

The wave shown in Fig. 2 was obtained by shunting the discharge circuit of a surge generator, arranged for the production of a smooth 1½-40 microsecond positive wave, with a needle gap having a resistance in series to ground. The needle gap flashing over near the crest of the wave results in an abrupt drop in potential across the insulators under test. However, the resistance in series with the gap limits the voltage to approximately 37 per cent of the crest voltage. Thereafter the voltage is reduced to a negligible value in five to six microseconds because of the energy dissipated by the resistor. In this manner, the wave shown in Fig. 1B was simulated.

The following crest voltages are the minimum values causing flashover of insulator assemblies made up of 8, 10, 12, and 14 units, 10-in. diameter and 5¾-in. spacing, when subjected to this type voltage wave.

No. units	Crest voltage (kv.)
8	1,050
10	1,240
12	1,550
14	1,800

It can be seen from volt-time curves shown in Fig. 5 that these voltages correspond to flashovers at some time lag lying between one and two microseconds on 1½-40 positive wave volt-time curves. The minimum flashover voltage decreases and corresponds to longer time lags when longer duration test waves are used. It has been shown in previous papers[4] that longer waves than shown in Fig. 1A result when tower footing resistances are high. These papers showed that effective ground wire protection can only be had with tower footing resistances below 10 ohms. The voltages appearing across the insulators with higher tower footing resistances become so high that it is uneconomical to insulate against even moderate intensity strokes. Because of these reasons, the comparison of insulators for use upon lines properly equipped with ground wires should be made at two microseconds as taken from volt-time curves.

It is realized that special field conditions may require a different wave upon which to base analyses; consequently, the insulator characteristic tests disclosed in this paper were made with a 1½-40 microsecond positive wave. The wave magnitude was varied to give time lag of flashover ranging from one to twenty microseconds. These results were then plotted in the form of volt-time curves. Insulator performance can be ascertained from these volt-time curves for any predetermined wave shape. In the event that some peculiar local condition necessitates the choice of an irregular wave as a basis of test, it is only necessary to test one combination of insulators with this wave. The flash-

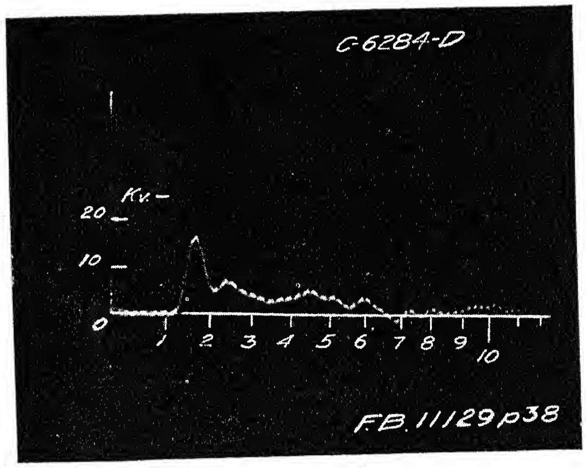

Fig. 1B—Actual Wave Shape from Field Tests

over voltage values then may be referred to the volt-time curve to ascertain the corresponding time lag. The comparison of various type insulators can be made at the corresponding time lag from their respective volt-time curves.

If the high-voltage transmission line is not protected by ground wires or if the ground wire protection is inadequate, then the voltage wave form across the insulators is not attended by reflections from tower footings; therefore, the wave shape approximates that of the lightning discharge. The wave form in this case is of longer

duration, ranging from 10 to 50 microseconds. Since the high waves will invariably cause flashover at short time lags the impulse strength under these conditions is not important and therefore insulator characteristics should be compared under long time conditions rather than under a wave form such as shown in Fig. 1B.

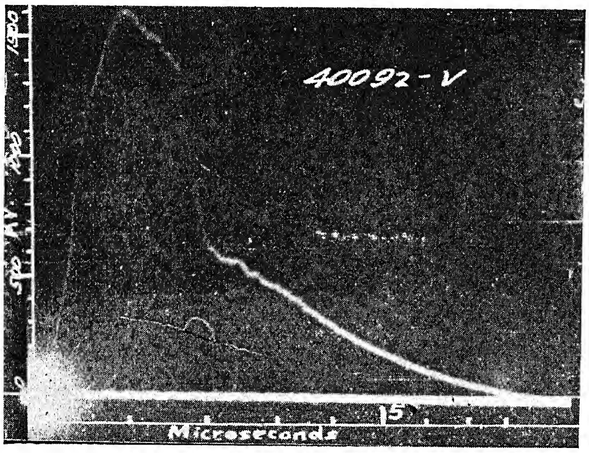

Fig. 2—Laboratory Test Wave

Diameter and Spacing Tests. Suspension insulators of the cemented cap and pin type were made up in a variety of shell diameters and unit spacings at the Westinghouse Porcelain Insulator Plant for pursuance of a series of tests to determine the effects these dimensions have upon the flashover characteristics of suspension

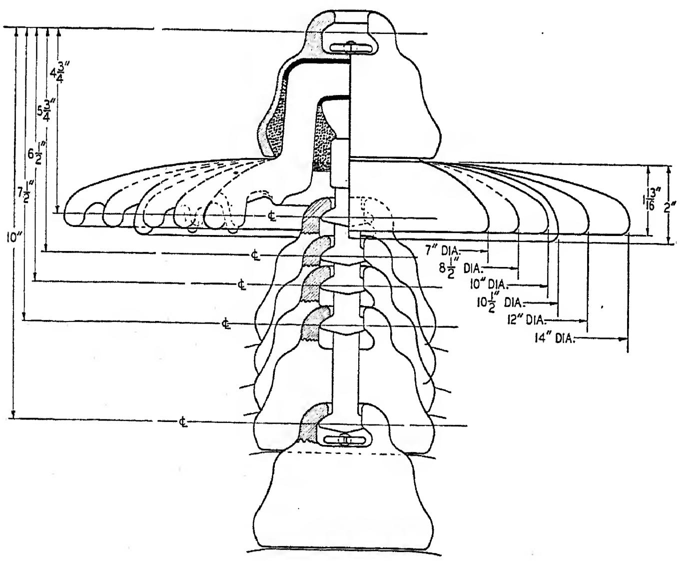

Fig. 3—Illustration of Insulators Tested

insulator assemblies. The porcelain shells were standard in head, petticoat, and shed droop design except a special 10½-in. diameter unit designed for coastal duty. This unit had 10 per cent greater shed droop than the standard design. The heads of all these units were the same size such that all hardware was identical. This head size was that of a standard strength unit now most commonly employed upon high-voltage lines. The fixation of these details in dimension and shape limited as much as possible any variations in electrical characteristics other than those due to the unit diameters and spacings. The specific shell diameters made up for test purposes were 7, 8½, 10, 10½, 12, and 14 inches. The detail construction of the various size units is shown in Fig. 3. The design of the porcelain shells is based upon the requirements of practical application. These designs of porcelain shells are either those in common use or those most likely to be used in commercial production.

The unit spacings ranged from a minimum value possible to a maximum value considerably greater than any spacing which has been employed for line insulation. The specific spacings used in the tests were 4¾, 5¾, 6½, 7½, and 10 inches. These variations in spacing were

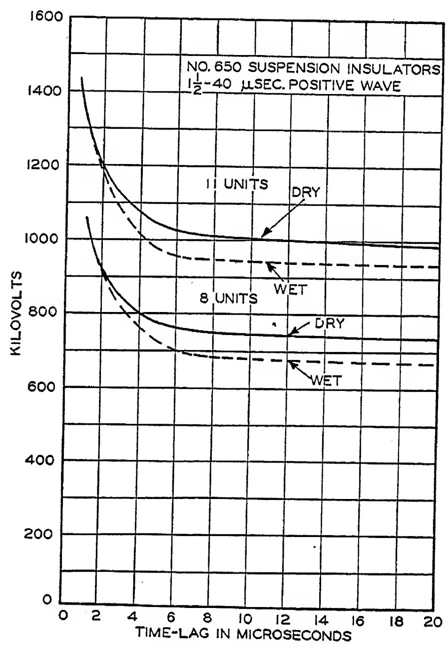

Fig. 4—Impulse Tests on Wet and Dry Insulators

obtained by pins of various lengths being screwed into a threaded socket provided on the stud. This arrangement materially reduced the number of units required for the tests and afforded more comparable data because identical shells of each diameter were tested at all spacings.

Conventional 60-cycle dry and wet flashover tests were performed upon strings of 4, 8, 12, and 16 units of each dimension combination. That is, the units in each string assembly were all of the same diameter and the same spacing. No non-uniform string assemblies were tested because these conglomerations have been found to be very impractical line assemblies. These tests were made in accordance to A.I.E.E. Standard No. 41 pertaining to the conduct of normal frequency flashover tests.

Impulse flashover tests were made upon the same string assemblies with a 1½-40 microsecond lightning

wave of positive polarity. A cathode ray oscillogram was made of each impulse flashover for accurate record. Over 700 oscillograph records were obtained in these tests. These data were corrected for atmospheric temperature and density, but no correction was made for the recorded relative humidity which ranged between 10 and 15 per cent. Correction factors for humidity have not been established. The question of values less than two microseconds, and that the effects of moisture upon impulse flashover become appreciable only for time lags of three or more microseconds.

A portion of the data obtained from the impulse tests on various insulators is given in the replotted curves shown in Fig. 5. These curves were obtained from time lag curves, each of which was plotted from an average of 13 points. Some combinations of unit dimensions

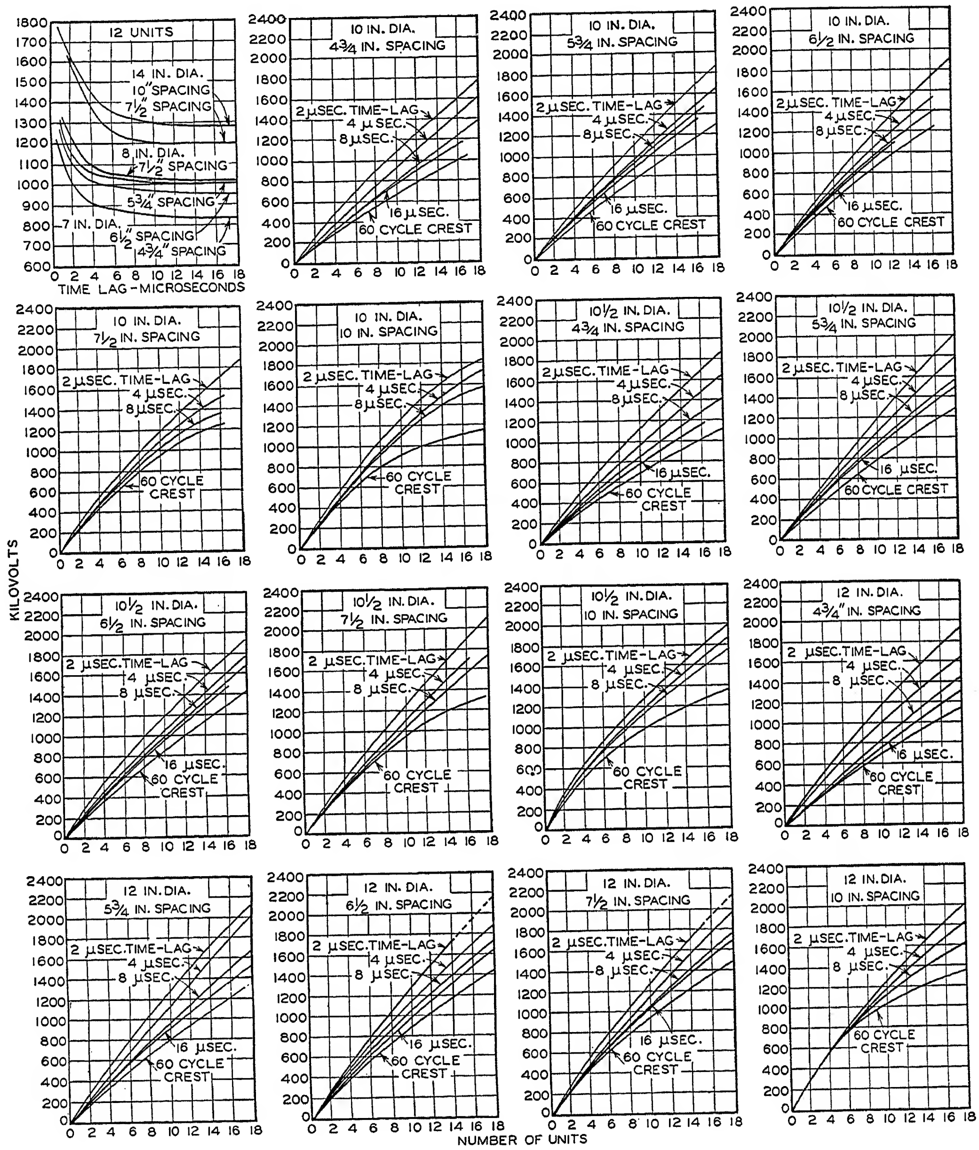

Fig. 5—Flashover Characteristics of Suspension Insulators

humidity correction factors is at present before the Lightning and Insulator Subcommittee of the A.I.E.E. It is the opinion of the authors that the variation in impulse flashover voltage caused by changes in humidity during the tests is negligible. This opinion is based upon wet and dry impulse flashover tests. The average variation in time lag curves caused by standard rainfall is illustrated in Fig. 4. These curves were plotted from more than 25 points each. It can be seen that the wet and dry curves practically coincide for were not tested under the impulse wave as the test would become too long. The omitted combinations were so selected that they could be readily interpolated from the other tests. A basis for this consideration may be readily seen in Figs. 6, 7 and 8. A large number of tests upon the 14-in. diameter unit was omitted because these units were found to be impractical with present service requirements and manufacturing knowledge.

It is to be remembered that in making high-voltage measurements it is difficult to keep the limit of error

below about 5 per cent, even though great care is exercised. For example, on the curve showing an insulator string to have a flashover of 1,000 kv., the flashover may, under some conditions, be 1,025 or 975 kv.

The units found to be most effective under impulse tests were later subjected to arc tests to check the behavior of these diameter shells under high current impulse and power follow arcs. An eight-unit string of 12-in. diameter shells was subjected, in addition to the normal impulse flashover characteristic tests, to more than 100 severe impulse flashovers at an approximate rate of voltage rise of 6,000 kv. per microsecond and a current of approximately 5,000 amperes. These same units were then subjected to 60-cycle arcs of approximately 3,000 amperes for two seconds duration. Since the 10-in. unit is the most common in usage it was chosen as a reference for comparison with the 12-in. unit. Under the impulse tests neither the 10-in. nor the 12-in. units failed either by puncture or breakage. Under the 60-cycle arc tests the performance of the 12-in. unit was as good, if not slightly better, than the 10-in. unit. These results are substantiated by available service data.

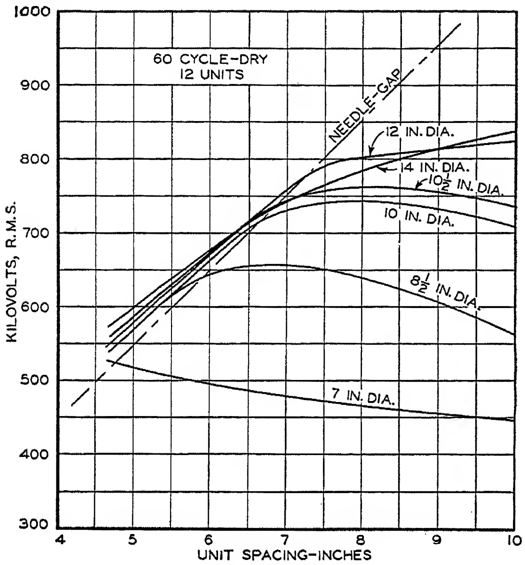

Fig. 6—Effect of Diameter and Spacing Upon 60-Cycle Flashover

Diameter and Spacing

Impulse. The effect of unit shell diameter and unit spacing upon the flashover characteristics of suspension insulator strings is shown by representative data in Figs. 7 to 11. The economic characteristics of these factors are shown in Fig. 12. The data shown in these figures are representative in that they conform to average steel tower line conditions. A two-microsecond time lag basis of comparison is used because that value of time lag corresponds to a tower footing condition of 10 ohms or less. Higher tower footing resistances permit the lightning voltages to rise well above any economic steel tower insulation.

Figs. 6 to 10 show flashover characteristics of string assemblies consisting of 12 units. A string of 12 units was arbitrarily chosen to give a representative example of the characteristics of an insulator string with reference to shell diameter and unit spacing.

Figs. 11 and 12 show impulse flashover and economic characteristics of the various unit combinations with regard to string length of the insulator assembly itself without recognition being given to the number of units. String length was chosen as the basis of reference as it is a controlling factor in supporting structure dimensions.

60 Cycles. At normal frequency there is but slight difference in the flashover voltage due to an increase in shell diameter above $8\frac{1}{2}$ in. It can be seen from Fig. 6 that for each diameter, except 7 in., there is a definite spacing beyond which any increase in the spacing will give but slight or no increase in flashover. It is probable that for the 7-in. diameter unit the knee of the curve occurs at or below $4\frac{3}{4}$-in. spacing. The head size of the unit forms a greater part of the total diameter with the small diameter shell and thereby reduces the flashover distance around the shell by a greater proportion at the small diameter than it does with the larger diameters.

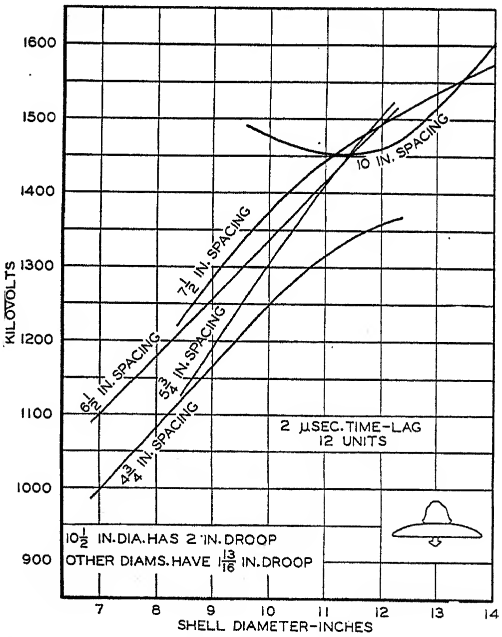

Fig. 7—Effect of Diameter Upon Flashover

It is to be noted from Fig. 6 that if the ratio of unit spacing to shell diameter is increased beyond 50 to 65 per cent, the flashover voltage falls below that of an equally spaced needle gap.

Wet flashover characteristics are of similar shape and relative position, being 60 to 75 per cent in voltage value.

It appears from Fig. 6 that at approximately $6\frac{1}{2}$-in. unit spacing the 60-cycle flashover voltage will be

about the same for all diameters from 10 to 14 in. Since the cost of larger diameter units is considerably higher, varying as some function of the diameter squared, it appears that large diameter units are not economical from a purely normal frequency standpoint. However, high-voltage insulation is no longer entirely judged upon its characteristics at normal frequency. The major consideration is now its behavior under impulse voltages.

Insulator Shields. The tests reported in this paper were made without auxiliary electrodes such as rings and horns. The effect of such devices has already been investigated.[11]

Cost Considerations. The designer is confronted by two major considerations in the selection of high-voltage transmission insulators. First, the permissible cost of

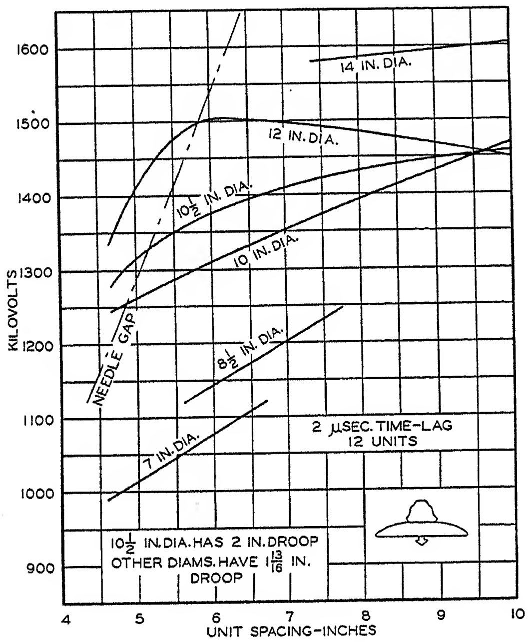

Fig. 8—Effect of Spacing Upon Flashover

line insulation. Second, the selection of the maximum insulation available for that cost

The first consideration comprises many variables, such as the importance of the circuit, the necessary reliability, and the character of the territory through which the lines pass. All these factors are characteristic of each installation, and require individual study of the combination peculiar to each line. The second consideration entails the selection of the proper insulator assemblies.

Steel Construction. It must be premised that a steel tower line be properly equipped with ground wires to provide lightning protection because it is not economically feasible to protect a line by over insulating.

Once the permissible cost of insulation has been decided upon, the economic suspension insulator unit can be selected from examination of insulation and cost characteristics similar to those shown in Figs. 11 and 12.

Fig. 11 shows the impulse flashover characteristics with respect to string length of various assemblies of 10-in. and 12-in. diameter shells. These flashover voltages were taken at a two microsecond time lag for reasons described previously. It is to be noted that for a given unit spacing and flashover voltage strings comprised of 12-in. shells are appreciably shorter.

As has been pointed out previously, the string length

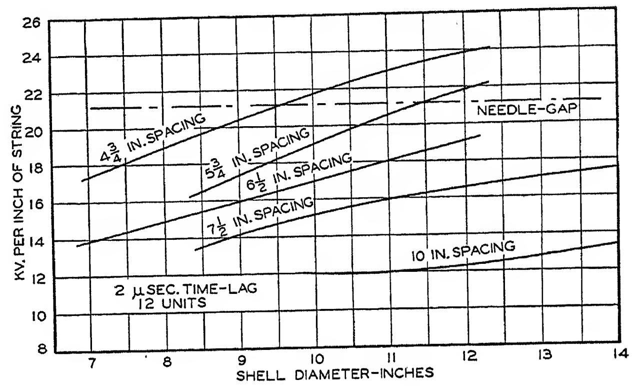

Fig. 9—Effect of Diameter Upon Flashover Efficiency

has an important bearing upon tower design. For a given flashover voltage, the shorter string length is more desirable on a tower cost basis as it may reduce the length of the arm and the height of the tower, or, with a given string length and the utilization of a more effective insulator, the flashover voltage will be increased and the number of outages due to lightning will be decreased. For example, in Fig. 11, a 1,500-kv. flashover at two microseconds time lag is provided by a string

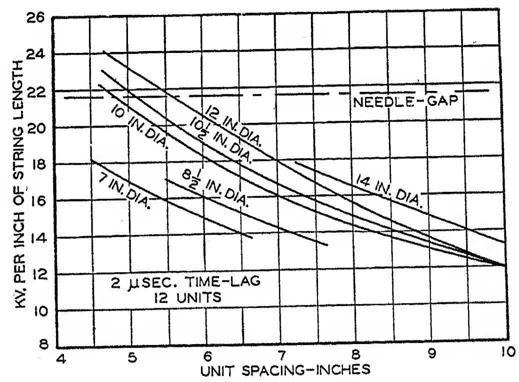

Fig. 10—Effect of Spacing Upon Flashover Efficiency

length approximately 65 in. when composed of 12-in. diameter units spaced 4¾ in. and by a string length approximately 91 in. when composed of 10-in. diameter units spaced at 6½ in. A difference of 26 in. exists between the length of these strings. A steel tower design having the same tower clearances and height required by the long string would, of course, be appreciably heavier and more expensive than a tower designed for the short string. Using a string length of 91 in., the twelve 4¾-in. units have a flashover of 2,050 kv. as

against 1,500 kv. for the ten 6½ units, or an increase of 550 kv. by using the more efficient combination of diameter and spacing. These factors should exert considerable influence upon the choice of properly dimensioned suspension units.

Economic Selection. The two-microsecond time lag

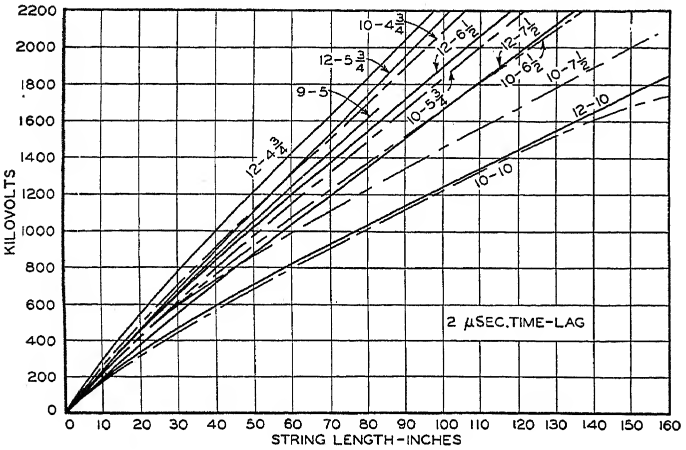

Fig. 11—Flashover Characteristics with Respect to String Length

flashover characteristics of Fig. 11 were incorporated with the cost of units to provide the economic comparison curves of Fig. 12. The cost values employed were those to a purchaser with packing and shipping expenses allowed to a shipping point 500 miles from the factory. This price is based upon the physical dimensions of the unit. The factors determining the price are, head

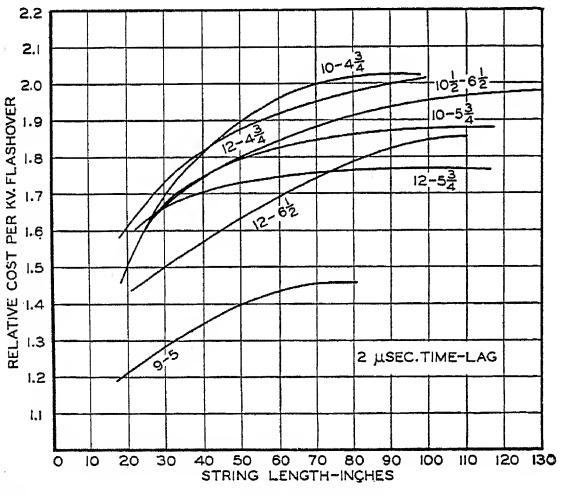

Fig. 12—Relative Insulation Cost of Various Sizes of Units

size, shell size, and total volume, as given by the maximum dimensions of the porcelain shell. The head size determines the hardware cost factor. The shell size determines the amount of plastic material and consequently the size of moulds, etc., used in manufacturing operations. The total volume indicates the necessary kiln volume for firing, type of packing or crating, and space for storage and shipping. Fig. 12 discloses that up to approximately 70 in. string length, a 12-in. diameter unit spaced 6½ in. provides two microseconds insulation at the least insulator cost. This point corresponds roughly to the amount of insulation considered as economically balanced for 138-kv. transmission upon steel towers adequately protected by ground wires.

Beyond a string length of 70 in., a 12-in. diameter unit spaced 5¾ in. shows the lowest insulation cost. Ten-in. diameter units spaced at 4¾ or 5¾ in. are those dimension combinations most commonly used at the present time. However, these units show much higher relative costs than 12-in. diameter units at the same spacing for string lengths above 30 in. Inci-

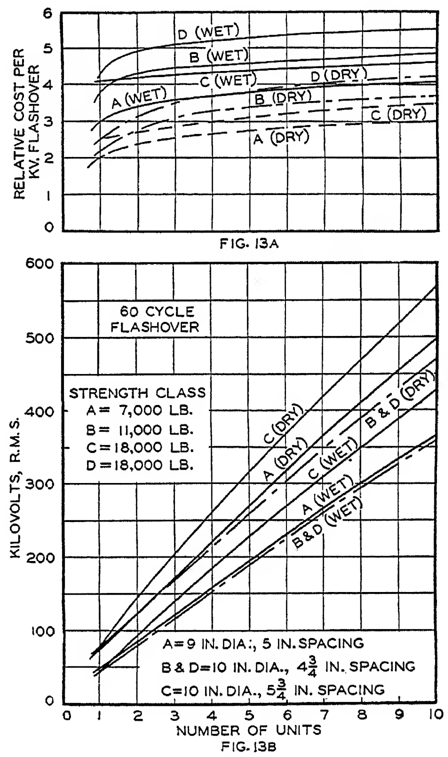

Fig. 13—Flashover and Cost Characteristics at 60 Cycles

dentally, a string length of 30 in. corresponds roughly to economically balanced insulation commonly employed for 69 kv. Examination of these flashover and cost curves clearly indicates a wide range of selection open to a high-voltage transmission line designer for choice of the proper insulation.

It must be noted that Figs. 11 and 12 must be employed simultaneously to determine the maximum insulation for the least cost. For example; at approximately 65-in. string length, a 12-in. diameter 5¾-in. spaced unit provides 1,500 kv. flashover at a relative cost of 1.76, or an arbitrary total cost of 2,640, whereas a 10-in. diameter 6½-in. spaced unit to provide 1,500 kv. flashover must have a string length of approximately 91 in. This gives an arbitrary total cost of 1.93 times 1,500 which is 2,900, approximately 10 per cent higher

insulation cost with 32 per cent undesirable increase in string length.

Very low cost units have been proposed for light duty purposes such as wood construction and lower voltage transmission lines. The economic characteristics of such a unit are shown by the 9-in. diameter, 5-in. spacing curve in Fig. 12. Obviously, for light duty service requiring insulation amounts corresponding to 60-in. string length (approximately 138 kv.) such units are economically desirable.

These unit dimensions were predicated upon the electrical and economic characteristics disclosed in these studies. The electrical characteristics compare favorably with those of much larger and more costly units as shown in Figs. 12 and 13. Impulse flashover string length characteristics of this unit are higher than a 12-in. diameter 6½-in. spaced unit and approach that of a 10-in. diameter 4¾-in. spaced unit.

Economically, for example, presume a 40-in. string of 9-in. diameter 5-in. spaced units. From Fig. 11 this combination gives 875 kv. at two microseconds time lag, and in Fig. 12 a relative cost of 1.35 per kv. is shown. A 12-in. diameter 5¾-in. spaced unit to provide 875 kv. must also have a string length of approximately 40 in. Fig. 12 shows a relative cost of 1.7. In equal comparison of electrical characteristics, the small unit shows a 20 per cent saving in cost. The economy of this smaller unit is a result of reduction in head size and consequently lower mechanical strength.

Wood Construction. The present practise on insulating wood pole lines is to use just sufficient porcelain to insulate the line for the normal operating voltage, and to depend upon the wood for impulse insulation. The insulator to be used for this type of service should have a low cost per kv. of 60-cycle flashover and mechanical strength slightly above that of the wood structure. Calculations of ultimate strengths of wooden structures now in use show that a unit having a mechanical strength of 7,000 lb. would be adequate. Fig. 13 shows the 60-cycle electrical and economic relationship of the various units most applicable to wood construction. The light duty unit cost is considerably below the other units, while the 60-cycle flashover per unit is only slightly below that of the popularly dimensioned unit of 10-in. diameter and 5¾-in. spacing.

Conclusions

1. The major factor of steel tower transmission line design is the shielding of the power conductors with properly located ground wires and maintaining the tower footing resistance as low as possible.

2. For towers of low footing resistance (on the order of 20 ohms) the wave is of short duration such that the minimum flashover voltage of the attached insulation corresponds to its breakdown voltage at two microseconds with a flat topped wave.

3. Suspension insulators should be judged upon their impulse flashover characteristics at two microseconds in applying insulation to well protected high-voltage lines.

4. The choice of insulation for lines not protected or inadequately protected by ground wires is based upon impulse flashover values at long time lags, of the order of 10 to 50 microseconds.

5. Impulse flashover space effectiveness of suspension insulators improves with increase in unit shell diameter at moderate spacings.

6. Impulse flashover space effectiveness for all unit diameters falls off with increase in unit spacing.

7. Flashover space effectiveness decreases with increase in string length.

8. The cost for a given impulse flashover voltage is generally lower for larger diameter units and is generally higher for larger unit spacings.

9. Available service data and laboratory tests show that the 12-in. unit is as serviceable as a 10-in. unit.

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Discussion

For discussion of this paper see page 697.

Normal Frequency Arcover Values of Insulators
as Affected by Size and Humidity

BY H. A. FREY*
Associate, A.I.E.E.

and

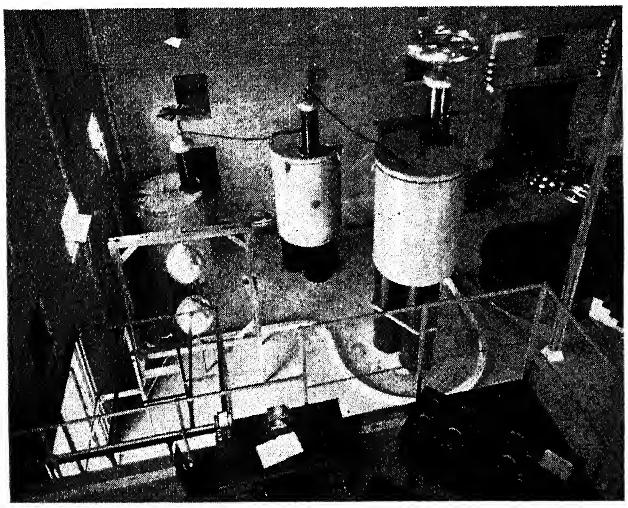
K. A. HAWLEY*
Member, A.I.E.E.

Introduction

DATA covering the electrical characteristics of comparable insulators as received from different sources are frequently conflicting. A study has been made to establish rules whereby such characteristics can be estimated from the dimensions of the insulators with a reasonable degree of certainty. A long series of tests has been made covering several months with repeated checks upon the same insulator units.

The variation of dry arcovers due to the influence of humidity changes also is considered with approximate correction ratios.

The equipment used for these tests in the High Voltage Laboratory of the Locke Insulator Corporation at Baltimore consists of three transformers in cascade to give 60-cycle voltages up to 1,050,000 r.m.s. Voltages less than 350,000 were obtained by using only one of the three transformers shown in the accompanying illustration. One meter spheres were used as the primary



Testing Equipment—High-Voltage Laboratory

standards for voltmeter calibration for subsequent observations. (A.I.E.E. Standard 4-53 to 57 inclusive.) The laboratory which is fully enclosed, can be kept at any desired humidity and temperature above that of the outside atmosphere. Special equipment is provided for this purpose. Routine tests upon insulators during cold weather were made at standard conditions, 25 deg. cent., (77 deg. fahr.), 65 per cent relative humidity; 0.608 in. mercury water vapor pressure. (A.I.E.E. Standard 410-100,—150). No air density corrections were made. (Standards 41-60 footnote.) Frequent calibrations from the spheres assured that insulators and sphere gaps, which are nearly equally affected by changes in air density, were under the same conditions of test. Thus insulator values were automatically referred to standard air density conditions.

Each recorded flashover measurement is the average of at least five successive arcover observations. Considerable variation was frequently noted.

TYPICAL OBSERVATION

Insulator	Pin type 35 kv. rating	Suspension 8–10 x 4¾
Total number observations	8.0	10.0
Minimum voltmeter reading	103.0	255.0
Maximum voltmeter reading	114.0	276.0
Average voltmeter reading	108.0	271.0
Per cent variation	11.0	8.2
Arcover value, kilovolts	111.0	428.0
Approximate transformer ratio	1,000 to 1	500 to 1
Barometer, inches mercury	30.14	30.01
Temperature, dry bulb	78.5 fahr.	80.0
Temperature, wet bulb	68 fahr.	68.0
Water vapor pressure	0.572	0.556

Suspension Insulators

Tests have been made upon suspension insulators of various spacings and diameters in the vertical position. (A.I.E.E. Standard 41-150, 151.) Insulators tested are:

1. Standard strength cemented 10 in. diameter, 4¾ in., 5⅛ in., 5¾ in. spacing.
2. High strength cemented 10 in. diameter 5¾ in. spacing.
3. Extra high strength cemented 12 in. diameter 7 in. spacing.
4. Hewlett insulators 10½ in. diameter 5½ in. spacing.

The 60-cycle dry and wet flashover values of 10-in. cemented type suspension insulators of all standard spacings are shown on Fig. 1, plotted against actual arcing distance, the shortest air path between electrodes. All dry values regardless of the insulator spacing, fall upon one graph within expected errors of observation. The wet values are affected slightly by design.

The characteristics of the high strength and extra high strength insulators are shown in Fig. 2. The graph for dry measurements is identical with that for the standard insulators, Fig. 1. The wet values again show variations due to design.

The characteristics of Hewlett insulators are shown in Fig. 3. Again the graph for dry values is the same as that of Fig. 1.

Dry arcover values, therefore, for all standard insulators depend solely upon the minimum arc length or tight string distance about the insulators. The wet values

*Locke Insulator Corp., Baltimore, Maryland.

Presented at the Great Lakes District Meeting of the A.I.E.E., Milwaukee, Wisconsin, March 14-16, 1932.

are affected by design. In general, insulators with the greatest clear arcing distance between shells show the highest wet values. The remarkable field performance of insulators with fish tail shaped hoods even in dirty locations suggests that insulators should not be judged as much by wet values as by dry. The dry arcover values are measurably affected by the proximity of grounded objects in the neighborhood of the insulator string. This is shown by the double graph at the upper end of the dry curves. The upper graph was obtained with clearances at least twice the length of the insulator string.

The wet values of Figs. 1, 2, and 3 are plotted as a function of the overall arc length. The most usual design basis is as a function of the wet striking distance or the summation of the air gap distances beneath the insulator assuming the wetted surfaces above are

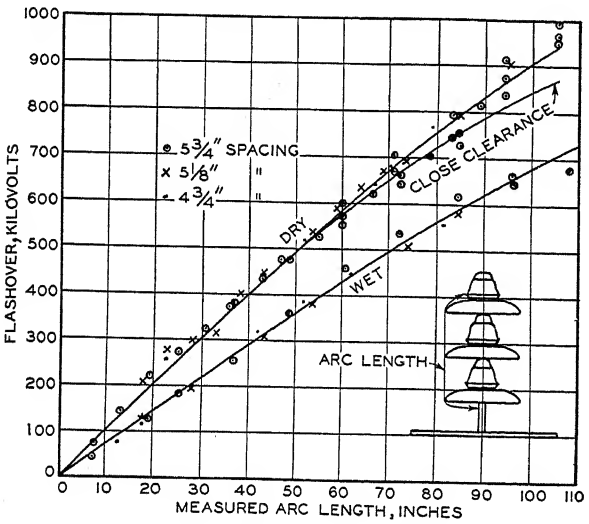


Fig. 1—60-Cycle Flashover Standard Strength Insulators 10 In. Diameter

short-circuited out by the water film. A more satisfactory check is so obtained as shown on Fig. 4.

Errors of individual observations and minor differences due to slight variations in manufacture are removed by obtaining insulator data from the above curves. Expected dry and wet arcover values of various standard and high strength insulators are given on Figs. 5, 6, and 7, plotted as a function of the number of units in the string, calculated from Figs. 1, 2, and 3.

Standard and high strength cemented insulators were tested also in the anchor position at an angle of about 10 degrees from the horizontal. The dry and wet values obtained also are shown upon Figs. 5, 6, and 7 in comparison with the arcover values for the vertical suspension insulators. The dry values are materially lower than for the insulators in the vertical. Wet arcover values are much higher in the horizontal than in the vertical. When the entire surface of the insulator is uniformly wetted a better voltage division between insulator units is obtained with the resultant higher voltage necessary to cause surface breakdown. The wet values in the horizontal are actually equal to, or in some cases with long strings, higher than the dry values in the horizontal.

Most of the tests in the horizontal were made without a jumper loop under the string or a conductor at right angles to it. Insulators in the horizontal position with

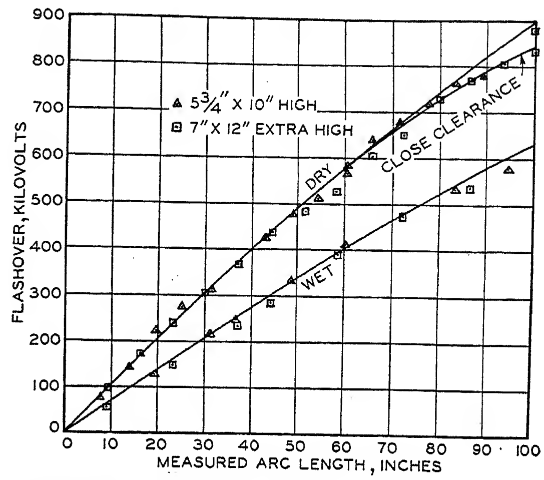


Fig. 2—60-Cycle Flashover High Strength Insulators

the conductor at right angles showed dry values materially higher than without it, yet not equal to the values obtained for the vertical suspension insulators.

Pin Type (Transmission) Insulators

Standard pin type insulators show fairly uniform arcover values in proportion to the distance between electrodes as shown upon Fig. 8. The small contact area of tie wires upon the porcelain causes heavy dielectric flux concentration upon the tie wires with resultant active corona streamer formation at voltages near arcover. Observed data show least uniformity of any of the insulator types because of the erratic action of these streamers. The results of this are shown in the above tabula-

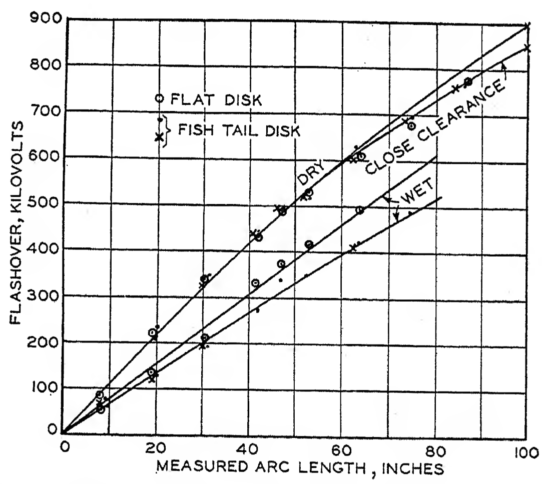


Fig. 3—60-Cycle Flashover Hewlett Suspension Insulators

tion of observed data and may be seen later on in the humidity studies. The dry flashover voltage per inch varies from 14 kv. for the small one piece and the smallest multipart insulators. It thence drops at a fairly uniform rate to 11.4 kv. for the largest multipart insulators nominally rated at 70 kv. For the single piece insulators the wet arcover value is uniformly 7 kv. per inch of total striking distance. A higher value is obtained for the standard multipart designs which are quite uniform at 8.9 kv. per inch of total distance. It is most interesting that more consistent comparisons are here obtained on the basis of total striking distance rather than wet striking distance. The low values for the small one piece insulators are due to the large proportion of wetted surface to the total striking distance.

Outdoor Apparatus Insulators

Outdoor apparatus insulators are pin types modified by adding cemented-on caps for attachments. In comparison with pin types these insulators show less variation between individual observations. The greater contact area of the cap causes less flux concentration and less corona streamer formation at voltages below flashover.

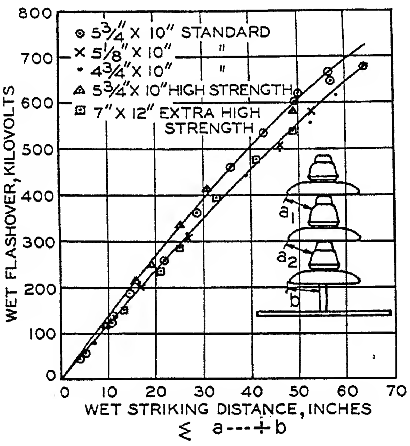


Fig. 4—Wet 60-Cycle Arcover All-Cemented Suspension Insulators

Fig. 9 for dry values covers single and stacked or multiple unit insulators to a height totaling 116 inches dry arcing distance. The lower part of the curve is practically the same as for pin types. The value at 100 inches of height is 7.3 kv. per inch.

The wet values plotted against total arcing distance show considerable variation but less when plotted against the wet striking distance. Both graphs are shown on Fig. 9.

Equipment Bushings, Solid Type

The performance of porcelain equipment bushings (not oil filled) is shown in Fig. 10, for ratings from 7.5 to 69 kv. The field established about the bushing determines the dry values. It is important that the conductor be in place inside the insulator as it will be in service. Much higher values will be obtained without it. The wet values, however, are the breakdown voltages for the water film upon the surfaces. For the larger bushings the wet value is higher than the dry.

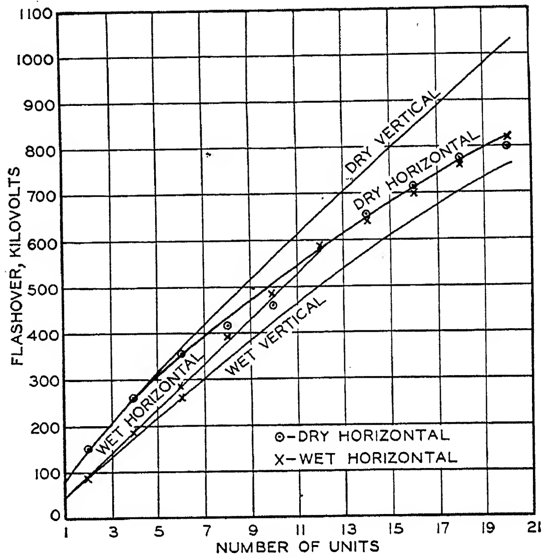


Fig. 5—60-Cycle Flashover Standard Strength Insulators

10 in. diameter, 5¾ in. spacing

Humidity Effects

The effect of humidity upon the dry arcover voltages of insulators may be compared on several different bases:

A comparison of flashover values on the basis of per

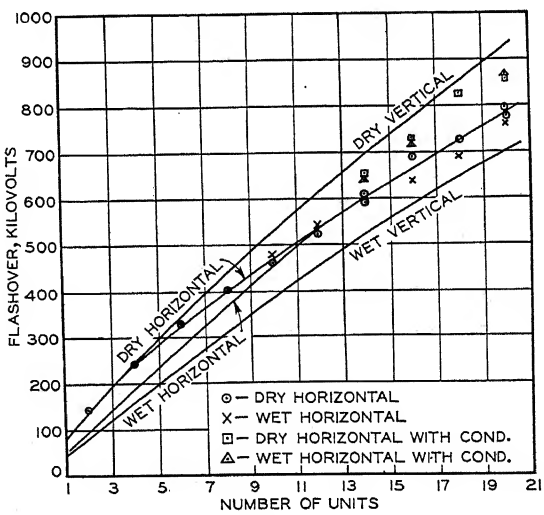


Fig. 6—60-Cycle Flashover Standard Strength Insulators

10 in. diameter, $5^1/_8$ in. spacing

cent humidity is not satisfactory, for it alone does not account for the total quantity of water in the air.

A comparison on the basis of absolute humidity or mass of water per unit of air volume may be satisfactory, but does not account for the molecular activity of the water.

A comparison of flashover may be made on the basis of water vapor pressure in some convenient units such as the barometric method, inches of mercury. This not only accounts for the number of water molecules present, but their velocity as well.

The water vapor pressure as the standard is recommended by A.I.E.E. Standards (41-100). All observations in the Locke Laboratory have been compared upon this basis.

No difficulty has been experienced in maintaining the laboratory at standard temperature (25 deg. cent.—77 deg. fahr.), during the cold months. The laboratory is not equipped to reduce the temperature below that of the outside atmosphere. All tests made during the winter are at the standard temperature and with humidity adjusted as desired. In warm weather tests were made at the outdoor temperature and humidity some-

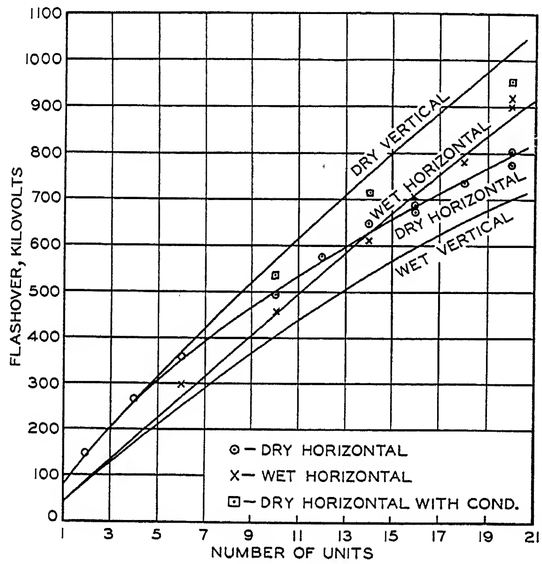


FIG. 7—60-CYCLE FLASHOVER HIGH STRENGTH INSULATORS

10 in. diameter, 5¾ in. spacing

times as high as 95 deg. fahr., and 85 per cent relative humidity.

The effect of humidity upon the dry flashover of standard suspension insulators is shown in Fig. 11. Average dry flashover curves have been drawn for strings of various numbers of units. By plotting upon semi-logarithmic paper equal ordinate distances give equal percentage variations. It is thus easy to compare percentage corrections for different strings and percentage variations from the average of individual observations.

The slope of all these average graphs is practically the same. When referred to 100 per cent at the standard water vapor pressure, 0.608 inches mercury the correction at 0.2 inches mercury is − 8 per cent. The correction, therefore, is approximately 2 per cent for each 0.1 inch of mercury variation of water vapor pressure.

The points of observation would be enclosed within an envelope of plus and minus 5 per cent from the curve of averages.

It, therefore, follows that even with the correction for humidity variations a variation of plus or minus 5 per cent in flashover values of suspension insulators must be

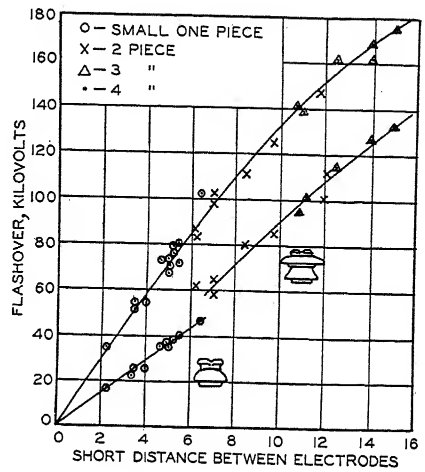


FIG. 8—60-CYCLE FLASHOVER PIN TYPE INSULATORS

expected. The insulators used were the same throughout the entire test.

The effect of humidity on flashovers of typical pin type transmission insulators is shown in Fig. 12. But one insulator of each type was tested. Variations shown are not due to manufacturing variations in different insulators. The individual observations fell

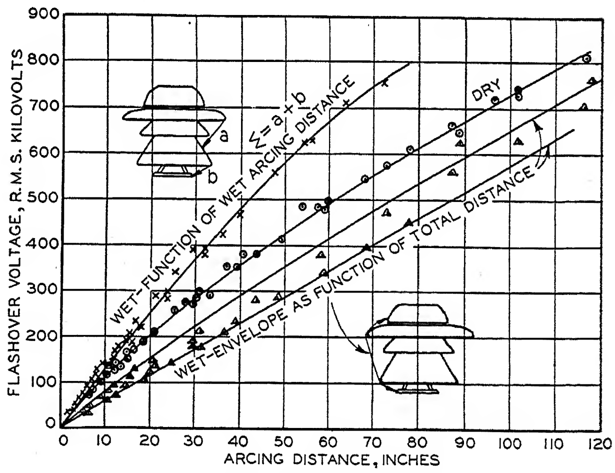


FIG. 9—60-CYCLE FLASHOVER OUTDOOR APPARATUS INSULATORS

within an envelope of plus or minus 6 per cent, the broadest of any insulator type tested. The variation in flashover of all insulators between vapor pressures of 0.2 and 0.608 is about 12 per cent. Frequently in testing at standard temperature with relative humidity above 65 per cent, erratic performance and marked fall in flashover was observed. A typical example is shown for the 27 kv. rated insulator. This is believed to be due to the formation of invisible water film upon the surface of the insulator at these higher humidities.

Fig. 13 shows the effect of humidity upon typical outdoor apparatus insulators. The points of observations shown upon these graphs in comparison with those for the pin types, Fig. 12, show much more stable performance.

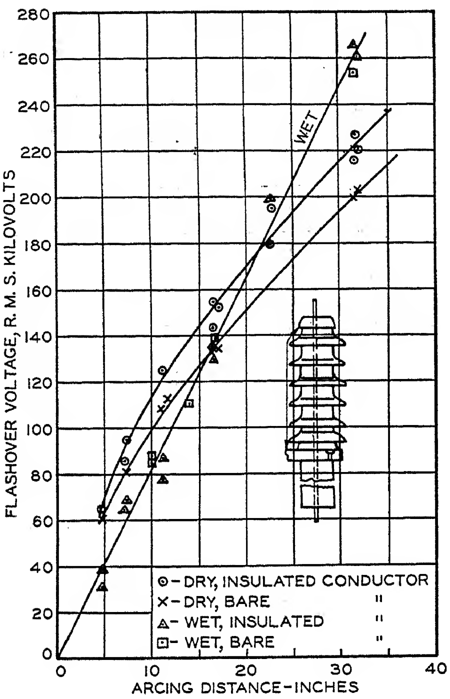


Fig. 10—60-Cycle Flashover Solid Type Equipment Bushings

ance. This is due to absence of corona streamers as already discussed. The variation from the average curve is not over plus or minus 5 per cent. The correction for the humidity effect is less than for pin types.

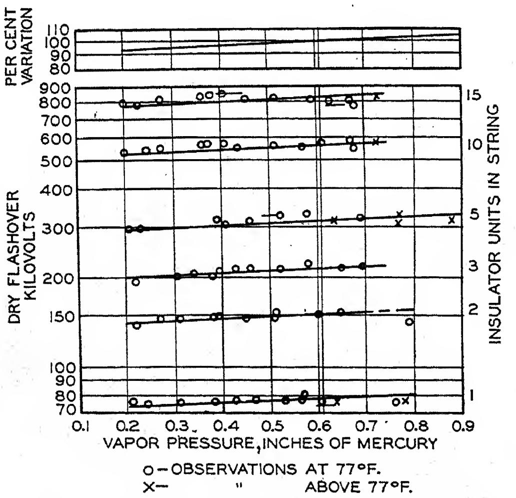


Fig. 11—Effect of Humidity Standard Suspension Insulators

10 in. diameter, 5¾ in. spacing

It is 8 per cent between vapor pressures of 0.2 and 0.608, the same as for suspensions. Points at the upper right hand ends of these graphs are for temperatures well above standard, yet falling closely upon the same graph, indicating that comparison on the basis vapor pressures is quite satisfactory.

A most interesting observation upon an apparatus insulator of the multiple unit type for stacking is shown in Fig. 14. The correction for humidity is the same for this insulator as for the other apparatus and for the suspension insulators, 8 per cent from 0.2 to 0.608 inches

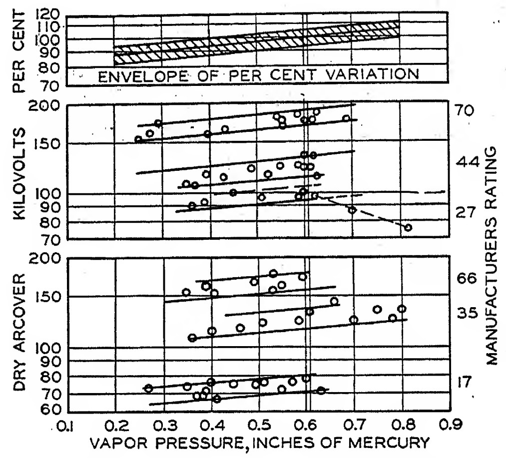


Fig. 12—Effect of Humidity on Flashover Pin Type Insulators

mercury water vapor pressure. The variation of observations from the average for the three and six unit stacks falls within an envelope of plus or minus 5 per cent. For a single unit, however, the variation is nearly plus or minus 10 per cent. This checks observations upon other insulators such as the tall apparatus and the suspension insulators, indicating that for great lengths

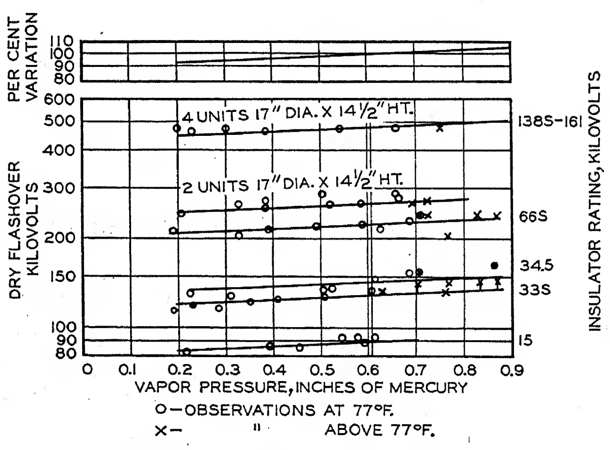


Fig. 13—Effect of Humidity on Typical Apparatus Insulators

the percentage variations are somewhat less and the performance of the insulators more stable.

The performance of porcelain equipment bushings with varying humidity is shown in Fig. 15. The correction for humidity with these bushings is the same as for all others except the pin types.

A tabulation has been made, summarizing the above data as follows:

TABLE I

Insulator type	Dry value — Arc length — Kilovolts/Inch	Wet value — Arc length — Kilovolts/Inch	Wet value — Wet striking distance — Kilovolts/Inch	Humidity correction per 0.1 in. mercury water vapor pressure
Suspension:				
0 to 100 in.				
Standard cemented	10.5 to 9	7 to 6.8		2 per cent
High strength cemented	10.5 to 9	7 to 6.3	14 to 10.6	
Hewlett flat disk	10.5 to 9	7.5		
Hewlett fish tail	10.5 to 9	6.5		
Pin types				
Small	14	7.0		3 "
Multipart	14 to 11.4	8.9	(Erratic)	3 "
Apparatus insulators	11 to 7.3	8 to 5.7	13 to 10.4	2 "
0 to 110 in. dry arcing distance		(Erratic)		
Equipment bushings				
Solid type	13.6 to 6.75	8.3		2 "
(With bare conductor)	12.4 to 6.1	8.3		

Where two figures are given above first applies to small insulators, second to the largest with fairly uniform steps in between, except as noted as erratic.

Effect of Surface Conditions

In an effort to determine how the above rules for insulator flashover might be affected by surface conditions, representative insulators were exposed above the smoke boxes of a battery of beehive kilns for a period of several months. The surfaces of these insulators at the expiration of this period were thoroughly covered with a dirt deposit consisting largely of carbonaceous material mixed with a certain amount of clay dust and ordinary floor dirt. The coating was rather hard, due to the baking action of the heat from the kilns.

Other insulators of exactly the same type were thoroughly coated with a salt crust by spraying with a concentrated salt solution and allowing to dry.

The insulators were then flashed over, first clean and dry, then salt coated and dry, and finally dirt coated and dry. Tests were repeated with the insulators in a steam chamber filled with steam at approximately 100 deg. temperature to simulate fog conditions. Finally some of the insulators were tested under A.I.E.E. Standard Wet Flashover Test. The results of the tests are shown in the following tabulation.

It will be noted that there is very little reduction in the dry flashover of the insulators when dirty. Dry tests on the salt coated insulators were made at two humidity conditions. The first group of tests yielding the higher values in the tabulation were made at approximately 77 deg. dry bulb temperature, 65 per cent relative humidity. The second group of tests made at approximately 77 deg. dry bulb temperature and 79 per cent relative humidity gave flashover values materially lower, probably due to the absorption of water by the salt, at this relative high humidity. At the lower humidity the dry flashover of the salted insulators is nearly the same as that of clean insulators.

When subjected to artificial fog the salt coated insulators showed in most cases very marked decline in flashover. The flashover of the dirt coated insulators was also decreased but not so severely.

In a further effort to study the effect of surface conditions on the flashover of insulators, other insulators were hung on the roof of the laboratory for a period of one year, each insulator being flashed over approximately once a month. Since the laboratory is in a heavy

TABLE II

Nominal rating or number of units	Type of insulator	Dry — Clean	Dry — Salt coated — 79% humid.	Dry — Salt coated — 65% humid.	Dry — Dirty	In A.I.E.E. standard wet flashover test — Clean	In A.I.E.E. standard wet flashover test — Salt coated	In A.I.E.E. standard wet flashover test — Dirty	In steam fog — Clean	In steam fog — Salt coated	In steam fog — Dirty
20 kv	Pin type A	67	41	72					22.5	23	
20 kv	Pin type B	69	62	77					30	20–26	
35 kv	Pin type	120	97	112	116		34–63		56	24	48
44 kv	Pin type	135	97	114	128	95	52–80	67–74	74.5	34	68
55 kv	Pin type	137	90	140	136				78–89	48	85
66 kv	Pin type	161		164.5					132	51	
70 kv	Pin type	178	185	189	178				132	55	83
1 unit	5¾ x 10 in. disk	77	34	75	69	48	27–44	36–53	33	23	32.5
3 units	5¾ x 10 in. disk	204			205				139		114
9 units	5¾ x 10 in. disk	535		479							
9 units	5⅛ x 10 in. disk	485		454							

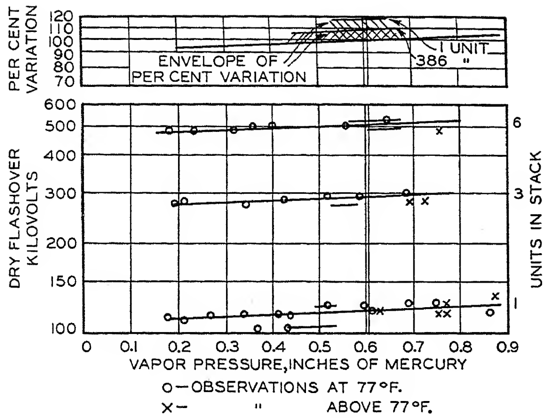


FIG. 14—EFFECT OF HUMIDITY ON APPARATUS INSULATOR MULTIPLE UNIT TYPE

10 in. diameter, 9¾ in. high

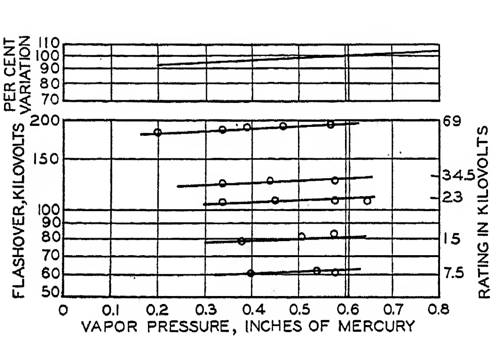


FIG. 15—EFFECT OF HUMIDITY ON PORCELAIN EQUIPMENT BUSHINGS

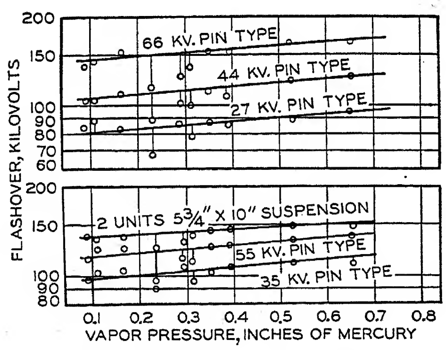


FIG. 16—FLASHOVER OF DIRTY INSULATORS AS A FUNCTION OF HUMIDITY

industrial district the insulators were subjected to soft coal smoke and industrial gas fumes. The results of the tests over a period of one year are tabulated. All flashover measurements were made at a time when the insulators were apparently thoroughly dry.

These tests were, of course, made at such humidity and temperature as happened to be existing at the time. The points have been plotted on Fig. 16. It will be noted that the curves are identical with those of Figs. 11 and 12. There are certain points on each of the curves which fall materially below the average. It is believed that in these cases the dirt film on the surface was very

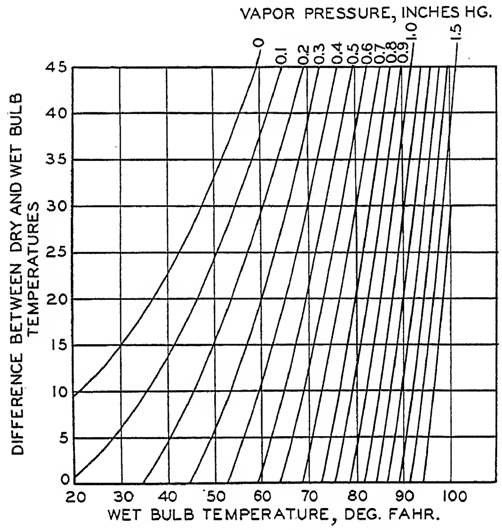


FIG. 17—PSYCHROMETRIC CHART

From Smithsonian Meteorological Tables. Barometer = 30.00 in. Hg.

slightly moistened. Since most of the tests were made in the morning it is quite possible that in these cases there was still a certain amount of invisible dew remaining on the surface of the insulators.

CONCLUSIONS

The relations found between arcing distance and insulator flashover and between humidity and insulator flashover are indicated in the foregoing curves and tabulations. At very high relative humidities there is in some cases a scattering of test results with points falling below the humidity curves. This is believed to be due

TABLE III

Time of exposure	0	1 Month	2 Months	3 Months	4 Months	5 Months	6 Months	7 Months	8 Months	12 Months
Dry bulb temp	82	65	50	43	27	30	44	52	61	74
Wet bulb temp	68	56	48	41	24	27	38	48	56	69
Vapor pressure	0.533	0.352	0.314	0.235	0.092	0.111	0.163	0.292	0.396	0.659
Barometer	30.04	30.15	30.25	29.91	30.09	29.94	29.57	30.06	30.00	29.96
Rating and type of insulator										
27 kv. pin type	89	86.5	78	68	83.5	88	82.5	86.5	85.5	95
35 kv. pin type	109	101.5	94.5	89.5	96.5	104.5	102.5	108	106	109
44 kv. pin type	123	111	100	88	103	105.5	110	101	108.5	127
55 kv. pin type	132	125	111	95	112	122	124.5	113	129	136
66 kv. pin type	164	152	135	116	136	141	152	126.5	151	164
2 units 5¾ in. x 10 in. suspension type	148	142	135	122	136	132	134	129.5	143	144

to the precipitation of a fine moisture film on the surface of the insulators.

Throughout the common range of temperatures (20 to 100 deg. fahr.) the relation between vapor pressure (expressed in manometric inches of mercury) and absolute humidity (expressed in grains per cu. ft.) is such as to make no practical difference which of them is used as the basis of measurement.

The same humidity corrections are applicable to dry dirty insulators as to clean insulators.

Dry dirt only slightly lowers the flashover of insulators. A small amount of moisture greatly reduces the flashover of dirty insulators, particularly if the dirt is of a saline nature.

At high relative humidities the flashover of salt coated insulators is materially reduced due to absorption of water by the salt.

Insulators can be checked for flashover from data given in Table I which also gives humidity correction factors. Deviation of individual flashover voltages from the average are not fully explained by humidity variations alone. After correction for humidity has been made a variation of about plus or minus 5 per cent remains in some cases.

NOTE: Vapor pressures may be obtained from barometric and dry and wet bulb thermometer readings with the assistance of the Smithsonian Meteorological Tables from which Fig. 17 has been drawn.

From this vapor pressures may be obtained with sufficient accuracy for the purpose.

Discussion

INSULATOR SPARKOVER
(LLOYD)

AN IMPROVED TYPE OF LIMITING GAP
(AUSTIN)

SUSPENSION INSULATOR ASSEMBLIES
(TOROK AND ARCHIBALD)

NORMAL FREQUENCY ARCOVER VALUES OF INSULATORS
(FREY AND HAWLEY)

A. O. Austin: The paper by Messrs. Torok and Archibald calls attention to a subject which is of great economic importance to the transmission system. It has been evident for many years that length efficiency was frequently of far more importance than any difference in the cost of the normal insulator. With the more efficient insulator it is possible to reduce the clearances and the height of tower. In the two-circuit tower this is exceedingly important as the height of the structure can be reduced so that the probable number of hits to the line will be less. With closer clearance the line reactance is reduced and capacitance increased so that more power may be transmitted and the stability increased.

Increasing the section length or the size of the metal parts to obtain a higher ultimate without increasing the diameter and flashover voltage of the disk lowers the length efficiency of the string. Impulse tests show that the wet flashover of the string as compared to the dry flashover voltage drops off materially as the insulation of the string is reduced. The ratio of flashover voltage in the disk to the voltage imposed upon same in the string under the most severe condition is the first consideration. As an increase in diameter raises the flashover voltage it will tend to improve the string performance providing this is not offset by increased drip water and section length. It would therefore seem that insulators having higher flashover voltages than the normal 10-in. diameter disks can be used to advantage.

The 10-in. disk insulator originally had a section length of 5¾ in., a flashover voltage of 92 kv. and a mechanical ultimate of 8,000 lb. The tendency has been to increase the mechanical ultimate and section length only, and in many cases the flashover voltage probably does not exceed 70 kv. This tends to reduce the time lag and length efficiency of the string under impulse.

It is feasible to use insulators which have a higher flashover voltage even though they are considerably more expensive. Where a whole string is made up of insulators of large diameter it will be found that the drip water has a material effect upon the flashover voltage. The large projected area of the insulators increases the amount of drip water, which has a material effect upon starting a flashover under impulse. By using insulator units

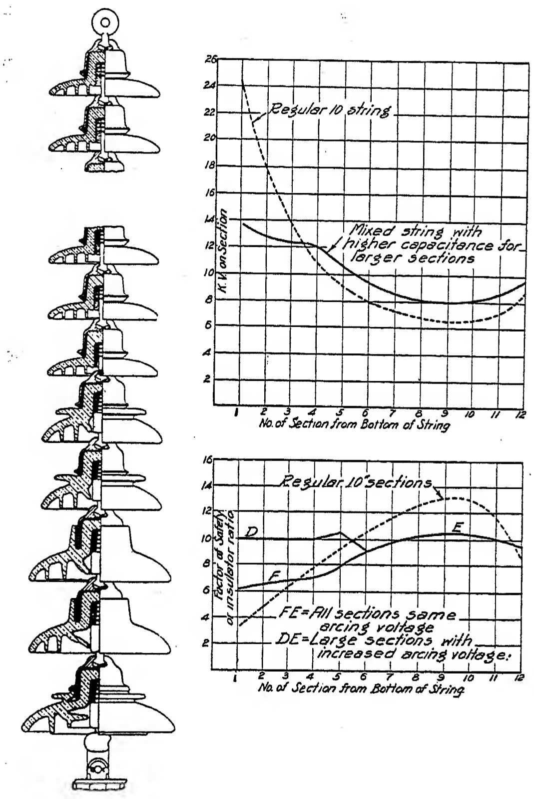


FIG. 1

of large diameter along with smaller intermediate units, not only is the drip water reduced but the short-circuiting effect of the stream of drip water is much less. Strings of this kind show a greatly improved performance, and when the advantage is given due credit it will be seen that the higher cost is more than offset.

The necessity of using mixed diameters in the shells of pin type insulators has long been recognized and the treatment is even more effective in the long string of suspension insulators, as accumulated drip water tends to short-circuit the lower portion of the string.

I believe that Mr. Torok should extend his investigation to include longer tailed waves, as many systems do not have a low effective ground resistance or capacitance, and an effective counterpoise may be considered too costly.

It must also be remembered that many strokes of lightning consist not of a single impulse but a number in rapid succession so that if tests are confined to a wave between one and two microseconds duration as shown by the oscillogram in the paper, the

insulator may give a poorer performance in service than would be expected from the tests.

Some interesting insulators of larger diameter and higher flashover voltage were shown in a paper presented at the Pasadena Convention in 1924. Fig. 1 shows some of these insulators and the effect upon the ratio of flashover voltage to duty imposed, which may be termed the unit factor of safety. The priming or shunting of the surface insulation of the disk under a very high overvoltage makes it possible to use insulator sections of higher flashover and improved characteristics to advantage. The insertion of a few disks of large diameter in a string of short spaced 10-in. diameter units will frequently increase the flashover voltage from 15 to 25 per cent even though the insulators used do not represent the maximum possibilities as to flashover voltage.

While larger and more efficient sections have been available for many years, their higher cost has prevented their general use. Recognition of the economic advantages of the high length efficiency will make it possible to use many of these insulators for at least a portion of the insulator string.

With higher operating voltages, the necessity of using a higher flashover voltage in the section becomes more important. The use of special units at the ends of strings of ordinary 10-in. diameter sections will frequently effect very material improvements in characteristics and prevent radio interference.

The economic advantage of a mixed string will offset any assumed difficulty in installation. It is not necessary that all strings be exactly the same to obtain the benefits of a mixed string, so that the difficulty of handling more than one type of section amounts to little. While units of improved design could be used for the entire string, the cost of the string would be greatly increased without necessarily improving the string materially over that of a string in which a few of these units were used.

The paper by Messrs. Frey and Hawley gives considerable information on the effect of humidity upon the flashover voltage of various types of insulators. It has long been recognized that humidity and air density are important factors in insulator flashover. There are so many factors involved which affect the flashover of an insulator that general laws cannot be established which apply to all types of insulators. The correction for humidity therefore must be determined largely by test on the particular insulator or one approximating its general characteristics.

In many cases the effect of humidity and air density upon the flashover voltage is approximately the same as that for a needle gap. For this reason a needle gap has given fairly consistent results for many insulators and is to be preferred to a sphere gap where corrections are not made for humidity or air density. In the past, corrections have frequently been made for a measuring gap without giving the insulator the benefit of the same corrections. This practise has led to more confusion than any other item in insulator testing. In making careful dielectric tests, corrections of course should be made so as to obtain the absolute voltage applied.

As pointed out in the paper of Messrs. Frey and Hawley, the field set up by a bushing has a considerable effect on the flashover voltage. It would seem, however, that the higher wet flashover shown in Fig. 10 applies only to bushings which have a very poor surface gradient.

Many properly designed bushings have a higher flashover voltage with the conductor in place then when the conductor is removed. This is due to the fact that the design of the bushing makes use of the field set up by the metal parts in screening the surface from shunting discharge streamers.

In regard to Mr. Lloyd's paper, where the humidity is high it is very important that the insulators be at room temperature in making the tests, otherwise large discrepancies may arise.

C. L. Fortescue: It will probably take some time for the industry to appreciate the significance of the work that has been done by Messrs. Torok and Archibald. The research was started purely with the idea of finding out the influence of diameter and spacing in the impulse strength of suspension insulators. Since the object of the research was to determine the insulating value of suspension insulators of different diameters and spacings to withstand lightning surges on transmission lines it was necessary to determine only a suitable wave form to represent the actual impulsive stresses to which these insulators would be subjected under operating conditions and the wave form shown in their paper was the result.

To those not familiar with the more recent theories resulting from the lightning field investigations of the last few years I wish to point out that the direct stroke theory has been quite generally accepted by all engineers. According to this theory the outages on high-voltage transmission lines, that is to say, 66 kv. and above, are solely due to lightning strokes which hit the ground wire or line wire; the induced surge due to a nearby stroke is too low to be of any consequence. The surge that appears across an insulator when a stroke takes place at a tower is of very steep front and the high part is of very short duration. This is due to the fact that the tower itself acts as a short length of transmission line and the lightning stroke builds up the potential at the tower top during the time required to travel back to the top of the tower. The approximate wave form may easily be computed by the method of reflections and the computations result in the series of waves which was shown by the authors for different values of tower footing resistance. It will be noted that as the tower footing resistance is increased the magnitude and duration of the high part of the wave is also increased. With a very low tower footing the high part will be of low magnitude and its duration will be quite short, as short as one microsecond.

The criterion set up was the ability to withstand surges of the wave form shown and the resulting data have been given in the plotted curves in the paper. These curves show that larger diameter, namely 12 inches, and medium spacing, namely 5¾ inches, will result in the most economic insulator to apply to high-voltage transmission lines. The important gain comes about by reduction in string length which with equivalent air clearances results in a lower and cheaper tower. I wish to emphasize also that there is nothing untried in the type of insulators proposed. Insulators of this diameter and larger have been in use on the Pacific Coast and have given good service. They are just as sturdy as the standard 10-inch suspension insulator.

In discussing the Torok-Archibald paper regarding the latest point of view in the application of insulators in transmission line design, Mr. Austin has laid great stress on what might occur after flashover takes place. We prefer to lay more stress on the ability of an insulator assembly to withstand an impulse without flashing over. The protection of transmission lines is now reaching the point where the protective level is so high that more than two outages per 100 miles per year is not considered a very good record. Under such conditions it seems to me that the result of a flashover occurring once or twice on a 100-mile line per year is not of much consequence since as a general rule after the flashover occurs the line goes into service at once. A broken shed on two or three insulators is not of any great importance and the insulators can be changed at any convenient time after the occurrence of the flashover. Therefore, we are more concerned with obtaining high economic efficiency in the application of insulators than in any other feature such as what occurs after flashover takes place. We may even reach the ideal where a flashover is so rare that it may happen, due to lightning, only once in every few years.

J. J. Torok: There are two very important points in Mr. Austin's paper, first in that he points out the difference between the positive and negative flashover characteristics of various gaps and its importance in coordination, and second, the relations of flashover between waves of various durations and the volt-time curve obtained by a flat-topped wave.

Reference will be made to several simple gap arrangements to illustrate the manner in which the above time lag and polarity principles are applied.

Time Lag. The typical property of a needle gap is the non-uniform voltage distribution between electrodes as shown in *A* of Fig. 2 of this discussion. On the other hand, the sphere gap *B* is known to have a much more uniform voltage distribution across it. As a result of the higher voltage concentrations at the electrodes of the needle gap, local breakdown or corona starts there and proceeds outward long before there is any sign of breakdown in the sphere gap. Consequently, for 60-cycle voltages, or impulse voltages longer than a fraction of a micro-second, the needle gap must be set at an appreciably greater spacing for the same sparkover voltage. This increased gap length that must be broken down is an important factor in increasing the time lag of a needle gap.

For impulse breakdowns occurring in a microsecond or less, a needle gap must be set at a spacing approximately the same

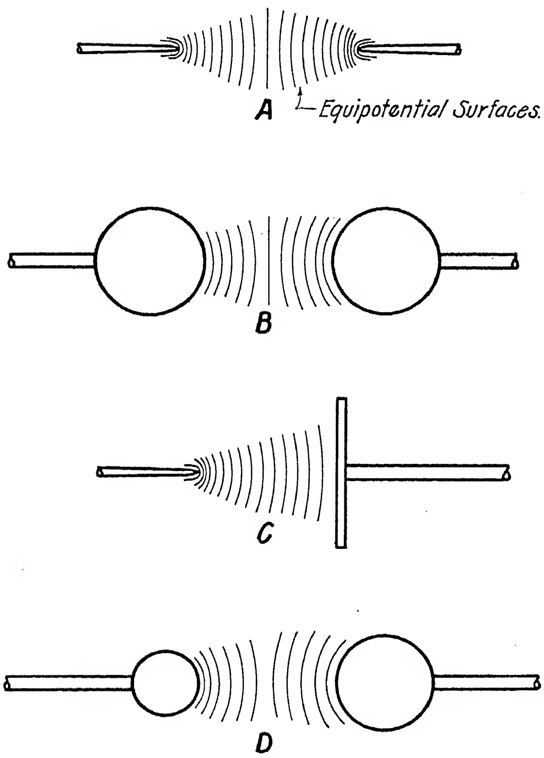


Fig. 2

or smaller than that of a sphere gap in parallel with it. This is also due to the non-uniformity of field and therefore the higher time lag of the needle gap, as the following reasoning will show:

When air breakdown starts at the electrodes of the sphere gap, it is only a negligible time later before the remainder of the gap is also broken down, due to the fact that practically the same voltage gradient prevails all the way across as shown in *B*. With the needle gap, on the other hand, the intense field near the electrodes causes an early failure of the air there, but until streamers from the electrodes have extended out appreciably and applied sufficiently higher voltage gradients to the center of the gap, the latter air remains undisturbed.

From the above, it is apparent that any tendency to approach a sphere gap condition by increasing the effective area of the electrodes of a gap tends to distribute the voltage gradient more evenly throughout the gap, allows the spacing to be reduced for the same sparkover voltage, and lowers the time lag. Mr. Austin accomplishes this in his control type of gap by arranging adjustable control shields about each electrode.

Polarity. It has long been recognized that sparkover between a point and a plane, as shown in *C*, always takes place at a lower value with uni-directional voltages when the point is positive; and with alternating voltages, always on a half cycle when the point is positive. In the same way the unsymmetrical field of the sphere gap *D* will cause it to display the same polarity effect and have a lower breakdown voltage when the smaller sphere is positive. The general rule for the above action is that sparkover always occurs at a lower voltage when the electrode in the more intense field is positive.

Mr. Austin develops this same polarity effect by arranging his control shields behind each electrode so as to increase or decrease its effective area and have the proper unsymmetrical electrostatic field condition across the gap.

It is apparent from Fig. 1 of Mr. Austin's paper that the insulating members of a station have line electrodes of small effective areas in comparison with the ground electrodes and, consequently, have appreciably lower positive than negative sparkover voltages. Also, Figs. 1 and 5 indicate that their time lag properties would lie between those of a needle gap and a sphere gap. With his adjustable field controls, Mr. Austin is able to secure any of the electrostatic field conditions of gaps *A*, *B*, *C* or *D* or variations of them and thereby arrange his gap to have the proper sparkover characteristics for protecting any or all station insulating members for all incoming transients, regardless of wave shape or polarity.

J. F. H. Douglas: Mr. A. O. Austin's paper shows how an analysis of the electrostatic fields can be used to achieve a sound solution of insulator problems. The different behaviour of points and spherical electrodes, with positive and negative electrification is clearly shown to be in accord with electrostatic field theory, and is verified amply by experimental test. Future experiment might be well directed toward some of the details which are not yet fully accounted for.

E. Hansson: Mr. Austin, in his paper, has outlined a method of controlling the electrostatic field around a relief gap so that the time lag of the gap will follow that of the insulation to be protected. That such coordination is necessary if full protection with a minimum of outages is to be obtained is indicated by the experience we had by reducing the line insulation ahead of some of our transformer banks.

In 1926 we reduced the insulation at both ends of two and at the station end of a third of our 66-kv. circuits from 8 to 5 units, O. B. Catalogue No. 25622. The transformer bushings which we wanted to protect were G. E. type F-1 rated at 88 kv. Our information as to the 60-cycle flashover values is 290 kv. for the bushings, and 267 kv. for the line insulators.

During 1926 and the early part of 1927, we had 22 cases of flashovers involving either bushings or reduced line insulation. Transformer bushings flashed in 7 cases, and the reduced insulation in 15 cases. Undoubtedly, the gap spacing was too large to protect the bushings under all conditions. We have no means of determining how many times the gaps operated unnecessarily. Insulation was restored to 8 units in June 1927.

Early in 1929, we installed fused arcing rings at both ends of one of the above mentioned circuits. The gap between the fuse and ring was set for 22 inches. During 1928 and 1929 we had four cases of bushing flashovers. In two of these cases, fuses were blown adjacent to the transformer and in two cases the fuses were intact.

During the same period another parallel circuit was equipped with a 12-in. gap between line and ground and in series with this gap we had an automatic fuse bank. We have had no bushing flashovers on this circuit since installation of this gap but we have, of course, blown a good many fuses. However, the fuse action is so rapid that the blowing of the fuse causes no disturbance on the system.

Joseph C. Rah: An explanation is given in the paper by H. A. Frey and K. A. Hawley that a better voltage division is

obtained if the whole surface of the porcelain insulator is wetted, and therefore in the horizontal position, higher wet flashovers are obtained. This is probably true under certain conditions, but if the authors believe a better voltage distribution is obtained, then in explaining a lower wet flashover on a string of insulators by the fact that the wet surface on the top may be considered as short-circuited and only the dry air space effective, makes the two statements contradictory.

If a wet surface on the top of the insulator is short-circuited by a film of water, then the horizontally placed string will be completely short-circuited by the water film and a much lower value of flashover should be obtained, which is contrary to test results.

The results obtained on insulators from experiences of many observers, including myself, indicated that air spaces are responsible for reduced flashover rather than wet surfaces, and therefore, the wet flashovers are entirely a function of design.

Dry flashovers to a certain extent are also a function of design, as indicated by the differences in flashovers of pin type and equipment type insulators. The metallic cap on pin type insulators makes the porcelain insulator more reliable and steadier in flashover value due to a decrease in streamers, which are local flashovers. Therefore, if an insulator is designed so that streamers do not appear until up to the flashover value of the insulator assembly, then the insulator is a function of design, and not only the striking distance alone, as was proven by recent designs.

Fig. 1 gives flashovers with respect to spacings indicating that there is very little effect of spacing between disks. This may be so if the variation of spacings is one inch only where the inaccuracies of observation are great enough to offset the spacing effect. With smaller spacings however, there is a pronounced difference in flashover. The disk diameter and spacings can be adjusted to obtain well observable and definite variations in flashover values.

In the part describing equipment bushings, the authors claim that the field established around the bushing determines the dry flashover value, and by putting a conductor inside the bushing, lower flashover values are obtained.

This appears inconsistent, as introducing a conductor inside the bushing should and does improve the field, provided the interior and exterior designs are harmonious and do not interfere with one another. I have obtained flashovers considerably higher with conductors inside the bushings than without conductors with the proper exterior and interior construction.

When testing glass insulators of the same shape as porcelain, in regard to the effect of humidity, a lower flashover is obtained. This is due to the increased dielectric constant, as air nearer the insulator is overstressed. If we can lower flashover by introducing a material of a higher dielectric constant, we can also increase flashover by introducing the porcelain insulator into a humid atmosphere, which has a resultant dielectric constant higher than air, and in some cases may be higher than porcelain. This is, in my opinion, one reason of the increased flashover due to humidity. I also believe the absolute humidity is responsible for the increase of flashovers. In testing, I have found that on short insulators, the change in humidity has practically no effect on flashovers. The longer the insulator, the more effect the absolute humidity has upon flashovers. This suggests that the air is laden with moisture particles which line themselves up in the field and produce a multiplicity of condensers. This multiplicity of condensers is only effective in the part of the field which is nearly uniform. Therefore, on shorter insulators where the field is not uniform, there is little effect of the moisture particles to increase flashover potentials. On long insulators with a much longer uniform part of the field, moisture content in the air has an effect. As we increase air humidity we get more condensers in series, and therefore, the air is rendered electrically stronger until a time where there are so many moisture particles that they touch each other, and this is the dew point at which the flashover potential falls down sharply.

Corroboration of this theory may be observed in the behavior of the needle gap. The farther apart needle points are set, the larger the variation due to absolute humidity is apparent, as with increase of length the uniform field is made stronger by particles of moisture.

As far as the effect upon the sphere gap is concerned, the following may explain the apparent lack of effect of humidity. When the sphere gap is put into an atmosphere laden with moisture, the dielectric constant of the air plus the moisture is increased, and therefore the field becomes more uniform. The equipotential surfaces straighten out and instead of a sphere gap of smaller dimensions, a gap of larger dimensions is produced. If it were not for the fact that moisture is being drawn into the densest part of the field at the center line of the spheres, actual flashover of the spheres in humid atmosphere would be higher. In other words, we get a resultant strength of the air, and this resultant strength appears to be very close to the original strength of dry air.

J. E. Clem: There is one factor which enhances greatly the value of any paper and which is not fully appreciated by the authors of many Institute papers. I have in mind the inclusion of actual test points in curves. When test points are given the reader can readily form an idea of the variation to be expected in the use of the material and can also make his own analysis and form his own conclusions. I think it is almost self-evident that papers giving curves actually drawn through test points are much more valuable than those in which curves are shown but with no supporting data.

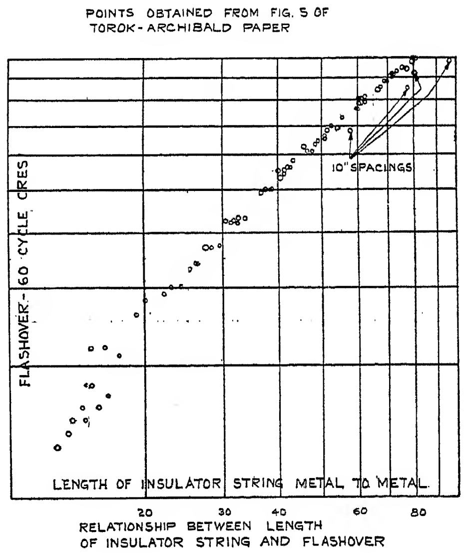


Fig. 3

The most important item of interest in the Frey-Hawley and the Lloyd papers is the effect of humidity on the flashover voltage of various types of insulators and gaps.

In Fig. 9 Mr. Lloyd shows a curve indicating that the flashover voltage of sphere gaps is not affected by changes in humidity for values of humidity of less than 11 grains per cu. ft. This is probably due to the fact that the field between the spheres is uniform and there is no point of flux concentration. It seems quite reasonable to believe that if the spacing between the spheres was large in proportion to the sphere diameter, this could no longer hold and the flashover voltage would be affected by humidity.

The type of gap which has characteristics departing most widely from those of a sphere gap is the needle gap. It would be

natural then to expect that the needle gap would be greatly affected by humidity and further that needle gaps of small spacings might show less effect than long gaps. Reference to Mr. Lloyd's Fig. 10 indicates that such are the facts. Over a range of humidity from zero to 10 grains per cu. ft. the 5-in. gap increases about 18 per cent in flashover and the longer gaps increase in the order of 33 per cent over the same range of humidity.

The effect of humidity on the flashover of insulators is very important and it is also highly desirable that some standard method of making humidity corrections be made available as soon as possible. It seems reasonable to believe that insulator assemblies having a long flashover distance and having end terminals relatively small in proportion to the arcover distance, should behave approximately as a needle gap; and that small insulators should behave approximately as a sphere gap as far as the humidity effects are concerned. This is confirmed by reference to the curves in both the Frey-Hawley and Lloyd papers where humidity effects as high as 33 per cent for a long string of suspension insulators and as low as 4 per cent for a 1-unit string can be found. Fig. 12 in the Frey-Hawley paper gives an increase of about 35 per cent for pin type insulators over the same range of humidity as previously mentioned.

Unfortunately, the degree of agreement between these two papers in regard to the relative effect of humidity is not as great as could be desired. After comparing and analyzing all the data in both papers the conclusion must be reached that the effect of humidity is not completely settled and that much further thought must be given to this problem.

The humidity curves in one paper were plotted with vapor pressure as the abscissa and in the other paper with grains per cu. ft. as the abscissa. It would be very desirable if humidity curves in future papers could all be plotted with the same abscissa. Institute Standards No. 41, covering insulator testing, recommend the use of vapor pressure as a basis for comparing humidity values. Until the Institute rules are changed, and they should be if good reasons can be offered, the present Institute recommendations should be followed.

In reading through the Torok-Archibald paper one would come to the conclusion that the spacing of insulator units and the shell diameter of the insulator units have a marked effect on the flashover of an insulator string of a given length. In Figs. 1 and 2 of the Frey-Hawley paper and Fig. 15 of the Lloyd paper there are curves indicating that the unit spacing and shell diameter have very little effect on the arcover voltage of an insulator string of a given length. However, further examination of the data given in Fig. 5 of the Torok-Archibald paper leads to the conclusion that the flashover voltage of an insulator string of a specified length is approximately the same with the spacings from 4¾ to 7½ in. or with the shell diameter from 10 to 12 in.

In Table I of this discussion there is shown the 60-cycle flashover voltage of the various insulator units as read from the curves of Fig. 5 of the Torok-Archibald paper. These data are plotted in Fig. 3 of this discussion. From this figure it is seen that there is no consistent variation in flashover for varying spacings between 4¾ and 7½ in. and shell diameter between 10 and 12 in. Therefore, since the 60-cycle flashover shows no variation for these factors, it is rather difficult to understand how the impulse flashover can show any variation for the same factors, as claimed in the Torok-Archibald paper. Until sound reasons can be advanced for a difference in effect of unit spacing and shell diameter on 60-cycle and impulse flashover, extreme caution should be exercised in designing transmission line insulation based on this difference.

Fundamentally, no great difference in the flashover of an insulator string of given length for units of different spacing or different shell diameter should be expected within the ordinary limits of these dimensions. The arcing distance is the metal-to-metal length from the top of the top unit to the bottom of the bottom unit. Increasing the spacing between units increases this distance or adding a unit increases it. Whichever way the distance is increased the effect should be the same. The effect of increasing shell diameter should be greatest for a single unit. However, if the shell diameter is increased too much without increasing the thickness, no improvement would be obtained on account of the overstressing of the porcelain.

TABLE I—ARCING DISTANCE AND 60-CYCLE FLASHOVER FOR SPACING INDICATED

No. units	Dia.	4¾ in. Arc Dis.	Kv.	5¾ in. Arc dis.	Kv.	6½ in. Arc dis.	Kv.	7½ in. Arc dis.	Kv.	10 in. Arc dis.	Kv.
2	10	12.75	130	13.75	150	14.5	180	15.5	220	16	230
4		22.25	290	25.25	330	27.5	370	30.5	425	32	450
6		31.75	425	36.75	490	40.5	530	45.5	625	48	640
8		41.25	550	48.25	630	53.5	680	60.5	800	64	800
10		50.75	660	59.75	760	66.5	840	75.5	950	80	910
12		60.25	780	71.25	900	79.5	980			96	1,000
14		69.75	880								
16		79.25	1,000								
2	10½	13.5	140	14.5	160	15.25	180	16.25	220	19.0	260
4		23.0	300	26.0	340	28.25	370	31.25	420	38.5	500
6		32.5	430	37.5	500	41.25	540	46.25	610	58.0	680
8		42.0	560	49.0	640	54.25	690	61.25	800	77.5	850
10		51.5	680	60.5	780	67.25	850	76.25	950	97.0	980
12		61.0	800	72.0	910	80.25	1,000				
14		70.5	900								
16		80.0	1,010								
2	12	14.85	150	15.75	160	16.5	170	17.5	210	20	280
4		24.25	300	27.25	340	29.5	375	32.5	420	40	550
6		33.75	430	38.75	500	42.5	560	47.5	610	60	760
8		43.25	580	50.25	650	55.5	730	62.5	790	80	920
10		52.75	700	61.75	800	68.5	880	77.5	940		
12		62.25	810	73.25	930						
14		71.75	920								

Based on readings from Fig. 5 of Torok-Archibald paper.

However, when the unit spacing exceeds the arcover distance of a single unit, conditions may be somewhat different. In this case each unit of the string serves as a flashover unit and there should be more liability for the strings to cascade. This is no doubt the reason for the rapid falling off in the flashover voltage of the insulators with 10-in. spacing shown in the Torok-Archibald paper.

Further evidence that the flashover of a string of insulators is independent within ordinary limits of the unit spacing or the shell diameter is given in Fig. 4 of this discussion. The plotted points represent the 60-cycle flashover of pin and pedestal type insulators. The units tested cover a wide range in design features, but it is clearly evident from the points that the total flashover voltage is a function of the total flashover length. If there were any fundamental effect on flashover voltage, of unit spacing or shell diameter, one would expect it to be as pronounced or even more pronounced in an assembly of pedestal insulators than in an assembly of suspension insulators. It seems reasonable then to conclude that within the ordinary limits of these factors there is no effect on the flashover of a string of insulators of given length whether the spacing varies from 4¾ to 7½ in. or the diameter varies from 10 to 12 in.

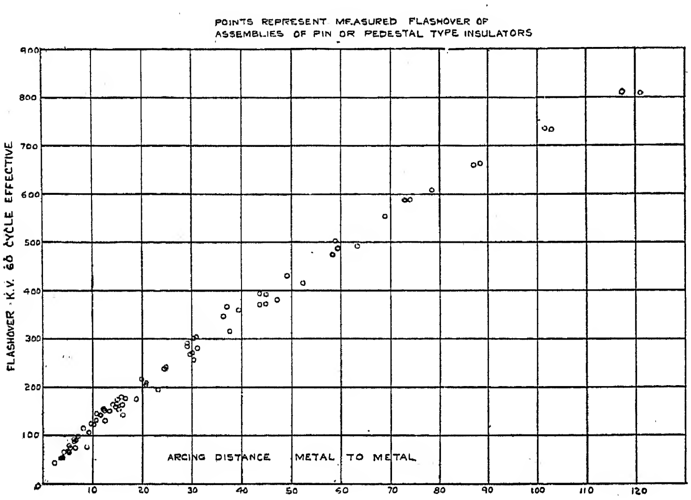


Fig. 4

In Mr. Austin's paper there is a lack of test points and oscillograms to show what impulse wave was used for obtaining the test data.

Mr. Austin would have us believe that the use of a gap for coordination leads to a very intricate and difficult problem. However, such is not the case. The problem in coordination is to select the gap setting so that the flashover of the gap will be less than the breakdown strength of the insulation for those waves which are considered most dangerous. The present tendency is to believe that the short waves of high magnitude are most likely to injure the insulation and further that there is little likelihood of obtaining long waves in sufficiently high voltage to be of much trouble. Therefore, in selecting the coordinating gap the thing to do is to base the setting on the gap characteristics for short time lags.

After comparing certain of the curves given in Mr. Austin's paper, I am doubtful as to whether his complex gap structure has any advantage over the common point gap. In Fig. 5 and Fig. 8 he shows the lag curves for the complex gap and in the text intimates that the adjustment is such as to give very desirable characteristics. For both figures about the same time lag curve could be obtained from the plain point gap, as can be verified by reference to Fig. 2 in Mr. Austin's paper. For instance, for comparison with Fig. 5, a 22-in. gap would have a flashover of about 470 at a 10 microsecond lag and about 600 at a 2 microsecond lag. Likewise for comparison with Fig. 8 a 60-in. gap would have a flashover of about 930 at a time lag of 100 microsecond and about 1,180 for 10 microsecond. It is quite clear that nothing has been gained by the use of a complicated gap structure for these two conditions.

W. L. Lloyd, Jr.: Absolute humidity may be expressed either in grains per cubic foot or in terms of the associated vapor pressure in inches of mercury. "Grains per cubic foot" appears to be the term more commonly used by engineers and it was for this reason that it was used in my paper. An absolute humidity of 6.5 grains per cubic foot corresponds to a vapor pressure of 0.608 inches of mercury at 77 deg. fahr. Therefore, where values in terms of vapor pressure in inches of mercury are desired approximate values can be obtained by dividing the grains per cubic foot by ten. The more exact figure is $\frac{6.5}{0.608} = 10.7$, but the difference in sparkover voltage associated with the resulting error in humidity value obtained is extremely small.

The A.I.E.E. Standards No. 41 dated March 1930, state that "flashover shall be determined at, or corrected to, a standard humidity corresponding to a vapor pressure of 0.6085 in (15.45 mm.) of mercury. This is equivalent to a relative humidity of 65 per cent at 77 deg. fahr. (25 deg. cent.) and a barometric pressure of 30.0 in. (76.2 cm.) of mercury." It might also have been added that this corresponds to 6.5 grains per cubic foot. In any future revision of A.I.E.E. Standards No. 41 it might be desirable to omit the reference to a relative humidity of 65 per cent at 77 deg. fahr. (25 deg. cent.) and a barometric pressure of 30.0 in. (76.2 cm.) of mercury since tests have since been made and data are given in this paper which indicate that the absolute humidity rather than the relative humidity is the determining factor.

Mr. Austin discussed the importance of maintaining the insulators at room temperature in making tests at high humidity. This is very important as indicated by Fig. 12 of my paper where a slight chilling of the insulators resulted in condensation on the porcelain surfaces and reduced sparkover voltages. Precautions against condensation were taken in making the tests up to 14 grains per cubic foot on 4 units in Fig. 3 and up to 16 grains per cubic foot on an oil-filled bushing in Fig. 8.

A. O. Austin: The several discussions make it appear that length efficiency in the insulator is a comparatively new thought, whereas the length efficiency or flashover voltage for a given length of insulator string is one of the oldest and most important considerations in the design and selection of insulators. The effect of length efficiency was treated in a paper before the World Engineering Congress in Tokio—November, 1929. Some examples showing the very material benefits possible with high length efficiency in the insulator string were treated in a paper before the International High Tension Congress in June, 1931.

Tower and conductor clearances are regarded as prime factors in transmission line costs. Unnecessary spacing not only increases the reactance of the line but increases the cost of the structure. High length efficiency in the insulator permitting of minimum tower clearance and height of structure therefore is of very great economic value. This matter was given very careful consideration by Mr. H. H. Cochrane on the lines of the Montana Power Company in 1909, the insulators and towers being selected on the basis of high length efficiency in the insulator.

Most of the work carried on at the outdoor laboratory of the Ohio Insulator Company has been devoted to increasing the flashover voltage or length efficiency of the insulator string, or in so constructing the tower as to provide maximum flashover between conductor and support. The economic advantages in many cases may be much greater than those pointed out by Mr. Obbard, particularly in the case of two-circuit towers. The use of an insulator of good length efficiency together with wood

arms may double or even treble the effective flashover voltage between conductor and ground for a given vertical spacing or size of tower. Taking advantage of the high flashover voltage will permit operation at a much higher operating voltage and the increased kva. of the line may save additional circuits. Under these conditions the corona point of the conductor is likely to be the limiting factor.

The possibility of using small conductors coated with an insulating paint or varnish promises very important economic advantages where it is desired to raise the voltage of an old system or to take advantage of the close conductor spacing or the lower stress due to smaller conductors. Under these conditions the additional cost of an insulator string of increased insulation would be rather small compared to the economic advantage.

In the two-circuit tower the high length efficiency in the string permits a material reduction in the height of the tower. This results not only in a lower cost of the structure but in fewer number of probable hits to the line which should receive due credit.

More attention should be given to low effective height, not only for the conductors but for the ground wires. Shorter spans, cross connection between ground wires, and the use of increased ground capacitance or counterpoises should all be given consideration, as well as the effective flashover in the string or the flashover between the conductor and tower. Savings due to increased length efficiency in the insulator and to the increased flashover due to the use of wood arms are the most important economies to be considered at this time.

With the exception of the height of the structure, the above advantages apply to practically any transmission line even though lightning is not a consideration. The design of the transmission line is a compromise and it will be found that any method which will increase the length efficiency and flashover voltage for a given clearance will result in a material saving when applied.

In recent years there has been a tendency to develop mechanical strength in the insulator at the expense of reduced insulation in the string. It is therefore gratifying to note that length efficiency in the insulator is likely to receive some of the attention it deserves.

Mr. Sporn has raised a very pertinent question as to the maximum working load of the insulator. An insulator designed for a 4,000 lb. maximum working load will have a higher length efficiency, lower thermal stresses and a longer economic life for the same cost than an insulator designed for a 7,000 lb. maximum working load. The use of an insulator having an unnecessarily high ultimate is therefore questionable from the standpoint of economy and reliability.

Mr. Fortescue apparently has misunderstood some statement, as I have always been in favor of increasing the effective insulation as the most economical means of improving reliability rather than the use of arcing devices such as rings or horns.

The use of wood makes it possible to develop very high effective flashovers for lightning transients even for the lowest voltage and most inexpensive lines. To design station equipment so it will match the flashover voltage of the line, however, would be economically impossible particularly for the lower voltage installations. It may therefore be assumed that the station equipment will be subjected to transients which will cause flashovers or damage to some of the equipment unless the voltage is limited by an arrester or some form of limiting gap. It is much better to provide a path for the discharge which will produce little or no damage than to permit the discharge at a point where considerable damage may be caused, even though a few more interruptions may take place due to the lowered flashover value of the gap. Mr. Hansson has covered this point very effectively and shows that by equipping the gap with a quick acting fuse the discharge may have little or no effect upon the operation of the system.

A long discharge path or a resistance in series with the discharge which tends to limit the current is very effective in causing a recovery of the resistance in a discharge path. This recovery tends to prevent the follow up of a power arc and is effective in eliminating interruptions either on the line or for station equipment. Increasing the insulation level of the station decreases the probable number of interruptions, but unfortunately high insulation levels are very costly. If a high insulation level could be obtained as easily for station insulation as for the line, there would be little need of protective equipment. However, since this is impossible some device which will limit the voltage is of great economic importance particularly for the lower voltage systems.

I quite agree with Mr. Fortescue that the damage of a few insulator sections is of little moment compared to reducing the number of interruptions. However, in the case of station insulation it is frequently very difficult and costly to replace a part damaged by a power arc such as a bushing. Therefore protective apparatus such as rings or horns which lower the flashover, while being economically wrong for the transmission line, may be justified for the station.

I wish to call attention to the fact that the flashover voltage of the disk for a given section length is of far greater importance than diameter. Some of the 10-in. disks shown in the slides, which were made in 1909, have flashover values as high as 150 kv. for a 7-in. section length. An insulator string made up of these high flashover disks will have a higher insulation ratio or factor of safety than an insulator string made up of units of the same section length and a flashover value of 90 kv. even though the diameter of the sections is 14 or 15 in.

Under rain, the performance of units of larger diameter will be much poorer, particularly for an impulse having a steep wave front, comparable to a direct hit on the line.

At the present time a high length efficiency or flashover voltage for lightning can be most economically provided by a mixed string. Once the advantages of a mixed string are appreciated, it will be readily seen that these more than offset any possible disadvantages even from the maintenance standpoint, as the few replacements required could be made with either diameter of section without material effect upon the operation of the system.

In order to reduce unnecessary interruptions due to the discharge of a gap to a minimum, it is essential that the gap will not flashover under oscillations or when wet at very low values. Mr. Lloyd apparently has overlooked the fact that the flashover voltage of a horn or point gap will vary considerably under these conditions. The flashover of a sphere gap will also be influenced by water, the familiar weather protection used for sphere gaps on lightning arresters being proof of this.

Reference to Fig. 6 shows that the controls make it possible to reduce the difference in flashover voltage for various conditions, the variation being much less than for a plain point or horn gap. While the sphere gap would have less variation, the presence of water makes this gap very erratic under some transients, particularly where fairly large spheres are used.

If the station insulation has a materially higher flashover voltage with a transient of one polarity, it is possible to take advantage of this by setting the control gap accordingly, which would be impossible with other types of gaps without making a change in the parts.

In installing a control type of gap the available information of course would be used in the setting of the gap. In the case of a plain gap it would not be possible to take any advantage of the difference in flashover under negative and positive impulses, whereas the control gap could be set to take advantage of any material difference. With the control type of gap it is also possible to reduce the time lag without lowering the flashover voltage of the gap unnecessarily for low grade transients by simply adjusting the control screens and the tips.

If a plain gap is used the only change possible will be to reduce the clearance and flashover voltage. If a favorable setting is obtained when the control gap is first installed, no further change will be necessary. However if it is desired to change the characteristics of the gap to reduce the number of discharges, means are available for making this change. It is also possible to change the characteristics of the gap should the insulation of the station be changed at any time. A control type of gap certainly would require no more attention than a plain gap and even with the very meager information available it would seem that a better result would be possible without any adjusting of the gap after the initial setting.

Inability to take advantage of additional information has rendered much insulation and protective apparatus obsolete. It would seem that the possibility of taking advantage of this information certainly would not be a detriment and could not be held against the control type of gap.

The intensity and magnitude of transients may vary greatly on different systems, and there certainly is no disadvantage in being able to conform to these conditions. The polarity of a stroke of lightning of course cannot be controlled. However, if the minimum flashover voltage of the insulation is much lower for one polarity than for the other, the control gap can be adjusted for this. What is really needed is more information as to the flashover or maximum voltage which may be imposed upon the equipment. Lacking this information the control gap may be used to obtain approximate information which otherwise might not be available.

J. J. Torok: Mr. Sporn in discussing Mr. Archibald's and my paper assumes that most of the work has been confined to the short wave which he has simulated to the ½-5 microsecond wave. It was the purpose of this investigation to analyze insulator performance on a broad basis rather than any specific condition such as an investigation with a short wave. A long wave was used, commonly termed the 1½-40, and the volt-time characteristics of the insulators obtained with this wave. With such a volt-time curve the characteristics of the insulators are known for short waves as well as for long waves and an analysis of the relative characteristics of insulators can be made for any length of wave desired. Thus if it is desired the relative characteristics of the insulators can be obtained for the three preferred waves from these volt-time curves. In the paper a comparison is made using the short waves for illustrative purposes and because short waves will be encountered in ground wire installations. Also, the comparisons are made for long durations such as 60 cycles.

Mr. Sporn pointed out that the authors did not make an investigation of insulators and arcing rings. In the body of the paper the authors very explicitly stated that arcing rings were not included in these tests because the effects of rings have already been investigated and reported in other papers and that inclusion of rings would introduce such a variable as to make the data required so large as to be out of the question.

Mr. Sporn commented that the cost analysis given in the paper was at variance with present price quotations and that the discrepancy in some cases was as much as 70 per cent. The cost analysis given in the paper is based on an average cost extending over a period of five or more years and is therefore only an average cost. At present costs of raw materials and finished products are undergoing a rapid change. It is therefore to be expected that the cost of porcelain products should also change. It is very likely that at the resumption of stable prices the ratios of costs as given in the paper will be approximately correct.

Mr. Lloyd questioned the nature of the wave shape used for the short time investigation. He stated that a chopped wave should be of the saw-tooth type. The reason why this wave is of altered shape is that with such high voltages the streamer formation prior to flashover requires such heavy currents that the voltage wave is considerably distorted. In lower voltage circuits the streamer formation is considerably less and the distortion of the voltage wave is negligible.

Mr. Lloyd stated that the points should have been placed on the curves. The authors in making the curves realized that because of space limitation they would have to be greatly reduced. Consequently, the significance of such points would be lost in greatly reduced figures. Again, these points are taken from oscillograms and since the oscillograms could not be printed it was logical to draw in the curves without the points.

H. A. Frey and K. A. Hawley: We would like to point out that the humidity data contained in Mr. Lloyd's paper and that in our paper may be compared very readily since, for the practical temperatures involved, the vapor pressure expressed in inches of mercury is numerically very nearly equal to the absolute humidity expressed in grains per cubic foot. Such a comparison shows that while there are some variations which will require further study the general agreement is remarkably close.

Mr. Austin has made some statements in his discussion with which we cannot quite agree. Mr. Austin expresses the opinion that general laws for humidity correction in insulator flashover cannot be established and that such correction must be determined,—"largely by tests on the particular insulator or one approximating its general characteristics." We believe that while this opinion may be true when viewed from a strictly rigorous standpoint yet even with the data now at hand it is possible to apply a humidity correction to the flashover value of any commercial insulator which is correct to within the necessary practical limits of accuracy.

Mr. Austin also states, in commenting on certain of the bushing data, that "many properly designed bushings have a higher flashover voltage with the conductor in place than when the conductor is removed." We have experimented with *solid type* bushings of several different types and makes and have failed to find any in which the presence of the conductor did not lower the flashover value. It is quite true, however, that oil bushings with built-in flux distributors may have a higher flashover with the conductor than without. Such bushings are of an entirely different type than the solid bushings for which data are given in the paper.

The Parallel Type Inverter

BY FREDERICK N. TOMPKINS*
Member, A.I.E.E.

Synopsis.—The parallel type inverter is one of many forms which are undergoing development at the present time. It gives promise of becoming of considerable importance and therefore the principles of operation should be better and more widely known. The object of this paper is to make a qualitative analysis of the operation of this type under different conditions of loading, and to present the results largely in the form of complete sets of oscillograms for further study.

Inverters of the types now being studied involve the use of hot-cathode mercury-vapor tubes called thyratrons. The characteristics of these tubes are given briefly and the data necessary for an understanding of their operation in these circuits are presented.

The principle of operation of the inverter circuit is developed by means of simple diagrams and then the actual conditions throughout the circuit are studied by means of oscillograms showing the voltages and currents in all of the essential parts. It is shown that the method of operation may be considered in a different manner from that usually assumed in the development of the circuit, inasmuch as the capacitor can be thought of as performing the function of giving the correct phase relation between the input current and the induced transformer primary voltage, rather than providing a sudden reversal of potential on the anode of the tube being stopped. This reversal is provided by the induced primary voltage. The current conditions for the various steps of operation are shown by simple diagrams.

The effects of low power factor loads with both leading and lagging current are shown in sets of oscillograms taken under these conditions. Another oscillogram gives the effects of improper circuit constants when operating at low power factor.

The output wave form under conditions of full load, unity power factor, is analyzed and it is shown that with the proper circuit constants the voltage wave is a fair approximation of sine wave. The output wave forms change, for any given set of circuit constants, with the character and degree of loading.

When reasonably good output wave forms are secured, the voltage regulation is poor but the efficiency is high. Changes in the circuit which give better regulation cause poorer wave form so that in the design of this type of inverter circuit a balance must be chosen between these two factors. Curves of efficiency and regulation, the latter showing the effects of different values of capacity, are given and briefly discussed.

Introduction

The development of the thyratron or three-element, hot-cathode gas-filled tube has given electrical engineering a new tool. Many interesting and valuable applications have been devised, among the most interesting of these being the inverter. Its purpose is the inversion or changing of direct current to alternating current, the reverse process of the more usual rectification. The inversion process is possible with the ordinary types of three-element vacuum tubes, but due to their low efficiency and high voltage drops it has not been practical to secure alternating current in this manner. With the advent of the thyratron, with its high efficiency, low voltage drop, and its ability to handle comparatively large amounts of power, the inverter gives promise of becoming of commercial importance. Hitherto, inversion from direct to alternating current has been possible only by means of rotating machinery. If the inverter is successfully developed, as it seems to give promise of being, a method of inversion will be provided which does not require rotating machinery, but which accomplishes its purpose by means of starting and stopping electron flows. This operation can be performed at high voltages, thus making high-voltage direct-current power transmission possible, the energy being generated as alternating current, rectified to direct current for transmission, and then inverted to alternating current at the receiving end of the line. It is easy to conceive of many other interesting applications of this circuit.

Inverter circuits may be divided into two main types, series and parallel. While considerable data have been published and many different circuits devised for the former type, very little beyond the fundamental circuit has been given for the latter. The purpose of this paper is to present the results of a study of the parallel type inverter which was undertaken to secure a better understanding of the function of each part and the operation as a whole. There is presented the necessary information concerning the operation of thyratrons in order that their operation in the circuit may be understood. The principles of operation of the inverter are developed by means of simple diagrams, and the actual operation under various conditions of loading is shown and discussed from complete sets of oscillograms. It is shown that the function of the condenser in the complete circuit may be considered to provide a phase relation necessary for operation, thereby differing from the description of its function that has hitherto been made. The regulation and efficiency will be indicated and briefly discussed.

Thyratron Characteristics

It is beyond the province of this paper to cover completely the characteristics of the thyratron but since the operation of the inverter cannot be understood without a knowledge of these, the essentials are given. Various types of thyratrons have been developed differing in their characteristics. The technical information concerning the more common and smaller types is listed in Table I. Much larger thyratrons than those listed are made for greater plate currents and higher anode voltages.

The thyratron, being a grid-controlled arc in low-pressure gas, differs considerably in operation from the

*Assoc. Prof. Electrical Engineering, Brown University.

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32-100

TABLE I

Type	FG-27	FG-57	FG-67
Main use	Controlled rectifier	Controlled rectifier	Inverter
Number of electrodes	3	3	3
Cathode—Volts	5.0	5.0	5.0
Amperes	6.0	5.0	5.0
Type	Coated	Indirect heater	Indirect heater
Max. peak inverse voltage	1,000.0 v.	1,000.0 v.	1,000.0 v.
Max. peak forward voltage	1,000.0 v.	1,000.0 v.	1,000.0 v.
Max. peak plate current	5.0 a.	15.0 a.	15.0 a.
Max. average plate current	2.5 a.	2.5 a.	2.5 a.
Max. tube voltage drop	24.0 v.	24.0 v.	24.0 v.
Min. tube voltage drop	10.0 v.	10.0 v.	10.0 v.
Deionization time (Microsec.)	1,000.0 approx.	1,000.0 approx.	100.0 approx.

ordinary vacuum tube. The characteristics essential for an understanding of its operation in the inverter circuit are given below.

1. The thyratron is a gas-filled tube dependent upon ionization of the gas by the passage of current to remove the space charge, thus allowing large currents to flow with very small and nearly constant tube voltage drops and consequently having a high efficiency with high anode voltage.

2. The potential of the grid controls the starting of the tube, but when the current has started and the gas has become ionized the grid has no further control. The grid has no control over the stopping of the flow of current nor of the amount of current flowing. The potential of the grid which will allow the current to start or that will prevent it from starting is dependent upon a number of factors, among them being the anode voltage, the temperature of the tube, and the inherent characteristics of the tube. Inverter tubes usually require the grid to be positive before the current will start.

3. Flow of current through the tube may be stopped in either of two ways; the anode may be made negative, causing the current to cease, deionization to take place and the grid to resume control, or the plate current may be momentarily interrupted giving the same effect. When alternating current is applied to the anode it becomes negative each half cycle, thus giving the first condition, and returning control to the grid each cycle. With direct current applied to the plate its potential must be made momentarily negative or the current interrupted for a sufficient length of time for deionization to take place before the grid can resume control.

4. The amount of current passed through the tube is dependent upon the characteristics of the external circuit, the average tube voltage drop of the order of fifteen volts having little effect. The safe amount of current is determined by the voltage drop across the tube and is limited by destructive ionic bombardment of the cathode. It has been found that as long as the current passed through the tube does not exceed the emission of the cathode, destructive bombardment will not take place.

The thyratron is inherently a rectifier, passing current in one direction only, thus it is often convenient to think of the device as a one-way contactor with an average drop of 15 volts irrespective of the amount of current. This view will help in the understanding of the operation of the inverter.

Fundamental Circuit

The development of the inverter from simple circuits involving direct-current-operated thyratrons has been described by several writers on the subject. A similar method will be used here with the aid of simple diagrams. The connections shown in Fig. 1 are indicative of a means of stopping the current in a direct-current-operated thyratron by momentarily making the anode potential negative. A condenser C is connected in series with a resistance across the load. The grid of the tube is normally biased negatively, thus no current can pass. If a positive impulse is applied to the grid circuit sufficient to overcome the bias the current will start. The grid has now lost control as the gas is ionized by the passage of the current. Assuming 500 volts being supplied, the drop across the load will be 485 volts as the tube drop is approximately 15 volts. The condenser being across the load will be charged at this potential. Now assume the switch S is closed; the right hand side of condenser C, which was at a positive potential of 485 volts, is suddenly dropped to zero volts. Due to the transient impedance of the load the left hand terminal of the condenser momentarily undergoes a similar drop which brings the anode potential to the value of 470 volts negative. If this negative voltage is maintained for a sufficient length of time, deionization takes place, the current is stopped, and the grid again assumes control.

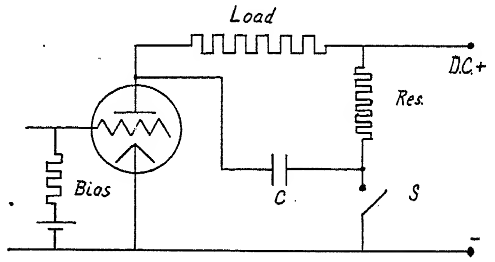

Fig. 1—Method of Stopping a Thyratron Operated on Direct Current

First stage in inverter operation

The next step in the development of the inverter circuit is the substitution of another thyratron for the switch. The connections are shown in Fig. 2. Here the grids are shown connected to a transformer in such a manner as to cause them to be alternately positive and negative. If the grid of No. 1 tube is positive, current will be passing through it and through the load, No. 2 tube not operating because its grid is negative and the conditions will be the same as in the previous discussion with the switch open. Now as the grid potentials change, that of No. 1 will become negative, which will

have no effect upon it, while that of No. 2 will become positive. As the potential applied to the anode of No. 2 is positive, current will flow through this tube and the conditions will be the same as in Fig. 1 with the switch closed, except that there is a tube drop of 15 volts. If the same supply potential is assumed as for the first case, the left hand terminal of condenser C, instead of falling to zero volts, now falls to positive 15 bringing the anode potential of No. 1 tube to 465 volts negative. Tube

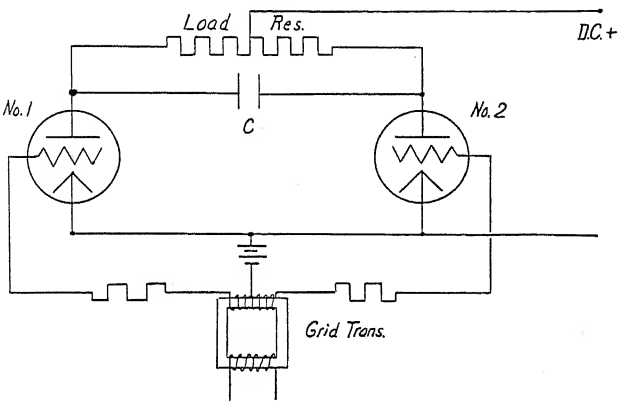

Fig. 2—Method of Stopping a Thyratron Operated on Direct Current by Means of Another Thyratron

Second stage of inverter operation

No. 2, it is seen, has performed the same function as the switch in Fig. 1. If the grids are made alternately negative and positive the current may be transferred back and forth between the two tubes, the major requirement for the operation being that the anodes are negative for a sufficient length of time for deionization to take place.

Substituting for the load and resistance of Fig 2, the primary of a transformer, gives the fundamental circuit of the parallel type inverter. This is shown in Fig. 3 with the addition of a choke coil in the supply lead to keep the alternating-current component out of the direct-current circuit. The actions taking place in this circuit are considerably more complicated than those in the circuit of Fig. 2, due to the addition of an inductance coil, the leakage reactance of the transformer, and the fact that considerable capacitance must be used. Another factor, which was not present in the circuit of Fig. 2, is the induced voltage of the transformer primary which greatly affects the operation of this circuit. A bias battery is shown so connected that the grids are biased negatively, but experience indicates that this is not needed. The frequency of the output is determined by the frequency applied to the grid transformer, which in this case is shown supplied from an external circuit. It is possible to make the circuit self-exciting, in which case the frequency is governed by the constants of the grid circuit.

Operation

The sequence of events in the operation of the circuit is illustrated by the oscillograms of Fig. 4 which give the voltages and currents in the different parts at approximately full-load unity power factor. In the tests herein described FG-67 inverter tubes were used, the condenser capacity being 4.5 μf., the inductance being 0.2 h., and the applied voltage of the order of 500 volts. While with this applied voltage the peak-inverse and peak-forward voltages were, under some conditions, rather more than the values given in the tube rating, no trouble was experienced from this cause. The circuit constants were chosen after much experimentation to give the best output wave form under widely varying conditions of load regardless of other factors such as regulation. In taking the oscillograms the input voltage was adjusted until the output voltage was 110, the grid voltage frequency being 60 cycles. The connection diagrams associated with the oscillograms of Fig. 4 show the positions of the oscillograph galvanometer elements in the circuit, and the various curves are numbered accordingly. The results are purely qualitative as no attempt was made to calibrate the elements.

It may now be of interest to analyze the conditions in each part of the circuit step by step throughout the cycle of operation. Referring to Fig. 4 at time A, curve 1 shows that the grid potential was zero, which at the applied plate potential was sufficient to permit the plate current to start as shown in curve 2. The condenser immediately charges through half of the primary in the transformer as shown in curves 3 and 4. The current in

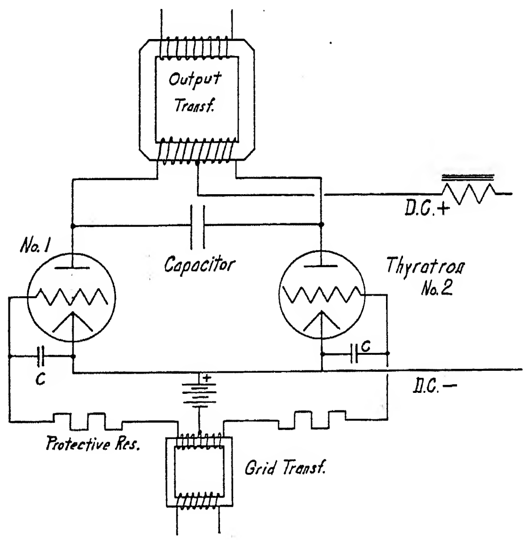

Fig. 3—Fundamental Circuit of the Parallel Type Inverter

this half would have become zero except for this charging current. The current in the other half of the primary, which was slightly negative, becomes positive as shown in curve 5. A high potential difference between the anode and cathode of the tube drops to a low value as soon as the current starts, as shown by curve 7, the anode being positive. This tube voltage drop of between 10 to 24 volts does not show on the oscillograms as the scale required by the inverse voltage was too

in succeeding sets of oscillograms does not differ materially from that just made. In the case just discussed one thyratron stopped only when the other started. Curve 2 of Fig. 9 shows that No. 1 tube stops before No. 2 starts. This is apparently due to the constants of the whole circuit including the load. When this happens the anode potential becomes negative due to the potential of one-half of the primary coil, the latter being shown by curve 9 and the sudden drop to this value appearing in curve 7. When the second tube starts putting the full primary potential on the first tube, the

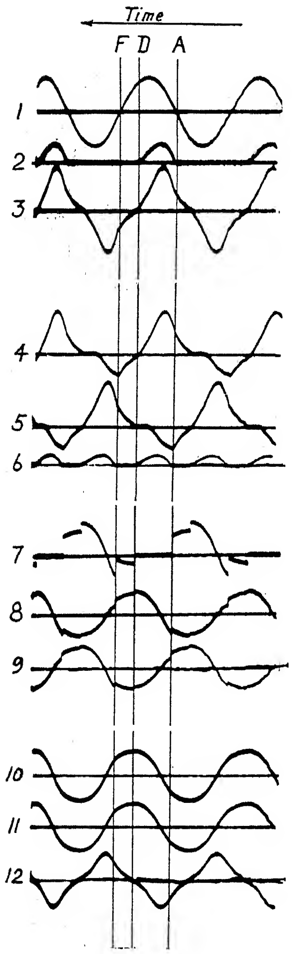

FIG. 10—OSCILLOGRAPHIC STUDY OF INVERTER OPERATION WITH INDUCTIVE LOAD

Output, 19 amperes, 110 volts, 11.5 per cent power factor, condenser 6¾ μf., inductance 0.20 h.

anode potential becomes still more negative as shown by the second drop in curve 7. In other words, the anode voltage becomes practically first that of curve 9 and then that of curve 10.

LOW POWER FACTOR LOADS

The parallel type inverter will carry low power factor loads when the circuit has the proper constants. An example of the operation with lagging current is given in the oscillograms of Fig. 10. The output voltage and current, curves 11 and 12, show that the current was lagging and that the power factor was very low. In this case, as in the case just described, the current through one tube stops some time before that through the other starts. This operation is characterized by the two distinct negative voltage drops appearing between the cathode and anode of the stopped tube as shown by curve 7.

With incorrect constants and low power factor loads the circuit may become oscillatory as is shown in the oscillograms in Fig. 11. All the stages from correct operation to that shown may be obtained by varying the values of the inductance and capacity.

Operation with a low power factor, leading-current load is shown in the oscillograms of Fig. 12. It is interesting to note that the condenser is not needed in this case, as is indicated by curve 3, the necessary phase relation between the input current and the induced primary voltage being caused by the load. The fact that this circuit will operate under this condition is proof that the condenser is needed only to give the necessary phase relation and not to fulfill the function assumed in the development of the circuit.

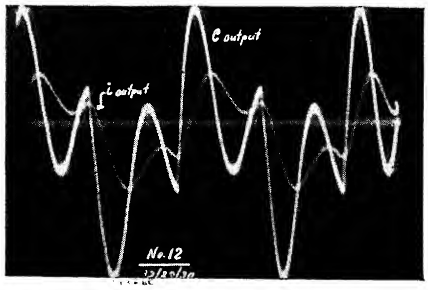

FIG. 11—OUTPUT WITH INDUCTIVE LOAD, INCORRECT CIRCUIT CONSTANTS

WAVE FORM

The output wave forms differ with the amount and power factor of the load when fixed values of capacity and inductance are used. With unity power factor, the best wave form is given at light load but as this is not as interesting a condition as full load, it was the wave form of the latter which was analyzed. When the curves for analysis were obtained the output voltage was not held at 110 volts as was done previously. The results of the analysis are given in Table II.

REGULATION AND EFFICIENCY

The value of the capacitor used in the circuit has great bearing on the voltage regulation of this type of inverter. It also has a large effect on the output wave form so that a reasonable compromise must be made. A low capacity gives the best voltage regulation and the poorest wave form and may cause instability. Capacity of too large a value reduces the output voltage to a considerable extent.

The efficiency of the inverter using thyratrons is high due largely to the high efficiency of the tubes them-

TABLE II

Effective voltage	122 volts
Average voltage	115 volts
Form factor	1.06
Peak factor	1.30
Maximum value of the 3rd harmonic	16.75 volts
Maximum value of the 5th harmonic	6.35 volts
Maximum value of the 7th harmonic	2.80 volts
Maximum value of the 9th harmonic	1.18 volts

selves. With an average tube voltage drop of only 15 volts and with reasonably high anode voltage, the losses in the tube are small and the efficiency high. Fig. 13 shows efficiency and voltage regulation curves

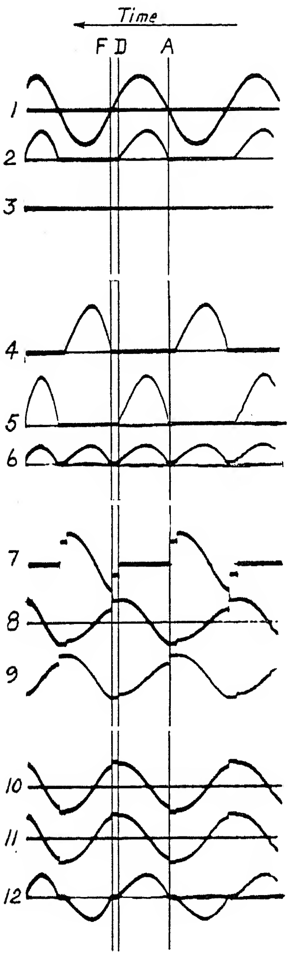

Fig. 12—Oscillographic Study of Inverter Operation with Capacitative Load

Output, 18 amperes, 110 volts, 10.1 per cent power factor, no condenser, inductance 0.20 h.

in which it is seen that the regulation with $4\frac{1}{2}$ μf. capacity is much poorer than with $1\frac{1}{8}$ μf. However, to secure a reasonably good output wave form it was necessary to use the higher value.

Conclusions

The oscillograms give in a graphical way the complete operation of the inverter under widely varying load conditions. An analysis indicates that the operation differs somewhat from the descriptions previously made, and it is felt that a more complete picture of the functions of each part and of the operation as a whole is here presented than has hitherto appeared. The circuit is a most interesting one and already has been applied in a commercial way for supplying alternating-current power to alternating-current radio sets where operation on direct-current distribution systems is required.

Nothing has been said about parallel operation of inverters with systems supplied by synchronous machines. This is a perfectly feasible method of operation and has the advantage that no condenser is required in the inverter circuit, thus making the operation more stable.

The two factors which may give difficulty are the poor voltage regulation and the lack of stability under certain conditions. The first requires, up to the present time, that some method of regulating the input voltage be used. The second, which largely disappears in parallel operation, will require the use of quick acting circuit breakers to protect the tubes in case the circuit stops operating. When the latter happens, the counter elec-

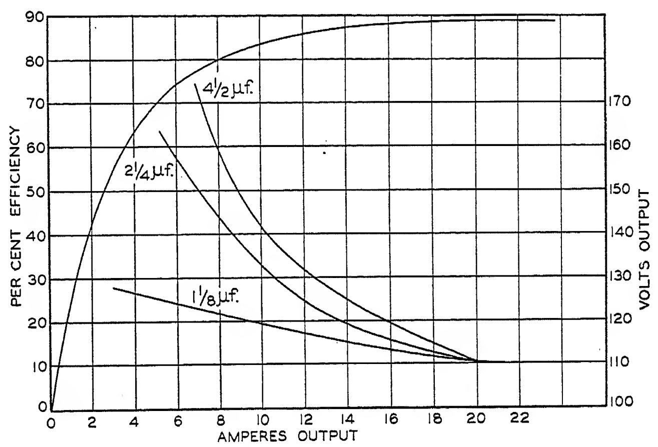

Fig. 13—Efficiency and Voltage Regulation Curves

tromotive force of the transformer disappears, and both the tube voltage drop and the resistance of the circuit being small, the tubes form practically a short circuit on the direct-current supply. Unless they are very quickly removed from the line they will be injured.

There is nothing to indicate that the difficulties of operation are insurmountable, and since the growth of electrical engineering seems to be in the direction of an increasing use of thermionic devices it may be expected that circuits of this nature will rapidly become of commercial importance.

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Discussion

W. M. Goodhue: Mr. Tompkins describes the operation of a thyratron inverter with static condensers and with an a-c. line, and stresses the fact that with an a-c. line the condensers may be omitted. Some experiments which I have carried on at the Harvard Engineering School show that static condensers may be omitted also when the a-c. line is replaced by an ordinary synchronous motor (of the salient-pole, damper-winding type). The circuit employed was a six-phase circuit with six thyratrons, it being preferable to operate a synchronous motor polyphase, instead of single-phase. The grids were excited by an induction phase-shifter of the polyphase type, consisting of a slip-ring induction motor (½ hp.), with the rotor turned by a handwheel and worm drive, and insulating transformers for the rotor output. *This phase-shifter was supplied directly from the terminals of the synchronous motor*, so that the inverter was permanently *locked in step* with the motor, and would automatically follow any speed changes. Fundamentally, the phase-shifter determines the *power factor* of the inverter, while the d-c. field of the motor enables adjustment of the a-c. voltage. The a-c. current is nearly proportional to the d-c. current, so that voltage, current, power factor, *and hence power* delivered to the synchronous motor may be adjusted, and if this power exceeds the requirements of the motor, the motor will speed up and vice versa. Hence the speed *and frequency* may be varied by the phase-shifter. The d-c. current and voltage are related by the volt-ampere characteristic of the d-c. supply circuit, so that the frequency *vs.* phase-shifter characteristic will be different, for example, in a constant-potential d-c. circuit than in a circuit of more nearly constant-current characteristics. In fact, the effect of the phase-shifter is often almost opposite for the two types of circuits.

The synchronous motor took load up to the maximum rating of the tubes, and *showed no tendency to hunt or to fall out of step*, even when the shaft load was suddenly changed. An a-c. load, single-phase or polyphase, may be supplied from the terminals of the synchronous motor, and in this case, if the shaft of the motor is unloaded, the motor may be called, as usual, a synchronous condenser, since the current leads, unless a highly capacitative load is applied in parallel.

Concerning the definition, "parallel-type inverter," some confusion arises. In polyphase work, a "series-type rectifier (or inverter) circuit" sometimes means that the different phases of a polyphase bank of transformers and tubes have their d-c. terminals connected *in series*, to accommodate the higher voltages, with the inference that when the various phases are connected in parallel for lower voltages (as in multiple-anode polyphase railway circuits), the term "parallel-type" should be employed. In fact, the title suggested a polyphase circuit, while Mr. Tompkins described a single-phase circuit, such as might be a single-phase unit of a polyphase system.

A General Theory of Systems of Electric and Magnetic Units

BY VLADIMIR KARAPETOFF*

Fellow, A.I.E.E.

Synopsis.—*A system of electric and magnetic units may be shown to be characterized by five parameters, namely: a numeric, n, which gives the ratio between the density of electric displacement used in that system and the so-called theoretical density; a numeric, p, which gives the corresponding ratio for magnetic flux densities; a physical conversion factor, k, which converts a given volume of current into the corresponding magnetomotive force; the absolute permittivity, κ; and the absolute permeability, μ. On the basis of these five parameters, the principal fundamental equations of electricity, magnetism, and electromagnetic waves are written in what the author calls the general system of units, without assigning definite values to these parameters. He also shows that the five parameters must satisfy the equation* $vk = \sqrt{np/\kappa\mu}$, *where v is the velocity of propagation of electromagnetic waves in that particular medium to which κ and μ refer; otherwise the five parameters may be chosen arbitrarily.*

By giving these parameters specific values, seven different systems of units are derived, namely, the electrostatic, the electromagnetic, the practical, the Gauss system, the Heaviside-Lorentz system, one which the author calls the compromise system, and the ampere-ohm system introduced by him some twenty years ago.

The characteristics, the advantages, and the disadvantages of each system are briefly discussed, and it is shown how to deduce a new system of units from the general system, to conform to certain desired specifications.

At the end of the article a discussion is given on the physical dimensions of various electric and magnetic units. The author states that four fundamental units are necessary for a system, so that if a certain length, mass and time interval are taken as three fundamental units, a fourth electric or magnetic unit must be added, for example, the permittivity or the permeability. In the ampere-ohm system two mechanical fundamental units are used, the centimeter and the second; and two electrical units, the ampere and the ohm.

* * * * *

IT IS generally conceded that the present situation in regard to the electric and magnetic units is far from being satisfactory, for the following reasons:

a. Two or three systems of units are in general use (c.g.s. and practical), at least two more are employed by some prominent writers (the Gauss system and the Heaviside-Lorentz system), and new ones are being proposed from time to time.

b. The absolute magnitudes of some units fixed by international agreement, or in general use otherwise, are considered as being too small or too large by some investigators, who urge the adoption of smaller or larger units.

c. There is an appreciable disparity between the "absolute" c.g.s. units and the corresponding "international" units.

d. The physical dimensions of the electric and magnetic quantities are believed by some scientists to be different in different systems of units. Some physicists even believe it legitimate to assign an arbitrary physical dimension to a quantity, for example, to assume the absolute permeability of any medium to be a numeric; see Section IX below.

It is not the purpose of the present investigation to extol one of the existing systems of units in preference to the others, or to propose a new system of units. The author merely intends to show that all the existing systems are special cases of a General System of Units and may be obtained from it by assigning definite numerical values to certain parameters. Moreover, new systems of units may be produced out of this General System, to satisfy certain desired conditions. In this manner, the argument about systems of units is reduced to that of the possible choice of most convenient values of parameters in the general system, and the burden of proof is shifted from a general discussion to that of particular convenience in a group of applications. It is hoped in this way to contribute to a satisfactory adjustment of the difficulty (a) above. Moreover, the relationship among the corresponding units in the different systems, which is one of the main stumbling blocks in actual use, follows directly from the manner in which a particular system of units is related to the General System. The question of the physical dimensions of a quantity is settled by reference to the dimensions of that quantity in the General System of Units. The same physical quantity can hardly be expected to have two different physical dimensions in two special systems derived from the same General System. This question is discussed more fully in Section IX below.

The difficulty (b) above has existed and always will exist in many branches of quantitative natural sciences. It is clear that atomic dimensions and interstellar distances cannot be conveniently measured with the same yardstick, any more than the current of a photoelectric cell and the current through a large metallurgical steel furnace. This difficulty is minimized by using a multiplying factor of ten to a convenient integral power. From the point of view of applications it is immaterial whether the millimeter or the meter is used as a primary unit, or whether a millisecond or a megasecond is chosen as the primary unit of time. The same applies to units of magnetic flux, power, etc. The use of the factor ten to an integral power is not only inevitable, but quite convenient in applications.

The difficulty (c) is in the process of being rectified.

*Professor of Electrical Engineering, Cornell University, Ithaca, New York.

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Due to a better experimental technic, it ought to be possible before long to reach a new agreement in regard to the slight corrections to the present international values of the ampere and the ohm, so as to bring them more closely in accord with the corresponding multiples of the c.g.s. values.

* * * * *

In order to derive a consistent set of equations of the principal phenomena in electricity and magnetism, we shall abandon the historical point of view and look upon these phenomena as being known to us without any chronological precedence of discovery. Besides, we shall introduce factors of proportionality in a general form, without ascribing to them definite values, such as 1, 4π, c, etc. In this manner, it will be possible to derive some general relationships first and obtain various existing systems of units as special cases later. The kinship of the systems and their salient features will then become apparent.

I. Electrostatic Relationships

We begin with the recognized fact that an electrostatic field may be established, say in vacuum, by suitably placing electric charges here and there. We agree to measure the intensity of such a field at a point by the mechanical force exerted by the field upon a small concentrated electric charge, say an electron, placed at that point. Thus, one of our starting points is the equation

$$f_e = Gq \qquad (1)$$

where f_e is the mechanical force, G the field intensity (voltage gradient) and q the probe charge.* The subscript e stands for "electrostatic." The force f_e is to be measured in any chosen mechanical units which are to go with the electric and magnetic units. G thus becomes defined as the mechanical force per unit charge. Of course, equation (**1**), by itself, is not sufficient to define both unknowns, q and G, and it is to be looked upon only as one of a set of simultaneous equations which together are sufficient for defining the principal electric and magnetic quantities; see Appendix. Consequently, it is immaterial whether equation (**1**) or equation (**8**) is introduced first, the latter being the familiar Coulomb's law of inverse squares. The fundamental quantities are not defined "step-by-step," but all the fundamental phenomena and the equations which express them are considered on a par in the set of final formulas.

For purposes of definition of units equation (**1**) may be said to contain two unknown quantities in the same sense as the familiar relationship in elementary mechanics

$$\text{force} = \text{mass} \times \text{acceleration}.$$

Here the acceleration may be assumed to be known from a more elementary subject, namely kinematics; however, the unit mass must be also defined before this equation, alone, can be used for the definition of unit mechanical force.

All the equations which are to be used in the deduction of the quantities in the proposed General System of Units must be based either on immediate results of some measurements, or on conclusions derived from observed facts and from an assumed mechanism at work. Some physicists may be inclined to consider equation (**8**) to be more fundamental than equation(**1**), yet the latter is being constantly verified in various measurements on electronic velocities, accelerations, and impacts. With the advance in the physical and chemical theories, certain ultra-microscopic mechanisms have to be postulated that lead to relationships which can be measured in bulk only. Thus, the Boltzmann-Maxwell distribution of molecular velocities in a gas, the mean free path, etc., can be verified only indirectly. From this point of view, equation (**1**) is also an experimental relationship and not an *a priori* assumption.

As another fundamental phenomenon, let us consider the dielectric flux and its density due to an electric charge, say Q, concentrated upon a sphere of very small radius ζ. It is an experimental fact that the total dielectric flux due to such a charge is the same through any imaginary concentric sphere or other surface which encloses the charge. In the language of theoretical physics, the divergence of an electrostatic flux in a space devoid of electric charges is equal to zero. The simplest assumption to make is that this flux is numerically equal to the charge which causes it, since experiment shows the two to be proportional to each other. We shall call the value of the flux expressed in such units as to be numerically equal to the corresponding charge the "theoretical" value. In a given system of units it may be necessary to multiply this value by a numerical factor in order to obtain the actual flux in that system of units. At a distance $R > \zeta$ from the center of our charge, the area of the imaginary concentric sphere is $4\,\pi R^2$, so that the "theoretical" density of displacement is

$$D_t = Q/(4\,\pi R^2) \qquad (2)$$

where the subscript t stands for "theoretical." In a chosen system of units another value of density of displacement, say D, proportional to D_t, may be used, so that, generally speaking,

$$D = nD_t \qquad (3)$$

where n is a numeric. Thus, for example, in the c.g.s. electrostatic system, as used by Maxwell, $n = 4\,\pi$.

The physical properties of a dielectric in which an electric displacement takes place are expressed by its so-called absolute permittivity, which we shall denote by κ. Except under very special laboratory conditions, there is no saturation in dielectrics, and experiment shows that G is proportional to D. We thus have, as a definition of κ,

$$D = nD_t = \kappa G \qquad (4)$$

As a special case, for vacuum,

$$D_v = nD_{vt} = \kappa_v G \qquad (5)$$

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*For complete Notation see page 726.

It is convenient to express the absolute permittivity of a medium by indicating the number of times by which it is greater than that of vacuum, in the same system of units. We may thus write

$$\kappa = K\kappa_v \tag{6}$$

where κ is the absolute permittivity of the dielectric in question, κ_v is that of vacuum, and K is known as the dielectric constant or specific inductive capacity. It is also proper to call K the relative permittivity of the dielectric. K is a numeric, whereas κ and κ_v are physical quantities which have, or may have, physical dimensions. Substituting for D_t its value from equation (2) in equation (4), gives

$$G = nQ/(4\,\pi\kappa R^2) \tag{7}$$

Equation (1) therefore becomes

$$f_e = nQq/(4\,\pi\kappa R^2) \tag{8}$$

This is the familiar Coulomb's law of inverse squares in an isotropic dielectric whose absolute permittivity is K. Instead of being accepted as a fundamental experimental fact, this law has been deduced here from two other general experimental facts, namely, (a) in a homogeneous isotropic dielectric which is free of charges an electrostatic flux is solenoidal, that is, its divergence is zero; and (b) an electrostatic stress at a point and the corresponding density of displacement are proportional to each other. For the purposes of this paper it is immaterial whether equation (8) is assumed as fundamental and the conclusion that the flux is solenoidal deduced from it, or *vice versa*. In either case, experimental facts are strictly adhered to, and the reader need not have an uneasy feeling that physical truths are being derived from mental ideas.

As a digression, it may be of interest to mention that the attraction or repulsion between two point charges in a three-dimensional space being inversely as the square of the distance, may be thought of as a consequence of the fact that the areas of two concentric spheres are as the squares of their radii. In other words, the density (and consequently the intensity) of the electric field due to one of the charges varies inversely as the square of the distance, because the flux has no divergence. Similarly, in a two-dimensional space with solenoidal fields, the mechanical force between two point charges may be expected to vary as the first power of the distance, because circles take the place of the spheres. This is confirmed by the fact that the mechanical force between two parallel infinitely long filamental charges, per unit length, varies inversely as the distance between them. If it may be assumed that the electric flux in a Euclidean space of Z dimensions is also solenoidal, the mechanical force between two concentrated charges should be taken as inversely proportional to $(Z-1)$st power of the distance, because of the geometric properties of the space.

Let Φ_e be an electrostatic flux expressed in some system of units and let Φ_{et} be the same flux in "theoretical" units, defined above by the condition that

$$\Phi_{et} = Q \tag{8a}$$

From equation (3) we then obtain

$$\Phi_e = nQ \tag{8b}$$

The latter expression is Gauss' theorem in the General System of Units.

II. Magnetic Relationships

On the assumption that free magnetic poles actually exist, a set of formulas may be written down by analogy with those developed for electric charges above. True, such an assumption is out of harmony with the modern ideas of molecular structure, and a theory of magnetism can be built up without bringing in the idea of free poles at all. However, the proposed General System of Units must include the c.g.s. electromagnetic system as a special case, and since the latter has the Coulomb law of mechanical force between free magnetic poles for its starting point, it is advisable to include the concept of magnetic pole in our General System of Units. The formulas containing such poles explicitly need not be used in applications.

By analogy with G, we define the magnetic (or magnetizing) intensity, H, at a point in a magnetic field as the mechanical force exerted at that point upon a unit magnetic pole. This gives the relationship.

$$f_m = Hm \tag{9}$$

analogous to equation (1). Here m is the strength of a probe magnetic pole, and the subscript m signifies that a magnetic phenomenon is meant. By analogy with equation (2) we have

$$B_t = M/(4\,\pi R^2) \tag{10}$$

where M is a concentrated magnetic pole and B_t is the so-called "theoretical" density of magnetic flux which M produces at a distance R from its center. Equation (5) becomes

$$B = pB_t = \mu H \tag{11}$$

Here B is the magnetic flux density used in a chosen system of units, p is a numerical coefficient which gives the ratio between this flux density and the above-defined theoretical flux density; μ is the absolute permeability of the medium. Of course, μ may have definite physical dimensions, although at present they cannot be expressed through those of length, time, and mass alone. By analogy with the absolute and relative permittivity, we introduce the notation

$$\mu = \mu_r\mu_v \tag{12}$$

Here μ is the absolute permeability of a medium, μ_r its relative permeability, and μ_v is the absolute permeability of vacuum. The quantity μ_r is a numeric. By analogy with equations (7) and (8), we may write

$$H = pM/(4\,\pi\mu R^2) \tag{13}$$

$$f_m = pMm/(4\,\pi\mu R^2) \tag{14}$$

The latter equation is Coulomb's law of inverse squares for magnetic poles. By analogy with equations (8a) and (8b), Gauss' law for magnetic poles becomes:

$$\Phi_{mt} = M \tag{14a}$$

$$\Phi_m = pM \tag{14b}$$

III. The Two Laws of Circuitation

When electricity is in motion along certain paths, magnetic lines of force are formed in such a manner as to be linked with the paths of the current. This general experimental fact may be quantitatively expressed by the following relationship:

$$k[(1/n)\ (\partial\Phi_e/\partial t) + IN] = \oint H\ ds \cos\ (H, ds) \quad \textbf{(15)}$$

In this equation the right-hand side represents the line integral of magnetic intensity H along a closed path; this line integral is known as the magnetomotive force. The relationship is similar to that between an electric intensity, G, and its line integral known as the electromotive force. On the left-hand side of the equation we have the total volume of current threading the closed curve chosen on the right-hand side. In the most general case, in an imperfect dielectric, the current is partly that of displacement of electric charges and partly a true conduction current. Φ_e is an electrostatic flux due to moving charges; dividing it by n converts it into the theoretical flux or actual charge, according to equations (**8a**) and (**8b**). The rate of change of this charge with the time gives the displacement current. The term NI represents the volume of the conduction current, where I is the current itself and N the number of turns through which it is flowing. The coefficient k characterizes a chosen system of units and converts a current into the corresponding magnetomotive force. In a perfect dielectric the IN term is equal to zero. In a metal conductor the term containing Φ_e may be omitted. The factor k may be assumed to have definite physical dimensions, because it converts an electric quantity into something different. What the dimensions of k are, we do not know.

In a special case of an infinitely long straight conductor carrying current I, the lines of magnetic force are concentric circles. At a distance R from the center line of the conductor, the length of a line of force is $2\ \pi R$, so that, with Φ_e = const. and $N = 1$, equation (**15**) becomes

$$kI = 2\ \pi RH \quad \textbf{(16)}$$

which is the general form of the law of Biot and Savart.

An equation similar to equation (**15**) may be written for a change of a magnetic flux with the time (Faraday's law of induction). In the General System of Units this law becomes:

$$-\ (k/p)\ (\partial\Phi_m/\partial t) = \oint G\ ds \cos\ (G, ds) \quad \textbf{(17)}$$

The right-hand side is the line integral of electric intensity G over a closed curve, in other words, the electromotive force induced along this curve. The magnetic flux threading through this curve is Φ_m, and its rate of change with the time is experimentally known to be proportional to this e.m.f. The factor p converts the flux expressed in a particular system of units into the "theoretical" number of lines of force, equation (**14b**). The factor k characterizes the system of units used and converts a time rate of change of a flux into an e.m.f. At first sight, no reason is evident why the factor k should be the same in equations (**15**) and (**17**), but this is proved below from energy considerations. In equation (**17**) the term corresponding to NI is omitted, because no magnetic current is known to exist which corresponds to an electric conduction current.

As a special case of equation (**17**), consider a uniformly wound torus ring connected to a source of direct current through a switch. The ring is also provided with a concentric secondary winding consisting of N turns and connected to a ballistic galvanometer. Let the primary circuit be initially open and then suddenly connected to a source of direct current. The absolute magnitude of the secondary induced voltage at an instant t will be $(k/p)\ N(\partial\Phi_m/\partial t)$. This voltage is used up in the resistance drop ir and the inductive drop $L\ di/dt$ of the secondary circuit. Thus, for an interval of time dt we have:

$$(k/p)N\ d\Phi_m = ir\ dt + L\ di \quad \textbf{(18)}$$

Integrating this expression with respect to time, from the instant at which the primary switch is closed until the flux has practically reached its final value and the secondary current vanished, we obtain:

$$(k/p)\ N\Phi_m = rQ \quad \textbf{(19)}$$

where Q is the total electric discharge through the ballistic galvanometer (integral of idt) in units of electric charge. The term with L vanishes because $i = 0$ at both limits. Equation (**19**) is more convenient in some applications than equation (**17**), and moreover it connects a magnetic flux with an electric charge, which is of importance in studying units.

IV. Stored Energy

The energy stored in a charged condenser may be looked upon either as concentrated on the metal electrodes, or as distributed in the dielectric in the form of electrostatic stresses and displacements. Of course, numerically the two expressions for the stored energy are identical, but the second form permits us to introduce the useful concept of density of stored energy, or energy stored per unit volume of the dielectric. Consider in particular a parallel-plate condenser which is being gradually charged from zero to a charge Q. At some intermediate instant, t, let the instantaneous accumulated charge be q and let dq be the charge added during the interval dt. If the stress in the dielectric is G and the distance between the plates a, the work done in bringing this additional charge will be:

$$dW_e = aG\ dq \quad \textbf{(20)}$$

The true electrostatic flux density is q/A, where A is the area of either plate, so that, in accordance with equation (**4**),

$$G = (q/A)\ (n/\kappa) \quad \textbf{(21)}$$

Consequently

$$dW_e = anq\ dq/A\kappa) \quad \textbf{(22)}$$

or, after integration,

$$W_e = anQ^2/(2\,A\kappa) \tag{23}$$

Dividing both sides of this equation by the volume of the dielectric, aA, gives the density of electrostatic energy

$$W_e' = W_e/(aA) = n(Q/A)^2/(2\,\kappa) = nD_t^2/2\,\kappa \tag{24}$$

From this expression we may immediately write

$$W_e' = nD_t^2/(2\,\kappa) = D^2/(2\,n\kappa) = GD_t/2 = GD/(2\,n) = \kappa G^2/(2\,n) \tag{25}$$

Since the magnetic concepts introduced above have been assumed to be formally analogous to those in electrostatics, we may also write

$$W_m' = pB_t^2/(2\,\mu) = B^2/(2\,p\mu) = HB_t/2 = HB/(2\,p) = \mu H^2/(2\,p) \tag{26}$$

V. Proof that the Factor k in Equations (15) and (17) is the Same

Let us calculate the magnetic energy stored in a torus ring excited with direct current. This energy is equal to the electric energy put into the exciting winding during the process of bringing the flux from zero to its final value. According to equation (17), that portion of the applied voltage which at an intermediate instant t balances the counter e.m.f. of the flux, is:

$$e = (k/p)N\,d\Phi_m/dt \tag{27}$$

Multiplying both sides of this equation by $i\,dt$, where i is the instantaneous value of the exciting current, we obtain

$$eidt = (k/p)Ni\,d\Phi_m \tag{28}$$

The left-hand side of this equation gives the energy contributed from the source of electric power. Consequently, the right-hand side represents the increase in the stored magnetic energy. In order to be able to integrate the right-hand side, between zero and the final value of the flux, we shall express both i and Φ_m through the magnetic intensity H. If the cross-section of the flux is A and the flux density B, we have

$$\Phi_m = BA = \mu HA \tag{29}$$

Let the average length of the lines of force within the torus ring be a; then the total magnetomotive force is aH, and according to equation (15) it is equal to $k'iN$. Here the value of k is temporarily denoted by k', and it is required to prove that it is equal to the value of k in equation (17). Thus,

$$Ni = aH/k' \tag{30}$$

Substituting the values from equations (29) and (30) in equation (28) and integrating between the limits $H = 0$ and $H = H$, we obtain for the stored magnetic energy the expression

$$W_m = (k/k')\mu H^2 \,.\, aA/(2\,p) \tag{31}$$

so that the density of energy is

$$W_m' = (k/k')\mu H^2/(2\,p) \tag{32}$$

Comparing this expression with formula (26), we see that $k' = k$.

VI. Relationship between the Parameters of a System and the Velocity of Light

So far the five parameters n, p, k, κ, and μ of a system of units have been assumed to be entirely arbitrary and independent of one another. It will now be shown that they must satisfy the experimental fact that the velocity of propagation of electromagnetic waves in a medium characterized by κ and μ is equal to the velocity of light in that medium. Consider a plane electromagnetic wave propagated in a perfect dielectric in the direction of the X-axis, the electric and magnetic intensities being parallel to the Y and Z-axes respectively. For any point in the medium, equation (15) becomes (references 12, 14, 16)

$$(k/n)\partial D/\partial t = \partial H/\partial x \tag{33}$$

while equation (17) gives

$$(k/p)\partial B/\partial t = \partial G/\partial x \tag{34}$$

In these equations, D may be replaced by κG and B by μH, so that only the functions G and H remain. H can be eliminated by taking a partial derivative of equation (33) with respect to t and a partial derivative of equation (34) with respect to x. The result of elimination is the following well-known partial differential equation of wave propagation for G as a function of t and x:

$$v^2\partial^2 G/\partial x^2 = \partial^2 G/\partial t^2 \tag{35}$$

An identical equation may be deduced for H. In these equations the new parameter, v, is determined by the relationship

$$vk = \sqrt{np/(\kappa\mu)} \tag{36}$$

The parameter v may be shown to be the velocity of propagation of electromagnetic waves in the particular medium characterized by the absolute permittivity κ and absolute permeability μ. For the vacuum the well-known magnitude of the velocity of propagation of light or of electromagnetic waves is usually denoted by c, so that we have

$$ck = \sqrt{np/(\kappa_v\mu_v)} \tag{37}$$

This is the relationship which must be satisfied by the parameters n, p, k, κ_v and μ_v of a system of units. In other respects the parameters may have arbitrary values. Thus, we are free to choose four out of five parameters, and the fifth must satisfy equation (37).

It must be understood that a choice of numerical values of four out of the five parameters merely singles out a group of systems of units, and not a specific system of units, because one is still at liberty to select desired "non-electrical" fundamental units, such as those of length, time and force (or mass). Therefore, it is not meant that in equation (37) the velocity of light should necessarily be measured in centimeters per second or in some such universally used units. Having chosen arbitrarily the values of four electromagnetic parameters, some units of length and time must also be chosen independently, and the velocity c

expressed in these units before equation (**37**) may be applied. Thus, the velocity of light is placed on no special or preferred basis.

The use of equation (**37**) will become more clear in the applications in the next section.

VII. Tabulation of the Parameters for some Existing Systems of Units

In Table I the values of the five parameters which characterize a system of units are shown for the principal systems in use or proposed. The "eruption of 4 π's," using Heaviside's expression (reference 5), is apparent in the first four systems, and has been one of the reasons for the introduction of the later systems indicated in columns 5 and 7. The table may be conveniently studied in connection with the list of the principal formulas given in the Appendix. These formulas are explained in the text above and the numbering is the same.

It is not the purpose of this paper to advocate or to defend any particular system of units, but only to show how all the existing systems and an infinite number of new ones can be derived by giving different values to the five parameters which satisfy equation (**37**) and also choosing desired units of length, time and, say, mass. Looking first at the values of n and p in Table I it will be seen that the existing systems may be divided into those in which the value of these parameters is 4 π, and those in which this value is equal to unity. The first group of systems may be called "classical," the second "rational," or "rationalized." The term rational has been often used by the defenders of the latter systems, and is employed here in that specific sense, without in any way implying that the 4 π systems are "irrational." The author is almost tempted to suggest the adjective "pi-less" in place of "rational."

The advantage claimed for a rational system ($n = p = 1$) is that the flux density in such a system (both electrostatic and magnetic) becomes equal to the theoretical flux density; equations (**3**) and (**11**). This means that the total electrostatic flux due to a charge is both numerically and dimensionally equal to that charge; equation (**8b**). The same applies to a magnetic pole and the flux issuing from it; equation (**14b**). Thus, Gauss' theorem simply becomes $\Phi_e = Q$ and $\Phi_m = M$.

As another advantage of a rationalized system it is pointed out that in dealing with geometric shapes of fields, the factor π appears with cylinders and spheres, as is to be expected, and does not appear with rectangular shapes. The opposite is the case with the classical systems. For example, in the c.g.s. electrostatic system of units the capacitance of a sphere in vacuum is equal to its radius R, and the capacitance of a plane-parallel condenser is $\kappa A/4\pi a$. On the other hand, in the Heaviside-Lorentz system these values are $4\pi R$ and $\kappa A/a$ respectively. Again, in the electromagnetic system, the density of stored magnetic energy in a nonmagnetic body is $B^2/8\pi$, whereas in the ampere-ohm system it is $B^2/2\mu_v$. The fact that μ_v in the latter system contains 4π is not supposed to weaken the argument. The user of this system of units simply knows that μ for ordinary non-magnetic bodies is equal to 1.257×10^{-8}, this being an experimental physical constant, and the presence of the factor 4π need not be explained theoretically, as is the case with the classical systems.

Of course, an expert who has been using the classical systems for a number of years finally learns to place the factor 4π where it belongs, but undoubtedly this factor is a stumbling block to a student and a young investigator. In a rational system, the factor 4π is retained in the expression for Coulomb's law of inverse squares, equations (**8**) and (**14**), for the reason that 4π enters in the formula for the surface of a sphere in equations (**2**) and (**10**).

Substituting the values of the five parameters of the electrostatic system in equation (**37**), we obtain

$$4\pi c = \sqrt{(4\pi)^2 c^2}$$

which is an identity. In the same manner it may be shown that the parameters of the other six systems given in the table satisfy equation (**37**).

It will also be seen that no particular units of length and time need be specified so long as it is only desired to express the electromagnetic parameters in terms of c

TABLE I—VALUES OF THE PARAMETERS IN SOME TYPICAL SYSTEMS OF UNITS

System number	1	2	3	4	5	6	7
Parameter	Electrostatic	Electromagnetic	Practical	Gauss	Heaviside-Lorentz	Compromise	Ampere-ohm
k	4π	4π	4π	$4\pi/c$	$1/c$	$4\pi/c$	1
n	4π	4π	4π	4π	1	1	1
p	4π	4π	4π	4π	1	1	1
κ_v	1	$1/c^2$	$1/c^2$ (not used)	1	1	$1/(4\pi)$	$10^9/(4\pi c^2)$
μ_v	$1/c^2$	1	1	1	1	$1/(4\pi)$	$4\pi/10^9$
Fundamental units	cm gram sec κ	cm gram sec μ	10^9 cm 10^{-11} gr sec μ	cm gram sec κ, μ	cm gram sec κ, μ	cm gram sec κ, μ	cm. ampere ohm sec.

algebraically. Thus, one may say that there is an infinite number of electrostatic systems of units, all characterized by the values $k = n = p = 4\pi$ and $\kappa_v = 1$. For all such systems $\mu_v = 1/c^2$. With the centimeter and the second independently chosen as fundamental non-electrical units, $c = 3 \times 10^{10}$, and therefore $\mu_v = (1/9) \times 10^{-20}$. With the kilometer and the microsecond as such units, $c = 0.3$ and $\mu_v = 1/0.09$. In some investigations it is convenient to choose $c = 1$; in the corresponding electrostatic system $\mu_v = 1$.

A similar reasoning applies to all the columns in Table I, each column (with the exception of the last) standing for an infinite number of systems of units all characterized by the same values of four out of five parameters, the fifth being expressed in terms of the velocity of light, c. The last horizontal row in Table I indicates which particular units of length, time, and mass have been chosen in the system of each type actually in use. These non-electrical units may be changed at will, still retaining the general electric and magnetic characteristics of the system.

The symbol κ written at the bottom of column 1 means that historically the electrostatic system of units was built on the assumption of a definite magnitude for the absolute permittivity of the vacuum, more particularly $\kappa_v = 1$. The symbol μ at the bottom of the second and third columns signifies that these systems were built on the assumption $\mu_v = 1$. In the systems shown in the columns 4 and 5 it was simultaneously assumed that $\kappa_v = 1$ and $\mu_v = 1$. In column 6 it is postulated that $\kappa_v = \mu_v = 1/(4\pi)$, for reasons of symmetry explained later. In the ampere-ohm system no attempt is made to fix the values of κ_v and μ_v, this particular system having been devised to take advantage of the existing values of the ohm and the ampere (irrespective of their historical origin) and of the simplicity of having $k = n = p = 1$.

It is left for the reader to substitute the values of the parameters given in Table I in the various formulas given in the Appendix and to satisfy himself that familiar expressions are obtained as a result. It may be of interest here to mention briefly the reasons for which the systems shown in the table have been proposed and used.

In the beginning, say late in the eighteenth century, no connection was known between electric charges and magnetic poles; therefore, there was no necessity for expressing the magnetic and the electrostatic quantities in one and the same system of units. Since the concept of "action at a distance" was prevalent among the French savants of that time, a deduction of formula (**8**) in a manner similar to that given in the text above was not possible. Two charges or two magnetic poles were found by experiment to exert upon each other a force inversely as the square of the distance, so that the natural assumption was (especially for the mathematical school of physicists) to write $f_e = Qq/R^2$ and $f_m = Mm/R^2$, for air or vacuum. The added assumption was that in the electrostatic system κ_v was equal to unity and in the electromagnetic system μ_v was equal to unity. Later the values of relative permittivity and permeability were added as a factor in the denominators of these formulas, to account for newer experimental results. These two formulas thus served as a basis for the c.g.s. electrostatic and electromagnetic systems of units respectively. In our notation, the omission of the factor 4π in the denominators of equations (**8**) and (**14**) is equivalent to the assumption $n = p = 4\pi$; it is in this way that the factor 4π came into the mathematical theory of electricity and magnetism and has kept physicists struggling with various systems of units ever since.

Later, Biot and Savart found experimentally that the magnetic field due to an infinitely long straight current-carrying conductor could be expressed by the formula $H = 2I/R$, and Laplace gave his familiar differential expression for the interaction between a magnetic pole and an element of current, in the form $H = ids \cos \theta/r^2$. This was equivalent to assuming $k = 4\pi$ in equations (**15**) and (**16**), and also meant $k/p = 1$ in equations (**17**) and (**19**), as was later found by Faraday. Of course, those investigators did not think in terms of the General System of Units here described, but their assumptions based on other arguments were equivalent to assigning the values of the parameters indicated in Table I. Having four out of five parameters fixed, the remaining ones (μ_v in the electrostatic system and κ_v in the electromagnetic system) also became determined.

Later it was found that the ratios of the various quantities in the two systems, 1 and 2, were simple functions of the velocity of light, c, although the reason for this did not become clear until after Maxwell had published his electromagnetic theory of light. This relationship between κ_v, μ_v and c is the one given by equation (**37**) which connects the five parameters with the velocity of light. It is because of this relationship that the absolute magnetic permeability of vacuum in the electrostatic system is found to be numerically (although not necessarily dimensionally) equal to $1/c^2$, and the same value comes out for the permittivity of the vacuum in the electromagnetic system.

Consider again equation (**8**), first written in an electrostatic system of units and then in an electromagnetic system, the fundamental units of length, time, and mass being the same in the two systems. Assume equal charges, $q = Q$ and $q' = Q'$, in vacuum; let primed letters refer to the electromagnetic system. Let R be the same in both cases, and let q and q' be physically the same charge, so as to have the same force on the left-hand side. We then have

$$q^2 = c^2(q')^2$$

where the factor c^2 comes in on account of the value of the permittivity in the second system. Thus, the unit of electric charge in an electromagnetic system is c times

$k/p = 1$, the unit flux is such a flux which induces one volt per turn when this flux disappears or is built up at a uniform rate exactly within a second. In other words, fluxes are measured in volt-seconds; the name "weber" has also been proposed for such a unit.

To deduce the value of μ_v in the ampere-ohm system theoretically, consider equation (**11**), $B = \mu_v H$, written in this system, and compare it with the same relationship, $B' = H'$, in the c.g.s. electromagnetic system. Since the unit of flux in the ampere-ohm system is 10^8 times that in the other system, we have $B' = 10^8 B$. The magnetizing force, H, in the ampere-ohm system is equal to NI/a, where I is in amperes. In the c.g.s. electromagnetic system, $H' = 4\pi(NI/a)\,10^{-1}$, because in this system $k = 4\pi$ and the unit of current is equal to ten amperes. Thus, we have

$$10^8 B = (4\pi/10)NI/a \tag{38}$$

and also

$$B = \mu_v NI/a \tag{39}$$

Comparing these two expressions, we find that in the ampere-ohm system

$$\mu_v = 4\pi/10^9 \tag{40}$$

To deduce the value of κ_v in the ampere-ohm system theoretically, use equation (**8**) in application to two equal charges at a distance R, in vacuo. In the ampere-ohm system

$$f_e = Q^2/(4\pi\kappa_v R^2) \tag{41}$$

whereas in the c.g.s. electrostatic system

$$f_e' = (Q')^2/R^2 \tag{42}$$

Here f_e is expressed in joulecens and f_e' in dynes. Since a joule is equal to 10^7 ergs, a joulecen is equal to 10^7 dynes; that is, $f_e' = 10^7 f_e$. We first convert Q into c.g.s. electromagnetic units (abcoulombs) by dividing it by 10, and then convert abcoulombs into electrostatic units by using the conversion factor c. Thus, $Q' = cQ/10$. Substituting this value in equation (**42**) and comparing the result with equation (**41**) we get

$$\kappa_v = 10^9/(4\pi c^2) \tag{43}$$

It will be seen that the values of the five parameters given in column 7 of Table I satisfy equation (**37**). In fact, having determined μ_v, the value of κ_v could have been deduced directly from equation (**37**).

The reader will thus see that with the general equations of electricity and magnetism written in the form given in the Appendix, new systems of units can be multiplied indefinitely, as shown on a few examples above. Each system so far developed satisfies certain "specifications" and may be the best system for a particular purpose or set of problems. It is hardly feasible to speak of the best all-around system of units; at least none has been universally accepted as such so far, despite numerous discussions and arguments. With the General System of Units expounded in this paper, all the systems now in existence, and possibly some of the new ones to be evolved in the future, may be looked upon as specific cases of this system. Therefore, the argument about systems of units becomes localized along a few definite lines, such as the following: (a) Is it desirable to have a "rationalized" system of units; (b) Should the best system be also "symmetrical"; (c) Should any of the existing international electric or magnetic units be made part of it; (d) What are to be the fundamental non-electrical units; (e) Should it be a pure or a hybrid system of units; etc. It will be readily seen that anything like a universal agreement among the physicists and engineers of the various countries on all these points is a remote possibility. A more hopeful initial program may consist in a gradual elimination of the older and less used system of units.

IX. Physical Dimensions of Quantities

The broad question of physical dimensions of units is not necessarily connected with the introduction of the General System of Units, and persons holding quite opposite views on dimensional analysis (ref. 21) may find the proposed derivation of various systems of units from a single more general system useful. A brief discussion of physical dimensions is included in the paper, partly for the sake of completeness and partly to bring out a subsequent discussion of conflicting views on the subject of dimensional formulas. The controversy hinges on whether the physical dimensions of a quantity, say mechanical force or inductance, are inherent in that quantity or are a matter of convenience and somewhat arbitrary. The author takes the point of view that for those investigators whose field of activity lies primarily in applied problems, it is both simpler and safer to consider the physical dimensions of a quantity as inherent in its nature (although perhaps unknown to us). With this assumption, the ratio of the magnitudes of the same quantity in two different systems of units can be only a numeric. It would be out of the question to state that the ratio of the units of electric charge in the two c.g.s. systems is equal to the velocity of light; this ratio is an abstract quantity and can only numerically equal the velocity of light. Otherwise it could be claimed that the ratio of a square meter to a square foot is equal to 10.76 gram-seconds!

The opposite point of view is ably defended by some leading physicists skilled in dimensional analysis, and the author can best indicate their general attitude by quoting an esteemed colleague who saw this paper in manuscript form: "Many people seem slow to realize that physical dimensions are purely a matter of convenience, as Bridgman has so ably shown in his little book on Dimensional Analysis. I think Professor Karapetoff becomes almost mystical in this last section of his paper. However, he has plenty of company in the attitude he takes. . . . In several places throughout the paper in speaking of dimensions the author speaks as if dimensions were inherent objective properties amenable to physical investigation. This attitude toward dimensions is similar to the attitude taken in a paper by Professor Kennelly on a related subject pub-

lished a year or two ago. It will appear to many readers, including the present one, that the assignment of dimensions is altogether arbitrary and subject to no physical investigation; it is suggested, therefore, that a discussion should be included of this matter, proving either that dimensions are inherent and amenable to physical investigation or else that they are purely arbitrary. . . . As an example, the dimensions of volume may be considered. It is customary to assign the dimension L^3 to volume for the reason that the volume of a cube, for instance, is obtained by raising its side to the third power. It would be equally correct to say that the volume of the cube equals a constant of numerical value unity but having some arbitrary dimension assigned to it multiplied into the cube of the side. In this case we are impelled to make the constant dimensionless by the feeling that the result will be greater simplicity rather than by the knowledge that volume inherently possesses the dimension L^3."

The author grants some strength to this argument and is the first one to welcome a generalization of our customary doctrines in the hands of scientists of the first rank who work upon an extension of the fundamental physical concepts and the underlying philosophy of ideas in physics. He only feels that for the rank and file of the profession it is safer and wiser to adhere to a more limited attitude of inherent physical dimensions. The remarks which follow represent distinctly this point of view, and the author hopes that more advanced and flexible views upon the subject may be contributed to the discussion of this paper.

In our present state of knowledge, it is impossible to express the physical dimensions of electric and magnetic quantities in terms of mechanical units alone, for example, of mass, length, and time. The dimensions of at least one electric or magnetic quantity have to be assumed as fundamental, in addition to the mechanical units. For example, in the electrostatic system of units the numerical value of κ_v is put equal to unity, but its physical dimensions should be taken as something fundamental and not reducible to the units of length, mass, and time. This is indicated in the last horizontal row of Table I by stating whether κ or μ is used as the fourth fundamental unit. In the systems 4, 5, and 6, the dimensions of some units are expressed through κ, of others through μ. In the ampere-ohm system the physical dimensions as well as the magnitudes of the ohm and the ampere are considered as fundamental. For this reason, only two mechanical fundamental units are necessary, those of length and time. Energy and force are expressed in terms of electric power, I^2r.

It is also possible to take the physical dimensions of voltage, inductance, electrostatic flux, or any desired electric or magnetic quantity as fundamental, and to express the physical dimensions of the rest of the quantities in its terms. For an engineer and applied physicist it is safer to adhere to the view that in nature the physical dimensions of an electric or magnetic quantity are intrinsic in that quantity and do not depend upon a human system of units or upon a choice of the fundamental units. While outwardly these dimensions may look different in different systems of units, this may be simply due to our ignorance of the intrinsic nature of the phenomena involved. It is therefore safer not to maintain (as is sometimes done) that in the electrostatic system of units absolute permittivity is a numeric, whereas in the electromagnetic system its physical dimensions are those of a velocity to the power of minus two.

From a practical point of view, a more conservative statement is as follows: In the electrostatic system of units the physical dimension of permittivity is a fundamental dimension. Therefore, calling the numerical value of an absolute permittivity (κ) and its physical dimension $[\kappa]$, we have

$$\kappa = (\kappa)\,[\kappa] \qquad (44)$$

In the electromagnetic system of units μ is a fundamental unit. From equation (36), the dimension of permittivity (assuming for the present k to be a numeric) is

$$[\kappa] = [\mu]^{-1}\,[v]^{-2} \qquad (45)$$

Therefore, in this system

$$\kappa = (\kappa)\,[\mu]^{-1}\,[v]^{-2} \qquad (46)$$

Intrinsically, however, the physical dimensions indicated in equations (44) and (46) are identical; their product has the dimensional formula (time/length)^2, to agree with equation (37).

In the ampere-ohm system, the dimensions of κ may be deduced from equation (5). The flux density has the dimensions of a quantity of electricity divided by an area, or $[D] = [i][t][A]^{-1}$. The dimensions of the voltage gradient are those of a voltage divided by a length, or $[G] = [I]\,[r]\,[a]^{-1}$. Consequently,

$$[\kappa] = [D][G]^{-1} = [t][r]^{-1}[a]^{-1} \qquad (47)$$

The physical dimensions of various quantities in the principal systems of units in use will be found in refs. 4, 11, and 17. For the corresponding tabulation in the ampere-ohm system see refs. 7 and 8. It may also be added that from the point of view maintained by the author the ratio of unit charges in the two c.g.s. systems is only *numerically* equal to the velocity of light in vacuo; the ratio itself can only be a numeric.

The problem of physical dimensions is further complicated by the question of the physical dimensions of the parameter k. In equation (15) this parameter changes a current into a magnetomotive force, and in equation (17) it converts a rate of change of flux into a voltage. Since the purely physical side of these transformations is still enshrouded in mystery, it may be just as well for the present *provisionally* to consider k to be a numeric. From the physical point of view this may be equivalent to the assumption that there are no transformations in kind, so that a current *is* a magnetomotive force and a change in flux *is* an induced e.m.f.

Nevertheless, it is well to keep in mind the possibility of a later discovery of the intrinsic nature of these circuital relations. For this reason, it seems both wise and considerate towards future generations not only to preserve this parameter, but to grant beforehand the possibility of its having definite physical dimensions. This may forestall for our posterity a difficulty similar to the one we are having with 4π.

The author therefore heartily seconds Dr. W. Jaeger's proposal, (ref. 3, table on page 35) of including the dimensions of k with those of the magnetic quantities. When studying his table the reader should keep in mind that Jaeger's k is different from the one used in this article, being defined by his equations (28) and (29), even though the general purpose of both parameters is the same.

Notation

- A cross-section of an electric or magnetic path
- a length, usually of a line of force
- B magnetic flux density
- c velocity of light in vacuum
- D dielectric flux density, or density of displacement
- e as a subscript, means "electrostatic"
- f mechanical force
- G potential gradient, or electric field intensity
- H magnetizing force, or magnetic field intensity
- I, i electric current
- k a parameter which characterizes a system of units with respect to the two laws of circuitation, equations (15) and (17)
- K relative permittivity, or dielectric constant
- L inductance of a circuit
- M, m strength of a magnetic pole
- m as a subscript, means "magnetic"
- N number of turns in a winding
- n a parameter which characterizes a system of units and gives the ratio of the dielectric flux density used in that system to the so-called theoretical flux density; equation (3)
- p a parameter which characterizes a system of units and gives the ratio of the magnetic flux density used in that system to the so-called theoretical flux density; equation (11)
- Q, q magnitude of an electrostatic charge
- R distance measured from the center of a charge or conductor
- r electrical resistance of a circuit
- r as a subscript, means "relative"
- s variable length along a closed curve
- t time
- t as a subscript means "theoretical"
- v velocity of electromagnetic waves in a perfect dielectric
- v as a subscript, means "vacuum"
- W stored electrostatic or magnetic energy
- W' same, per unit volume
- x direction of propagation of a plane electromagnetic wave
- κ absolute permittivity of a dielectric
- μ absolute permeability of a magnetic medium
- Φ electrostatic or magnetic flux
- ζ radius of a small sphere

Appendix

Tabulation of the Principal Formulas in the General System of Units

The equation numbers are the same as in the text.

$$f_e = Gq \quad (1)$$
$$D_t = Q/(4\pi R^2) \quad (2)$$
$$D = nD_t \quad (3)$$
$$D = \kappa G \quad (5)$$
$$\kappa = K\kappa_v \quad (6)$$
$$G = nQ/(4\pi\kappa R^2) \quad (7)$$
$$f_e = nQq/(4\pi\kappa R^2) \quad (8)$$
$$\Phi_e = nQ \quad (8b)$$
$$f_m = Hm \quad (9)$$
$$B_t = M/(4\pi R^2) \quad (10)$$
$$B = pB_t = \mu H \quad (11)$$
$$\mu = \mu_r\mu_v \quad (12)$$
$$H = pM/(4\pi\mu R^2) \quad (13)$$
$$f_m = pMm/(4\pi\mu R^2) \quad (14)$$
$$\Phi_m = pM \quad (14b)$$
$$k[(1/n)(\partial\Phi_e/\partial t)+IN] = \oint Hds \cos (H, ds) \quad (15)$$
$$kI = 2\pi RH \quad (16)$$
$$-(k/p)(\partial\Phi_m/\partial t) = \oint Gds \cos (G, ds) \quad (17)$$
$$(k/p)N\Phi_m = rQ \quad (19)$$
$$W_e' = nD_t^2/(2\kappa) = D^2/(2n\kappa) = GD_t/2 = GD/(2n) = (\kappa G^2/(2n) \quad (25)$$
$$W_m' = pB_t^2/(2\mu) = B^2/(2p\mu) = HB_t/2 = HB/(2p) = \mu H^2/(2p) \quad (26)$$
$$(k/n)\, \partial D/\partial t = \partial H/\partial x \quad (33)$$
$$(k/p)\, \partial B/\partial t = \partial G/\partial x \quad (34)$$
$$v^2\partial^2 G/\partial x^2 = \partial^2 G/\partial t^2 \quad (35)$$
$$v^2\partial^2 H/\partial x^2 = \partial^2 H/\partial t^2 \quad (35a)$$
$$vk = \sqrt{np/(\kappa\mu)} \quad (36)$$
$$ck = \sqrt{np/(\kappa_v\mu_v)} \quad (37)$$

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Discussion

Lloyd Champlin Eddy: Such statements on the general relations of units as were presented by Mr. Karapetoff may prove to be of ultimate value for more thoroughly understanding universal relations, even of certain forms of life also. Who knows what manifestations of life are not fundamentally electric in nature? Perhaps two physical phenomena most resembling life are radio-activity and magnetism, both electrical phenomena. Both of these phenomena apparently depend on certain relations or numbers of electrons or atoms much as animal life itself appears mainly where carbon is present in more or less definite relations with other elements. The extension of our knowledge of the general relations of units in electrical engineering as well as other fields, therefore, may be of considerable interest also to those interested not only in the fundamental units of science and electrical engineering but in universal relations of physical life.

A Proposal to Abolish the Absolute Electrical Unit Systems

BY ERNST WEBER*

Associate, A.I.E.E.

Synopsis.—*The specification of a physical quantity must include numerical value and unit. The expression "dimension" is defined as the general unit of a quantity and it is shown that applied mathematics has to deal with physical quantities rather than pure numbers, so that each equation yields a relation between numerics and dimensions. It is possible to reduce all dimensions to a set of a few fundamental dimensions, called a "dimension system." The application of this concept to the various fields of physics is shown, and it is proved that the so-called "absolute" dimension systems in electromagnetism are incorrect from this point of view and lead to inconsistent results. The proposal is made, therefore, to abolish the "absolute" dimension systems in electromagnetism and to introduce two new dimension systems as extensions of the two well-known mechanical systems by adding just one electric fundamental dimension. The new proposal retains all the familiar units but puts them on the sound basis of correct dimension systems, which, it is hoped, eliminates the unfortunate discrepancy in electromagnetism which has caused so much trouble ever since the "absolute" systems were introduced. The adoption of the new proposal, however, requires the cooperation of physical as well as electrical engineering societies, especially with regard to possible international action.*

* * * * *

INTRODUCTION

THE fundamental discovery that the ratio of the "absolute" electrostatic unit to the "absolute" electromagnetic unit of electric charge, predicted as a velocity, was apparently identical with the velocity of light,[1] brought about an unforseen confusion in the field of electrical units. The use of various "absolute" dimension systems, expressing the electric and magnetic quantities by means of the three mechanical fundamental dimensions, influenced the writing style of formulas and caused the introduction of factors in various places where they seemed quite arbitrary. At the same time that A. E. Kennelly delivered a paper on "magnetic reluctance"[2] and caused a long discussion as to the position of the factor 4π, Heaviside in London prepared a paper on "the eruption of the 4π's."[3] Heaviside was apparently the first to notice that the dielectric constant and the permeability of free space ought to be considered as physical quantities, but he made no proposals as to their definite dimensions.

Since then a veritable flood of articles and papers has been published in the important periodicals of the world, concerned either with a new system of electric and magnetic units or dimensions, or with a comprehensive basic system of mechanical units. None of them has succeeded as yet, because of the *attitude of respect toward the classical absolute systems,* which have exerted an unfortunate power of tradition, especially in the field of physics. This paper aims to prove conclusively the incorrectness of these classical dimension systems and to propose *natural systems* which, it is hoped, may finally settle the differences and put the electrical unit systems on a sound basis.

In order to provide the means for a logical proof and to furnish the necessary elements for building up the new systems, the paper deals first with the fundamental conception of the dimension of a physical quantity, in so far as it is necessary for the scope of this paper.

I. Physical Quantities, Their Units and Dimensions

The aim of all physical science is to obtain a description of the inorganic world in terms of mathematics. The scientific type of human mind tends to bring all relations into forms which are more or less mathematical and in doing this is concerned fundamentally with the process of measurement.

A. The Dimension of a Physical Quantity

The following statements are a brief summary of the fundamental conception of physical quantities in so far as they differ from pure numbers and must form the true basis of all considerations concerning the problem of units and unit systems. In order to emphasize the importance of each statement, the special arrangement in form of letter paragraphs has been chosen.

α. To determine definitely the value of a physical quantity, it is necessary to make *two statements*; one indicating the *unit* which is used in measuring the quantity, and the other giving the *number of units* involved in the result of that measurement. For example, to state "the length of the path is 4" is an incomplete statement because the unit may be *m*, cm., inch, foot, mile, or so on, each of the units being widely different. The theoretical number of possible units, obviously, is infinity.

β. However, all the various units of length form an infinite group of similar quantities, each one being a definite length itself, so that $[l]$ may stand as a symbol for any one unit of length. The statement, "the length of the path is $4 \cdot [l]$ "is still incomplete, but in this case there is at least the indication that a physical quantity is involved and it is therefore necessary to specify a length unit.

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*Graduate Research Prof. of Elec. Engg., Brooklyn Polytechnic Institute, N. Y.

1. For references see Bibliography.

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γ. At this point the *dimension* of a quantity is defined by the infinite reservoir of possible units for that quantity. A physical quantity is then generally designated by

$$N \cdot [D], \tag{1}$$

where N is the numeric and $[D]$ is the symbol for the dimension. A special value of that quantity is obtained by specifying the values of N and $[D]$. The conception of a physical quantity as the product of numeric times dimension corresponds to the process of measuring, which essentially consists in *first* fixing $[D]$ as a definite unit (usually with a definite name), and *then* comparing the given quantity with this unit.[4,5]

From this point of view, dimension is as necessary as the numerical value and can not be abstracted from a physical quantity without reducing it to a pure number.* It should be noted that the word dimension is often used with other meanings, as for example: three-dimensional space, four-dimensional world, and so on. This should not be confused with the definition given above.

δ. Comparison of the same quantity, measured with two different units leads to the mathematical relation

$$N_1 \cdot [D]_1 = N_2 \cdot [D]_2,$$

where $[D]_1$ and $[D]_2$ designate definite units. If the following unit relations exist

$$[D]_1 = n_{12} \cdot [D]_2, \; [D]_2 = n_{21} \cdot [D]_1 \tag{2}$$

then

$$N_1 \cdot [D]_1 = N_1 \cdot n_{12} \cdot [D]_2 = N_2 \cdot [D]_2 = N_2 \cdot n_{21} \cdot [D]_1 \tag{3}$$

illustrating the process of changing units. It must be emphasized, however, that the conversion factors n_{12}, n_{21} *must be pure numbers*, in order to make the units comparable at all, *i. e.*, all units of the same quantity must have the same dimension. This statement is identical with that in β or γ and is extremely important in passing on the correctness of unit systems.

B. Applied Mathematics and Physics

Pure mathematics operates with symbols designating numbers and numbers only. It is closely related to geometry in so far as all expressions may be interpreted in terms of coordinates, and their relations producing geometrical forms, such as surfaces, curves or points. The extension of these interpretations to hyper spaces is well known.

Applied mathematics, however, uses the laws and methods deduced by pure mathematics in order to solve the problems arising in physics, chemistry, and the related engineering sciences. All physical relations involve physical quantities, since they can be found or proved only experimentally. The only means for finding quantitative laws is by measuring the various quantities involved. This conclusion is thus identical with the statement at the beginning of the previous chapter and it is obvious now, that before experimental investigations can proceed, it is necessary that the means for measurement, *i. e.*, the units, must be available.

*However, the dimension has nothing to do with the "ultimate nature" of physical quantities. See also reference 37, chapter 2.

In this paper no attempt is made to give a historical review of the development of measuring quantities and determining units in the various fields of physics. However, a brief introductory review of the fields of physics may be given in order to prepare for the following presentations.

The dominating law of physics is undoubtedly the law of conservation of energy. It is known that energy appears in various forms, but that the total quantity of energy can not be changed. Energy is therefore the *only true fundamental quantity* which ties together all the various fields of physics.

A fundamental scheme for distinguishing between the various fields of physics would appear to be the subdivision according to the different forms of energy. This method would lead to the following classification:

Mechanics. The mechanical energy appears in the two forms—potential energy and kinetic energy.

Heat. In this case there is only the single form energy which may be called heat energy or molecular energy.

Electricity. Here the energy manifests itself as the electric field energy and magnetic field energy.

These three fields of physics are the only ones which have been discovered so far. It is unnecessary at this point to speculate as to the possibility of a common origin of these various forms of energy.

It is obvious from what has been said in (A) that, whatever unit we choose for energy, all such units in the various fields of physics must have the same dimension and can be converted into one another. Since the research work in the various fields was carried on independently it is but natural that different units of energy were chosen. Table I gives a short review of the units most frequently used and the relations existing between them.

TABLE I

Mechanical energy	1 kg force m	= 9.806 joules	= 9.806 × 10^7 ergs
Heat energy	1 cal.	= 10^3 cal	
Electromagnetic energy	1 wattsec		

Inter-relations

1 kg force m = 2.34 cal = 9.806 wattsec.

II. The Mathematical Forms Used in Physics

Theory in any respect is descriptive. The mathematical type of human mind searches for relations between quantities, progressing in each field toward the fundamental laws upon which the final theory may be built. The laws are expressed in mathematical forms and the whole chain of relations, *the theory*, is then the *mathematical picture* of the respective physical field.

Only when experiment and theory conform exactly can the theory be said to be satisfactory.

The fundamental mathematical form is the equation. There are but two different types of equations* which occur in theoretical work: the arbitrary definition and the statement of experimental proportionality. Because of the importance of these two kinds of equations they will be considered in some detail in this section.

A. The Arbitrary Definition

Many physical laws can be expressed in simpler form, if use is made of defined quantities rather than measured quantities. Thus, the quotient of two physical quantities, Q_1 and Q_2

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \cdot \left[\frac{D_1}{D_2} \right] = N \cdot [D],$$

results as the product of a numerical value N and a dimension $[D]$, *i. e.*, as a physical quantity itself which may or may not have a true physical meaning. At any rate it can be named and used in physical relations as an arbitrarily defined quantity.

The arbitrary definition can also take the form of differential and integral relations, as for example

$$v = \frac{ds}{dt},$$

the definition of velocity being the differential quotient of path and time. Even differentials are parts of the physical quantity, although infinitely small. In general, mathematical operations on physical quantities again give physical quantities.

It is this fact which justifies the comparison of the dimensions on the two sides of any physical relation, or more exactly, any relation in applied mathematics, and such a comparison is a direct consequence of dealing with physical quantities rather than with pure numbers. Thus, each equation can be split up into a relation of numerics, and a relation of dimensions or units depending on whether or not definite units are specified.

B. The Statement of Experimental Proportionality

Our knowledge of natural phenomena is due primarily to the research of experimental physics, and the mathematical expression of these experimental investigations is always the statement of a proportionality. For example, the relation

$$F = k \cdot \frac{Q_1 Q_2}{r^2}$$

represents the experimental relation describing the force existing between two masses, two electric charges, or two magnetic poles, if their centers are separated a distance $r \cdot [l]$. In all three cases the proportionality factor k serves to express the proper quantitative dependence. This means, however, that k has *two* functions, namely, to balance the numerical values as well as the units used for the known physical quantities on both sides of the equation. It is obvious, therefore, that generally k itself appears as a *physical quantity*.

In many cases this fact is recognized by giving a characteristic name to the proportionality constant. In some important cases attempts were made to specify quite arbitrarily the meaning of the proportionality factor, thereby emphasizing the least important quantity in the whole equation. It will be shown that this procedure leads to inconsistent as well as incorrect results.

*E. Bennett[6] distinguishes 4 types of relations; however, for the purpose of this paper it is but necessary to distinguish according to the mathematical types.

III. On Dimension Systems and Unit Systems

A. Definitions of Dimension Systems and Unit Systems

In order properly to express its value, a physical quantity must be definitely specified as to its numerical value and unit. However, it would be too confusing to use as many unit names as there are physical quantities. As pointed out above, there are many relations between the various quantities, either as definitions or as experimental proportionalities, which permit a reduction in the number of unit names.

Since each relation between quantities can be split up (see chapter II A) into one relation of numerics and another one of dimensions (as general units), it is possible to combine all known relations of dimensions. In setting up these relations, all proportionality factors must be taken as physical quantities. If there are m *independent* relations known, and $(m + p)$ dimensions involved, it indicates that m dimensions can be expressed in general by any p "fundamental" dimensions chosen arbitrarily.

This set of p "fundamental" dimensions is then called a *dimension system*. From the theory of numbers therefore, it is known that one generally has a choice of $\binom{m+p}{p}$ possible dimension systems. Thus, if $p = 3$, $m = 3$, then one has $\binom{6}{3} = 20$ different possibilities. A necessary condition, however, is that *each* independent relation involves at least $(p + 1)$ dimensions. If this is not the case, then the number of possible dimension systems is less, so that $\binom{m+p}{p}$ indicates the *upper* limit.

Furthermore, in order to arrive at some method of properly expressing physical quantities it is necessary to choose a unit for each of the p fundamental dimensions. Each set of p units so obtained is a "*fundamen-*

tal unit system." Any unit of the $(m + p)$ quantities can then be expressed in terms of the p fundamental units.

Since each fundamental dimension yields an infinity of possible units, there result "infinity to the p, raised to the $\binom{m+p}{p}$ power," possible fundamental unit systems. This vast array of possibilities must emphasize the desirability of international agreement before too much use is made thereof.

Any dimension system chosen in the described manner, is consistent, as well as correct and never leads to ambiguity with respect to the expression of physical quantities.

B. The Dimension Systems in Mechanics

No attempt is made to give a complete history of the mechanical units. The investigations of this chapter only serve to develop a thorough basis for the derivations of the following ones, and present selected points of interest.

It follows from Appendix A that there are at least $\binom{6}{3} = 20$ possible dimension systems. Since space and time are admittedly the fundamental perceptions for investigating physical phenomena, let $[l]$ and $[t]$ be two of the three possible dimensions. The third dimension could be chosen from $[k]$, $[m]$, $[f]$, $[W]$, thereby restricting the number of possibilities to four. There is no dimension system known, which uses the proportionality constant $[k]$ as a fundamental dimension, but the use of $[W]$ as the third fundamental dimension has been proposed by W. Ostwald[7] without much success. If, then, $[f]$ is taken as the third fundamental dimension, the system becomes the so-called gravitational system, in which the fundamental dimensions are $[l]$, $[t]$, $[f]$. Because of the fact that this system is used to a large extent in engineering, it will be referred to in this paper as the *technical dimension system.* On the other hand, if $[m]$ is chosen for the new dimension there results the so-called dynamical system whose dimensions are $[l]$, $[t]$, $[m]$. Because of the frequent use of this system in physics it will be called the *physical dimension system.* In the latter system the dimension of force becomes

$$[f] = [m] \cdot [l]\,[t]^{-2}. \tag{5}$$

These two dimension systems of mechanics are consistent because they are composed of the required three fundamental dimensions which can be used to specify all the other quantities.[37]

Table II shows, for convenience, the dimensions of various mechanical quantities in both dimension systems, if use is made of the relations given in Appendix A.

The conversion from one system into the other is easily made if the unit systems are specified within the dimension systems. Consider, for example, the same amount of work measured in both dimension systems. In the technical dimension system the equation could be written in the form

$$W = a \cdot ([f]\,[l])_u,$$

where a is the numerical value, and the symbol $(\ )_u$ designates the unit. In the physical dimension system the work could be given by

$$W = b \cdot ([m]\,[l]^2\,[t]^{-2})_u,$$

where b is now another numerical value and $(\ )_u$ the other unit. Since the same amount of work is involved, equating these two quantities gives the unit relation

$$1\,([f]\,[l])_u = \frac{b}{a}\,([m]\,[l]^2\,[t]^{-2})_u.$$

As emphasized above, units have the same dimensions as the quantities themselves. Equating the dimensions on both sides there results

$$[f] = [m]\,[l]\,[t]^{-2},$$

a dimension relation which is also given in Table II. *No new dimensional relations are ever obtained by measuring quantities in various dimension-systems as long as such systems are consistent.*

It follows from chapter III A that there are "infinity cubed" unit systems possible in each dimension system. In spite of this fact, however, when the two systems were introduced, exactly the same unit names were chosen in both systems. This startling coincidence is perhaps comprehensible, but it is the cause of much confusion in attempting to distinguish the *kg*, *g*, etc., as a unit of force in the one system and as a unit of mass in the other. It has been proposed to call the technical system unit, *kg*-force, and for the sake of clarity this designation will be used here. (See Table I.)

C. The Dimension Systems in Heat

It is surprising to note that in the field of heat a single dimension system has been developed and no confusion whatsoever exists as to its fundamental dimensions. There are, however, many inconveniences as to the units; these difficulties may be settled internationally when scientific minds grow international.

TABLE II

Symbol	Quantity	Technical system	Physical system
	Fundamental dimensions	$[f],[l],[t]$	$[m],[l],[t]$
m	Mass	$[f][l]^{-1}[t]^2$	$[m]$
f	Force	$[f]$	$[m][l][t]^{-2}$
k	Gravitation constant	$[f]^{-1}[l]^4[t]^{-4}$	$[m]^{-1}[l]^3[t]^{-2}$
W	Work, energy	$[f][l]$	$[m][l]^2[t]^{-2}$
P	Power	$[f][l][t]^{-1}$	$[m][l]^2[t]^{-3}$

As may be seen from Appendix B the two fundamental laws in heat involve 6 dimensions, thus requiring 4 fundamental dimensions for a dimension system.[8] Again we assume the two basic perceptions of space and time to be natural fundamental dimensions, and since temperature had been measured long before a theory of heat existed, it is justifiable to take the

temperature as a third fundamental dimension $[\theta]$. The fourth dimension can be chosen from

$$[k], [R], [m], [f], [W],$$

if use is made of the definitions in Appendix B whereby $[W]$ can be replaced by either $[m]$ or $[f]$.

Of all these possibilities, the only important system is the one involving $[m]$ and it may be referred to as the "thermophysical" dimension system, since it is based upon the mechanical-physical system. No use has been made of the system involving $[f]$ which might similarly be called the "thermotechnical" dimension system. As mentioned above, the system using $[W]$ had been proposed by Ostwald[7] without success.

The proportionately constant R of Boyle-Charles's law is known as the universal gas constant and its dimension can be read from the law in Appendix B as[9]

$$[R] = [l]^2 \cdot [t]^{-2} \cdot [\theta]. \quad (6)$$

In concluding this chapter it may be emphasized that, thus far, none of the proportionality factors has been mistreated or taken as fundamental dimensions.

possibilities are the only ones of practical importance as seen from chapter III B.

The fourth fundamental dimension in electromagnetics can then be chosen from

$$[Q], [I], [k_e], [k_m].$$

The last two quantities are the proportionality constants of Coulomb's and Ampere's laws and as such, have no special physical significance. However, even though such a specification of this dimension system is entirely natural and quite convincing, it has been customary to specify as fundamental dimensions[12] those of the proportionality factors k_e and k_m, so that the electric charge and current appear as derived dimensions. This entirely arbitrary procedure is hardly justifiable on the grounds of a logical development of dimensions.

As previously mentioned, each of the four quantities, Q, I, k_e, k_m, can be taken as the new fundamental dimension. The four dimension systems thus obtained are shown in Table III, where the first two columns give

TABLE III—COMPLETE ELECTRICAL DIMENSION SYSTEMS

		Additional fundamental dimension			
		$[Q]$	$[I]$	$[k_e]$	$[k_m]$
	Dimension systems	$[m],[l],[t],[Q]$	$[m],[l],[t],[I]$	$[m],[l],[t],[k_e]$	$[m],[l],[t],[k_m]$
Quantity	Symbol				
Electric charge	Q	$[Q]$	$[I][t]$	$[m]^{1/2}[l]^{3/2}[t]^{-1}[k_e]^{-1/2}$	$[m]^{1/2}[l]^{1/2}[k_m]^{-1/2}$
Current	I	$[Q][t]^{-1}$	$[I]$	$[m]^{1/2}[l]^{3/2}[t]^{-2}[k_e]^{-1/2}$	$[m]^{1/2}[l]^{1/2}[t]^{-1}[k_m]^{-1/2}$
	k_e	$[m][l]^3[t]^{-2}[Q]^{-2}$	$[m][l]^3[t]^{-4}[I]^{-2}$	$[k_e]$	$[l]^2[t]^{-2}[k_m]$
	k_m	$[m][l][Q]^{-2}$	$[m][l]\ [t]^{-2}[I]^{-2}$	$[l]^{-2}[t]^2[k_e]$	$[k_m]$
Electric field strength	E	$[m][l]\ [t]^{-2}[Q]^{-1}$	$[m][l]\ [t]^{-3}[I]^{-1}$	$[m]^{1/2}[l]^{-1/2}[t]^{-1}[k_e]^{-1/2}$	$[m]^{1/2}[l]^{1/2}\ [t]^{-2}[k_m]^{1/2}$
Magnetic field strength	B	$[m][t]^{-1}[Q]^{-1}$	$[m][t]^{-2}[I]^{-1}$	$[m]^{1/2}[l]^{-3/2}[k_e]^{1/2}$	$[m]^{1/2}[l]^{-1/2}[t]^{-1}[k_m]^{1/2}$

IV. The So-Called Absolute Electric and Magnetic Dimension Systems

Confusion seems to have reigned unconfined in the dimension systems of electricity and magnetism, even to the present day. It is the purpose of this section to apply the fundamental principles deduced above to an analysis of these absolute dimension systems.

A. The Electromagnetic Dimension Systems

As indicated in Appendix C, the three independent laws of electromagnetics give relations between seven dimensions, so that four, and not less than four, fundamental dimensions compose an electromagnetic dimension system. Again, the basic perceptions of length and time would seem to justify the choice of $[l]$ and $[t]$ as fundamental dimensions. Since Coulomb's and Ampere's laws are closely related to mechanics, the choice of a mechanical dimension as a third one suggests itself. Inasmuch as both of these laws involve only the force $[f]$, the two definitions given in Appendix C can be used to express $[f]$ in terms of either $[m]$ or $[W]$, so that the third fundamental dimension can be chosen from $[m]$, $[f]$, $[W]$, as in the mechanics. The first two

the dimensions with integer exponents, and the last two columns have exponents of fractional orders. This again makes it preferable to choose either $[Q]$ or $[I]$ as fundamental dimensions instead of $[k_e]$ or $[k_m]$.*

It is possible to convert from the one electrical system into the other if the units are fixed. Since there are four different systems, four relations for the dimensions of the units are obtainable, as for example, in the case of current:

$$[Q][t]^{-1} = [I] = [m]^{1/2}[l]^{3/2}[t]^{-2}[k_e]^{-1/2} = [m]^{1/2}[l]^{1/2}[t]^{-1}[k_m]^{-1/2}$$

which fits perfectly into the respective columns of Table III. There is not the least ambiguity as to the fundamental dimensions; *i. e.*, the *unit relations are unique*. This is also the reason for calling these systems complete dimension systems, and as such, they cannot have less than four fundamental dimensions.

It may appear possible to obtain a complete set of four fundamental dimensions by choosing *two electrical fundamental dimensions* in addition to length and time. Such an "electrical" set of dimensions has been used by several authors in which current, resistance, length

*In the field of electrical engineering, E. Bennett[6] used a complete practical system, as given in the third column.

and time are the four fundamental dimensions.[13,14] In this case it becomes necessary (see Appendix C) to introduce the mass as a derived electrical dimension, and so one would "electrify" the mechanical system. From the point of view of international agreement between physicists and engineers, however, such an arrangement would appear to be quite unsatisfactory and undesirable.

B. The "Absolute" Systems and the System of Gauss

The mechanical system of dimensions composed of the three fundamental dimensions, $[m]$, $[l]$, $[t]$, was often called the "absolute system" and it was thought possible to reduce all the dimension systems in the various fields of physics to the same absolute system. This was shown to be impossible in the case of heat, and from the discussion above (see Table III) it ought to be impossible in the case of electromagnetism. However, the first experimental law (Appendix C) presents an almost irresistible temptation to be made the basis of an absolute system. It was thought possible to assign any dimension whatsoever to the proportionality factor and consequently the choice was so made that $[k_e] = 1$; *i. e.*, k_e *degenerated* into a pure number. The third column in Table III gives then the absolute dimensions of the electromagnetic quantities if $[k_e] = 1$. For convenience the result is shown again in the first column of Table IV. Since Coulomb's law occurs in electrostatics, the dimension system has been called the *absolute electrostatic dimension system.*

Another absolute dimension system was specified on the basis of the fundamental law of magnetism, or steady currents. By choosing $[k_m] = 1$ the proportionality factor became a pure number (similar to k_e in the electrostatic system) and Ampere's law (Appendix C) could then be used for determining the absolute dimension of the current. The resulting dimensions may be found in the last column of Table III, introducing $[k_m] = 1$ and is given for convenience in the second column of Table IV. The dimension system based upon this arbitrary choice of k_m is called *absolute electromagnetic dimension system.*

Although each of the above dimension systems is quite arbitrary, it is apparent that they are unsymmetrical, in spite of the fact that the two proportionality factors have the same respective position and are supposed to play the same role. A comparison of the last two columns in Table III will show that the ratio of the dimensions for any quantity in both systems is given by

$$\frac{[k_e]}{[k_m]} = [l]^2 [t]^{-2}, \tag{7}$$

which is evidently the dimension of the square of velocity.

In order to obtain a symmetrical system, Gauss introduced a new proportionality factor defined by the equation

$$k_m' = k_m \cdot v^2, \tag{8}$$

where v denotes a quantity having the dimension $[l]/[t]$. Because of this substitution the factors k_e and k_m' have then the same dimensions. By specifying $[k_e] = [k_m]' = 1$, *i. e.*, defining both constants as pure numbers, the remarkable result is obtained that all the electric quantities, including the current, are in the electrostatic dimension system, and the magnetic quantities are in the electromagnetic system, as shown in the last column of Table IV.

C. Objections to the Absolute Electrical Dimension Systems

In introducing *arbitrary statements about the proportionality factors* k_e *and* k_m, *absolute systems have been obtained but the dimensional relations are incorrect.* The proof of this statement can be presented by summing up the more important objections to the various attempts at reducing the number of required fundamental dimensions to less than four.

α. *The dimension of the proportionality factors* k_e *or* k_m *can not be chosen arbitrarily.* It has been shown in Appendix C that there are three independent relations between mechanical and electrical quantities, involving, however, four new electrical dimensions which must be determined. This means, as explained above, that it is necessary to introduce a fundamental electrical dimension. Defining k_e or k_m as pure numbers, however, means exactly the same as defining the gravitation constant k, in law 2 of Appendix A, as a pure number. On the basis of this supposition, then, the three laws in mechanics would involve only 5 dimensions and there would be only two fundamental dimensions. This means, in other words, that mass, force and energy could be expressed in powers of $[l]$ and $[t]$ alone.

TABLE IV

		Absolute dimension systems		
Quantity	Symbol	Electrostatic $[k_e]=1$	Electromagnetic $[k_m]=1$	Gauss $[k_e]=[k_m']=1$
Electric charge	Q	$[m]^{1/2}[l]^{3/2}[t]^{-1}$	$[m]^{1/2}[l]^{1/2}$	$[Q]_e$
Current	I	$[m]^{1/2}[l]^{3/2}[t]^{-2}$	$[m]^{1/2}[l]^{1/2}[t]^{-1}$	$[I]_e$
	k_e	1	$[l]^2[t]^{-2}$	1
	k_m	$[l]^{-2}[t]^2$	1	$[k_m]_e$
Electric field strength	E	$[m]^{1/2}[l]^{-1/2}[t]^{-1}$	$[m]^{1/2}[l]^{1/2}[t]^{-2}$	$[E]_e$
Magnetic field strength	B	$[m]^{1/2}[l]^{-3/2}$	$[m]^{1/2}[l]^{-1/2}[t]^{-1}$	$[B]_m$

However, it would not be necessary to stop there, since other arbitrary statements could be made so that the number of dimensions could be further decreased. There is no logical reason for such a procedure and it never can be justified. The theory of dimensions and dimension systems as developed in these previous chapters give, so it is hoped, a definite guide for obtaining proper dimension-relations.

β. *The arbitrary designation of k_e and k_m as pure numbers actually converts the absolute electrical systems into the mechanical dimension system.* Hence, absolute electric, as well as absolute magnetic dimension systems, are in fact parts of the mechanical system. Consequently, since all quantities are measured in the same system, unit relations of the type $1m = 100cm$ ought to hold. However, in both systems *the dimensions are different.** There is thus an *obvious contradiction* which can not be reconciled. It would be similar to a situation, for example, in which velocity had different dimensions if measured in the physical or in the technical mechanical dimension systems. This would be immediately recognized as being incorrect, so that the same conclusion should be drawn in the case of the absolute electrical systems.

γ. *The absolute electrical dimension systems as parts of the mechanical dimension system lead to strange relations.* Since all the absolute electromagnetic systems are based upon the same three fundamental dimensions of mechanics, all electric and magnetic quantities appear as mechanical quantities of the same type. As in the case of the two mechanical dimension systems (chapter IIIB), and as in the case of the complete electrical systems (chapter IVA), the dimensions in the absolute systems may be compared. If these dimension systems are perfect, no new dimensional relations should be obtained.

However, equating the dimensions of Q, I, E, and B, as given in Table IV, gives the strange result

$$[l] \cdot [t]^{-1} = 1,$$

and equating the dimensions of k_e and k_m there results $[l]^2\,[t]^{-2} = 1$. Or, in other words, it ought to be possible to express length by time or vice versa. Needless to say again, the arbitrary statements of k_e and k_m as pure numbers are quite incorrect.

As may be seen from Table IV, even Gauss' System does not succeed in obtaining a mechanical system in electromagnetics. Since the electric quantities are measured in the electrostatic system and the magnetic quantities in the electromagnetic system, exactly the same objections can be made.

δ. *The ratio of the electrostatic to the electromagnetic unit of charge is not equal to the velocity of light.* It has been explained in almost every textbook† that the ratios of electrostatic to electromagnetic units involve a fundamental quantity—the velocity of light, approximately equal to $c = 3.10^{10}\,\dfrac{\text{cm}}{\text{sec}}\cdot$ It is known that within the mechanical system, the multiplication of two physical quantities changes the dimension and in general the physical meaning. It is, therefore, a surprising statement that in the absolute electric systems the same electric quantities have units which differ in their *dimensions by a velocity* or some power thereof.

However, introducing the absolute electrostatic unit of charge as defined by Coulomb's law with $k_e = 1$ and $f_e = 1$ dyne, $r = 1$ cm, it follows that

$$1\,([m]^{1/2}\,[l]^{3/2}\,[t]^{-1})_u = 1g^{1/2}\,\text{cm}^{3/2}\,\text{sec}^{-1},$$

which is a very small unit. For practical purposes, the *coulomb* has been introduced so as to give

$$1 \text{ electrostatic unit of charge} = 1g^{1/2}\,\text{cm}^{3/2}\,\text{sec}^{-1} = \frac{1}{3.10^9}\text{ coulomb.} \qquad (9)$$

On the other hand, the absolute electromagnetic unit of current follows from Ampere's law with $k_m = 1$ and $f_m = 1$ dyne, $r = 1$ cm, $l = 2$ cm, as

$$1\,([m]^{1/2}\,[l]^{1/2}\,[t]^{-1})_u = 1g^{1/2}\,\text{cm}^{1/2}\,\text{sec}^{-1}.$$

The practical unit is defined as the *ampere*, so that

$$1g^{1/2}\,\text{cm}^{1/2}\,\text{sec}^{-1} = 10 \text{ ampere,} \qquad (10)$$

and converting to the electromagnetic unit of charge by means of the relation $1cb = 1a \cdot$ sec, there results

$$1 \text{ electromagnetic unit of charge} = 1g^{1/2}\,\text{cm}^{1/2} = 10 \text{ coulomb.} \qquad (11)$$

Hence, the ratio of (21) to (23) becomes

$$\frac{1 \text{ electrostatic unit of charge}}{1 \text{ electromagnetic unit of charge}} = \frac{\text{cm}}{\text{sec}} = \frac{1}{3.10^{10}}\cdot \qquad (12)$$

and this surprising result shows that the ratio of the units of electric charge gives *the unit of velocity;* but *the ratio of the numerical values of electric charge* gives the *reciprocal of the numerical value of this velocity.*

The last two terms of equation (12) may be written as[18]

$$3.10^{10}\,\frac{\text{cm}}{\text{sec}} = 1. \qquad (13)$$

This rather unusual result is a direct consequence of using absolute unit systems. These contradictions have been sanctioned by physics since their introduction and almost no protest has been raised against such relations.

(ϵ) *The form of Faraday's induction law.* In Appendix C, Faraday's induction law is given as a dependent relation and the proportionality factor β which was arbitrarily introduced, turns out to be a pure numeric, if use is made of the general dimension relations given by the 3 independent laws. This indicates that the

*The different dimensions always caused difficulties. See references 17, 18, 37.

†As illustrations, some of the most recent valuable textbooks on physics and electrical engineering may be named. See bibliography references 19, 30, 21, 22, 23, 24.

factor $1/c$ which is necessary in Gauss' dimension system in order to balance both sides of the induction law, has that position only because of the incorrect dimension of either B or E. Again, the inconsistency shows up quite clearly and brings to light the various ingenious ways of correcting equations in order to maintain artificial dimension systems.

D. Conclusions as to the Absolute Electrical Dimension Systems

All attempts to reduce the required four fundamental dimensions of electromagnetism to the three fundamental dimensions of mechanics by arbitrary statements as to the proportionality factors have failed. Moreover, the consequences of such statements are not in agreement with the normal laws of mathematics as shown in chapter IIIA. The results, therefore, lead to the following conclusion:

The absolute electrostatic, electromagnetic and Gauss' dimension systems in the field of electromagnetics are incorrect and have to be abandoned finally and completely. Reference can *no longer* be made to *c.g.s. systems or to "absolute units" of any quantity in electromagnetics,* since the proper electrical dimension system must have four fundamental dimensions. The notation, *c.g.s. system, occurs only in mechanics* and *has to be restricted to this field of physics,* where it represents just one possible system of units within the "physical" dimension system.

If this denial of the possibility of c.g.s. systems in electromagnetics is granted—and the above proof justifies this conclusion—it must follow at once that the proportionality factors k_e and k_m can not be considered as pure numbers. Their dimensions and numerical values follow from the special type of the dimension system which shall be adopted, if possible, internationally. If k_e and k_m are physical quantities then the *dielectric constant* (permittivity) and the *magnetic constant* (permeability) of free space (being linearly depending on k_e and k_m) are physical quantities. As such they must have numerical value and dimension, similar to the *gravitation constant k,* and the gas constant R, and must be considered to be *of the same universal importance.*

Due to the fundamental importance of the subject and because of the fact that even the most recent textbooks of physics and engineering are using these absolute electrical systems,* international authority in physics and engineering will be required to banish the absolute systems and to introduce a dimension system appropriate to the field of electromagnetism. The various committees could consider the specification of units and unit names only after the absolute systems have been abandoned and a proper dimension system introduced.

There has been much discussion as to the "rationalization" of units; *i. e.,* shifting the factor 4π, involved in Maxwell's field equations, into more natural positions. However, it is meaningless to talk of "rationalized absolute" or "unrationalized absolute" units when the very existence of such absolute units is unjustifiable.

*See as examples, references 19, 20, 22, 23, 24.

V. On Proper Dimension Systems in Electromagnetism

The fact that the absolute system has been proven to be incorrect makes it necessary to propose a proper dimension system in electromagnetics—one which presents at least some promise of being considered internationally. A selection has to be made between the four complete dimension systems given in chapter IVA, since these systems are directly deduced from the fundamental laws of electromagnetics. However, before making a definite proposal, it will be feasible to consider the so-called practical unit system.

A. The So-Called Practical Unit System

The "absolute electrostatic" unit of charge and the "absolute electromagnetic" unit of current (see chapter IVB) were the basis of two unit systems, the c.g.s. electrostatic and the c.g.s. electromagnetic unit systems. Later, both unit systems were found to be quite inconvenient and for practical purposes other units were introduced and defined quite arbitrarily, as shown by the following examples.

1 coulomb	$= 3.10^9$	c.g.s. electrostatic units of charge
	$= 10^{-1}$	c.g.s. electromagnetic units of charge
1 farad	$= 9.10^{11}$	c.g.s. electrostatic units of capacitance
	$= 10^{-9}$	c.g.s. electromagnetic units of capacitance
1 henry	$= \frac{1}{9} \cdot 10^{-11}$	c.g.s. electrostatic units of inductance
	$= 10^9$	c.g.s. electromagnetic units of inductance

Only a few relations are given here for a special purpose, and even these few show a surprising variety of numerical factors.

The practical units are but definitions of units without regard to their dimensions. And, indeed, it would be difficult to state dimensions for these practical units. Take for example, the charge. As shown in equations (**9**) and (**11**) the *coulomb* is related to both dimension systems with the numerical factors given above. Equating (**9**) and (**11**), since the *coulomb* has to be the same unit in both cases, gives

$$3.10^9\, g^{1/2}\, cm^{3/2}\, sec^{-1} = 1 \text{ coulomb} = \frac{1}{10}\, g^{1/2}\, cm^{1/2}. \quad (14)$$

The question naturally arises as to which dimension may now be chosen for *coulomb*. Furthermore, equation (**13**) may be read again if the extreme left-hand and the

extreme right-hand terms are considered. Similar contradictions can be obtained for the dimensions of each practical unit.*

Another strange relation can be obtained for the dimensions of *farad* and *henry*. The following dimension relations can be taken from any conversion table of the absolute electric and absolute magnetic dimension systems:

[c.g.s. electrostatic unit of capacitance] = cm,

[c.g.s. electromagnetic unit of inductance] = cm.

Using the above numerical factors it is permissible to write

$$1 \text{ cm} = \frac{1}{9} \cdot 10^{-11} \text{ farads} = 10^{-9} \text{ henrys} \qquad \textbf{(15)}$$

so that it should be possible to express *farads* in terms of *henrys* and vice versa—another proof of the inconsistency of the absolute dimension systems.

In many cases the absolute c.g.s. units were designated as abcoulomb, abohm,[26,27,36] or the abbreviation e.s. volt, and e.m. volt were used for the c.g.s. unit of voltage in the electrostatic and electromagnetic dimension systems.[6] Again, the "practical" unit of magnetic flux was called the pra-maxwell,[28] but an extension of this method would have resulted in complications. All these attempts were quite unsatisfactory but serve as an indication of the many difficulties which were caused by the simple statements that k_e and k_m were to be considered as pure numbers.

The *conclusion stated in chapter IVD* has to be supplemented, therefore, by the condition *that the practical units be completely separated from the absolute systems* and that the conceptions of the dimensions of the practical unit systems be clarified. It is a unique fact that *here is a unit system in daily use and well known* even to nonexperts, but *without the background of a proper dimension system.* In no other field of physics is there such a well-built unit system with such distinguished unit names as in electromagnetics, and yet there is no fundamental basis to justify this system.

Although the practical unit system is complete as to the unit names, it has not been customary until recently to use it as a complete system in the case of Maxwell's field equations. A mixture of the absolute electrostatic, absolute electromagnetic and practical units has been used most frequently, especially in the literature of electrical engineering. Accordingly, Maxwell's field equations are amply supplied with numerical factors in the technical literature.[29] This was the cause of an attempt to choose the practical units ampere, ohm, centimeter and second as a basis for a separate and independent set of electrical units.[13,14] However, the above four units are not a unit *system* and consequently, they are not at all a dimension system as frequently assumed. (See chapter IVA).

This proposal resulted as a reaction against the mixture of absolute systems and as such, took the extreme position of separating the field of electromagnetics completely from mechanics. These attempts have the invaluable merit of having caused a spirited discussion for over twenty years, without, however, achieving a definite result.

B. Requirements of New Proper Dimension Systems

Thus far it has been explained and proved that all existing dimension systems are quite unsatisfactory, the classical systems themselves being incorrect. Greatest care has to be taken in proposing new dimension systems, since such systems should be satisfactory for international consideration.

The *main points in preparing new dimension systems,* according to the results indicated in this paper, can be enumerated as follows: *First,* a single fundamental electrical dimension has to be combined with the three mechanical fundamental dimensions in order to obtain a proper dimension system. There is no possibility of transposition of a purely mechanical dimension system into electromagnetics. *Second,* the expressions of *absolute* units, *absolute* dimensions and *absolute* systems have to be entirely suppressed. *Third,* it is necessary to adhere to the trend in electrical engineering of using the so-called practical *unit* system, but with due regard to a proper and adequate *dimension* system. *Fourth,* the independently defined absolute units have to be introduced into a physical dimension system. These physical units must be related to the so-called practical units by numerical factors. The velocity of light as an important multiplier must disappear finally and completely. *Fifth,* it should be possible to write Maxwell's field equations in such a general way that, by introducing the different unit systems, all the various schemes for writing the equations are obtained. In all cases only numerical conversion factors are involved—the factor 3.10^{10} is in no case to be considered as a velocity. *Sixth,* the dielectric constant and the permeability of free space are not to be taken as numerics, but rather as physical quantities with dimensions derived from the dimension system specified.*

At first glance it would seem almost impossible to find a solution containing all these requirements, especially since no comprehensive solution has been suggested throughout the last forty years of active discussion. However, a satisfactory solution of the problem is possible and the following section contains proposals for such "natural dimension systems."

Before submitting a plan of solution of the problem, it will be convenient for future use to indicate the funda-

*Tables of such comparative types may even be found in the literature. See reference 25.

*O. Heaviside[3] emphasized this point of view, but did not assign definite dimensions.

mental equations of the electromagnetic field in homogeneous media.

$$f_e = k_e \frac{Q_1 Q_2}{r^2} \qquad f_m = k_m \frac{I_1 I_2}{r} l \tag{16}$$

$$k_e = \frac{1}{4\pi} \cdot \frac{1}{\Delta} \qquad k_m = \frac{1}{2\pi} \cdot \Pi \tag{17}$$

$$\Delta = \Delta_0 \cdot \epsilon \qquad \Pi = \Pi_0 \cdot \mu \tag{18}$$

$$\bar{\mathbf{D}} = \Delta \cdot \bar{\mathbf{E}} \qquad \bar{\mathbf{E}} = \rho \bar{\mathbf{G}} \qquad \bar{\mathbf{B}} = \rho \cdot \bar{\mathbf{H}} \tag{19}$$

$$\psi = \iint D_n d\sigma; \quad I = \iint G_n d\sigma; \quad \phi = \iint B_n d\sigma \tag{20}$$

$$W_e = \frac{1}{2} \iiint \bar{\mathbf{E}} \bar{\mathbf{D}} d\tau; P_R = \iiint \bar{\mathbf{G}} \bar{\mathbf{E}} d\tau; W_m = \frac{1}{2} \iiint \bar{\mathbf{H}} \bar{\mathbf{B}} d\tau \tag{21}$$

$$= \frac{1}{2} CV^2 \qquad = RI^2 \qquad = \frac{1}{2} LI^2 \tag{22}$$

$$\text{curl}\, \bar{\mathbf{H}} = \bar{\mathbf{G}} + \frac{\partial \bar{\mathbf{D}}}{\partial t} \tag{23}$$

$$\text{curl}\, \bar{\mathbf{E}} = - \frac{\partial \bar{\mathbf{B}}}{\partial t} \tag{24}$$

These forms hold in *any case* where the *same unit system* is used throughout. If units of different systems are employed on both sides of any equation, numerical factors enter the relations, and it will be shown later that these numerical factors are the same as those found in the various forms of equations used for decades. The above *given forms* may be called *rationalized*; but it will be seen later, that the factor 4π can be shifted around simply by choosing the respective combination of units of the various systems in order to obtain all the familiar forms. The factor 4π and the problem of rationalization is not an essential problem for correct expression of the equations; but it is essential that the velocity of light no longer appears in the fundamental laws.

It should be emphasized at this point that the proposals made in the following section are in no sense to be considered as the ultimate or final solution of the problem; but that any other complete analysis must follow this same line of thought. This first discussion then, will confine itself to the fundamental requirements—no final proposal being made as to the notation and symbols to be adopted.

C. Proposal as to Natural Dimension Systems

In all the attempts to settle the confusion of the various unit systems, one fact seems to have been consistently disregarded; namely, that there *already are two mechanical systems* in general use, a technical and a physical one (see chapter IIIB) with their respective fundamental dimensions [*f*], [*l*], [*t*] and [*m*], [*l*], [*t*]. It is but natural to ask—why not supplement both systems by a fundamental electrical dimension and call them respectively, the *electrotechnical and electrophysical dimension systems?* As has been shown in chapter IIIB, both mechanical dimension systems are perfect. If, therefore, an electrical dimension is added, it is to be expected that perfect electrical dimension systems will be obtained (see chapter IVA). This is the substance of the following proposal, for which the dimensional relations and units are given in Tables V and VI.

The *new fundamental electrical dimension* is taken to be the *electric charge*. As previously pointed out in chaper IVA, there are many advantages in specifying the electric charge as a fundamental dimension. In addition to the *natural unit* existing in the electron, the electrotechnical unit *coulomb* is fixed internationally and can be realized electrochemically quite readily. The basis of the electrophysical definition of unit charge is Priestley's law (**16**). By taking the numerical value of k_e for free space equal to unity (but still having a dimension assigned to it), a unit of charge is defined as such a charge, acting on an equal charge at a distance of 1 *centimeter* with a force of one *dyne*. This unit is here called the *priestley* according to the true discoverer of the law[10] and is the fundamental unit.

The dimensions of the true electrical quantities in the third group of the tables are the same in both systems and are expressed in [*l*], [*t*], [*Q*], only. In addition, the dimensions of all other mechanical-electrical quantities (related to the mechanical forces in the electromagnetic field) as well as the parameters are integer powers of all four fundamental dimensions. Finally, from the physical point of view, electricity is nothing more than electric charges at rest or in motion and as such are the cause of the electric and magnetic fields.

The separation into two dimension systems extending the two mechanical systems into electromagnetics gives *the solution* in so far as avoiding the interference due to the requirements of physics and engineering. In the former field the fundamental units *cm*, *gram*, *sec*, *priestley*, represent the extension of the extremely valuable c.g.s. system into the c.g.s.p. system for electrophysics. On the other hand, the fundamental units *cm*, *newton*, *sec*, *coulomb*, represent the extension of the similarly valuable technical-mechanical system into the c.n.s.c. system. The electrical units employed in the electrotechnical system are all the familiar, so-called "practical" units; while the units employed in the electrophysical system are all of the so-called absolute units, using the latest decisions of the International Electrotechnical Commission.[28] It is surprising to note, that during the last few meetings the International Electrotechnical Commission considered the completion of unit names in the electrophysical system. It is

TABLE V—ELECTROTECHNICAL DIMENSION SYSTEM

Fundamental Dimensions: $[f]$, $[l]$, $[t]$, $[Q]$

Group	Quantity	Symbol	Dimension	Electrotechnical unit	Expressed in electrophysical units
1. (Fundamental)	Length	l	$[l]$	centimeter	1 centimeter
	Time	t	$[t]$	second	1 second
	Force	f	$[f]$	newton = 10.2 kg force	10^7 dynes
	Electric charge	Q	$[Q]$	coulomb	$3 \cdot 10^9$ priestleys
2. (Mechanical)	Energy	W	$[f]\,[l]$	joule = 1 newton cm	10^7 ergs
	Power	P	$[f]\,[l]\,[t]^{-1}$	watt	10^7 ergs/sec
	Mass	m	$[f]\,[l]^{-1}\,[t]^{-2}$	gram-seventh = 10^4 kg	10^7 grams
3. (Electrical)	Electrostatic flux	ψ	$[Q]$	coulomb	$3 \cdot 10^9$ priestleys
	Displacement	D	$[Q]\,[l]^{-2}$	coulomb/cm²	$3 \cdot 10^9$ priestleys/cm²
	Current	I	$[Q]\,[t]^{-1}$	ampere	$3 \cdot 10^9$ priestleys/sec $= \frac{1}{10}$ gilbert
	Current density	G	$[Q][l]^{-2}[t]^{-1}$	ampere/cm²	$\frac{1}{10}$ gilbert/cm²
	Magnetizing force	H	$[Q][l]^{-1}[t]^{-1}$	ampere/cm	$\frac{4\pi}{10}$ oersted $= \frac{1}{10}$ gilbert/cm
	Magnetomotive force	F	$[Q][t]^{-1}$	ampere	$\frac{4\pi}{10}$ oersted cm $= \frac{1}{10}$ gilbert
4. (Electrodynamical)	Magnetic flux	ϕ	$[f][Q]^{-1}[l][t]$	voltsec	$\frac{1}{300}\;\frac{\text{erg sec}}{\text{priestley}} = 10^8$ maxwell
	Induction	B	$[f][Q]^{-1}[l]^{-1}[t]$	voltsec/cm²	10^8 gauss
	Electric field intensity	E	$[f][Q]^{-1}$	volt/cm	10^8 maxwell/cm sec
	Voltage	V	$[f][Q]^{-1}[l]$	volt	10^8 maxwell/sec
5. (Electromagnetic parameters)	Capacitance	C	$[f]^{-1}[Q]^2[l]^{-1}$	farad	10^{-9} sec/kirchhoff
	Absolute dielectric constant	Δ	$[f]^{-1}[Q]^2[l]^{-2}$	farad/cm	10^{-9} sec/kirchhoffcm
	Resistance	R	$[f][Q]^{-1}[l][t]$	ohm	10^9 kirchhoff
	Inductance	L	$[f][Q]^{-2}[l][t]^{-2}$	henry	10^9 kirchhoffsec
	Absolute permeability	Π	$[f][Q]^{-2}[t]^2$	henry/cm	10^9 kirchohffsec/cm

UNIVERSAL CONSTANTS

Absolute dielectric constant of free space: $\Delta_0 = \frac{1}{4\pi} \cdot \frac{1}{9.10^{11}}$ farad/cm $= 0.884 \,.\, 10^{-13}$ farad/cm

Absolute permeability of free space: $\Pi_0 = 4\pi \,.\, 10^{-9}$ henry/cm $= 1.256.10^{-8}$ henry/cm

necessary, however, that a final agreement be reached between physicists and engineers.

The unit relations contain numerical factors and not factors with dimensions of velocity or any power thereof. This is due to the clear specifications of four fundamental dimensions.

In order to simplify the notations, three new unit names have been suggested: priestley, newton and kirchhoff. Whether or not these names are adopted is irrelevant, but it is necessary that these or other names be chosen to complete the tables. For some time the need has been felt for a unit of force equal to 10.2 *kg-force*,* which provides a simple relation between the technical and physical force units, as well as the respective energy and power units. With respect to the *priestley*, it is obvious that a name should be given to such a fundamental unit of the entire electrophysical system. The *kirchhoff* has been introduced due to its importance for the last group of parameters in Table VI.

It remains to show how the various forms of writing Maxwell's field equations merely depend upon the units employed. Consider, for example, the first field equation in the simplified form:

$$\text{curl}\,\overline{\mathbf{H}} = \overline{\mathbf{G}}. \tag{25a}$$

In the electrotechnical system the units are amp/cm for $\overline{\mathbf{H}}$, amp/cm² for $\overline{\mathbf{G}}$, and no numerical factor is involved. Now, if $\overline{\mathbf{G}}$ is left in amp/cm² and $\overline{\mathbf{H}}$ is given in *oersteds*, Table VI gives the relation

$$1 \text{ oersted} = \frac{10}{4\pi} \text{ amp/cm},$$

so that

$$\left.\begin{array}{ll} \text{curl}\,\dfrac{10}{4\pi}\,\overline{\mathbf{H}} = \overline{\mathbf{G}} & \\ \text{or} \quad \text{curl}\,\overline{\mathbf{H}} = \dfrac{4\pi}{10}\,\overline{\mathbf{G}} & \end{array}\right| \begin{array}{l} \overline{\mathbf{H}} \text{ in oersteds,} \\ \overline{\mathbf{G}} \text{ in amp/cm}^2. \end{array} \tag{25b}$$

In this case, where different units are to be used on the two sides of the equation, it has to be written explicitly as shown in (**25b**), and a numerical factor enters the equation.

Consider now the complete relation in (**23**) and introduce $\overline{\mathbf{D}}$ in *priestley*/cm². If $\overline{\mathbf{H}}$ is expressed in *oersted*, it is necessary to use the unit relations

*This has already been emphasized by V. Karapetoff.[14]

TABLE VI—ELECTROPHYSICAL DIMENSION SYSTEM

Fundamental dimensions: $[m]$, $[l]$, $[t]$, $[Q]$

Group	Quantity	Symbol	Dimension	Electrophysical unit	Expressed in electrotechnical unit
1. (Fundamental)	Length	l	$[l]$	centimeter	1 centimeter
	Time	t	$[t]$	second	1 second
	Mass	m	$[m]$	gram	10^{-7} gram-seventh
	Electric charge	Q	$[Q]$	priestley	$\frac{1}{3}\cdot 10^{-9}$ coulombs
2. (Mechanical)	Energy	W	$[m][l]^2[t]^{-2}$	erg	10^{-7} joule
	Power	P	$[m][l]^2[t]^{-3}$	erg/sec	10^{-7} watt
	Force	f	$[m][l][t]^{-2}$	dyne	10^{-7} newton
3. (Electrical)	Electrostatic flux	ψ	$[Q]$	priestley	$\frac{1}{3}\cdot 10^{-9}$ coulombs
	Displacement	D	$[Q][l]^{-2}$	priestley/cm^2 = $\frac{1}{3.10^{10}}$ gilbert sec/cm^2	$\frac{1}{3}\cdot 10^{-9}$ coulombs/cm^2
	Current	I	$[Q][t]^{-1}$	gilbert = 3.10^{10} priestley/sec	10 amperes
	Current density	G	$[Q][l]^{-2}[t]^{-1}$	gilbert/cm^2	10 ampere/cm^2
	Magnetizing force	H	$[Q][l]^{-1}[t]^{-1}$	oersted = $\frac{1}{4\pi}$ gilbert/cm	$\frac{10}{4\pi}$ ampere/cm
	Magnetomotive force	F	$[Q][t]^{-1}$	gilbert = 4π oersted cm	10 ampere
4. (Electrodynamical)	Magnetic flux	ϕ	$[m][Q]^{-1}[l]^2[t]^{-1}$	maxwell = $\frac{1}{3.10^{10}}\frac{\text{erg sec}}{\text{priestley}}$	10^{-8} voltsec
	Induction	B	$[m][Q]^{-1}[t]^{-1}$	gauss	10^{-8} voltsec/cm^2
	Electric field intensity	E	$[m][Q]^{-1}[l][t]^{-2}$	maxwell/cm sec	10^{-8} volt/cm
	Voltage	V	$[m][Q]^{-1}[l]^2[t]^{-2}$	maxwell/sec	10^{-8} volt
5. (Electromagnetic parameters)	Capacitance	C	$[m]^{-1}[Q]^2[l]^{-2}[t]^2$	sec/kirchhoff	10^9 farad
	Absolute dielectric constant	Δ	$[m]^{-1}[Q]^2[l]^{-3}[t]^2$	sec/kirchhoff/cm	10^9 farad/cm
	Resistance	R	$[m][Q]^{-2}[l]^2[t]^{-1}$	kirchhoff = $\frac{1}{9.10^{20}}\frac{\text{erg sec}}{\text{priestley}^2}$	10^{-9} ohm
	Inductance	L	$[m][Q]^{-2}[l]^2$	kirchhoff sec	10^{-9} henry
	Absolute permeability	Π	$[m][Q]^{-2}[l]$	kirchhoffsec/cm	10^{-9} henry/cm

UNIVERSAL CONSTANTS

Absolute dielectric constant of free space: $\Delta_0 = \frac{1}{4\pi}\frac{\text{priestley}^2}{\text{erg cm}} = \frac{1}{4\pi}\cdot\frac{1}{9.10^{20}}$ sec/kirchhoffcm

Absolute permeability of free space: $\Pi_0 = 4\pi\cdot\frac{1}{9.10^{20}}\frac{\text{erg sec}^2}{\text{priestley}^2\text{ cm}} = 4\pi$ kirchhoff sec/cm

$$1\text{ priestley/cm}^2 = \frac{1}{3.10^{10}}\text{ gilbert sec/cm}^2$$

$$= \frac{4\pi}{3.10^{10}}\text{ oersted sec/cm,}$$

which lead to

$$\text{curl }\overline{\mathbf{H}} = \frac{4\pi}{10}\overline{\mathbf{G}} + \frac{4\pi}{3.10^{10}}\cdot\frac{\partial\overline{\mathbf{D}}}{\partial t} \tag{25c}$$

Where $\overline{\mathbf{H}}$ is in *oersteds*, $\overline{\mathbf{G}}$ in *amp./cm*^{2} and $\overline{\mathbf{D}}$ in *priestleys/cm*^{2}. It appears that the degree of familiarity of the result depends upon the variety of units involved.

D. The Problem of a Unique Comprehensive Unit System

Quite a long series of papers has been devoted during the last three decades to a problem which may be formulated as: *Find, among the "infinity to the fourth" various unit systems* possible in electromagnetics, *a single one* which satisfies all requirements of practicability in all fields of physics, by means of merely taking powers of 10 of the "fundamental units."*

No such system has been found as yet and it would appear that *there is no real need for such a comprehensive system*. The two systems as proposed here may, however, approximate quite closely such a comprehensive system, if the *newton* is introduced as a new force unit. Then the electrical and mechanical quantities are connected by simple relations involving powers of 10, but the mass unit is somewhat large. This objection is not so serious since the mass is only a *derived* unit in the electrotechnical system.

The electrophysical system, with the unfortunate tendency of introducing factors 4π and 3.10^{10} naturally does not involve such simple relations. If there were,

*For information pertaining to the various attempts to solve this problem see references 26, 30, 31, 32, 25, 33, 6, 34, 35.

however, the possibility of defining a fundamental unit of electric charge as

$$1 \text{ cavendish} = 3 \text{ priestleys},$$

then the factors 3 and 9 or $\frac{1}{3}$ and $\frac{1}{9}$ would disappear in all unit relations but would combine with the universal constants as

$$\left.\begin{aligned} \Delta_0 &= \frac{1}{4\pi} \cdot \frac{1}{9} \frac{\text{cav}^2}{\text{erg cm}} = \frac{1}{4\pi} \cdot \frac{1}{9} \cdot 10^{-11} \text{ farad/cm} \\ \Pi_0 &= 4\pi \cdot 10^{-20} \frac{\text{erg sec}^2}{\text{cav}^2 \text{ cm}} = 4\pi \cdot 10^{-9} \text{ henry/cm.} \end{aligned}\right\} \quad \textbf{(26)}$$

Then, if the unit of magnetizing force be taken as 1 gilbert/cm instead of the new oersted, the relations between the two electrical systems of units would only involve integer powers of 10. This would require that, for free space $k_e = 9 \frac{\text{erg cm}}{\text{cav}^2} = 1 \frac{\text{erg cm}}{\text{priestley}^2}$; or the unit charge 1 *cavendish* would be defined as a charge exerting a force of 9 *dynes* on an equal charge at a distance of 1 *cm;* or a force of 1 *dyne* at a distance of 3 *cm.* Such definitions might not meet with the approval of physicists but they do indicate the possibility of a comprehensive unit system.

Conclusion

The absolute dimension systems expressing the electric and magnetic quantities by the 3 dimensions of mechanics have been shown to be incorrect, since the consequent use of proper mathematical statements leads to impossible results. However, the definitions of the absolute units, as well as the practical units in electromagnetics have sense quite independent of the dimension systems. It is shown that the simplest and as it seems apparently, the only true solution, is to extend the two mechanical dimension systems, known as technical and physical systems, into the field of electromagnetics, by adding a new fundamental electric dimension. This gives two electrical systems—the electrotechnical dimension system with all the practical units and the electrophysical dimension system with all the absolute units.

The continuous use of the *absolute* dimension systems in physics as well as electrical engineering textbooks, requires final international action and the proposals summarized in Tables V and VI may be considered as a basis for such action. However, for the present, it would seem desirable to restrict the discussion to the fundamental idea outlined in this paper. The introduction of new unit names and other such details may well be deferred to some later time. The achievement of final international agreement is highly desirable, but such a goal will require the cooperation of physicists and the International Electrotechnical Commission.

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Appendix A

The Dimension Systems in Mechanics

Fundamental Measurements:

Distance $[l]$

Time $[t]$

Important Definitions

Velocity $\quad v = \frac{ds}{dt} \therefore [v] = \frac{[l]}{[t]}$

Acceleration $\quad a = \frac{dv}{dt} \therefore [a] = \frac{[l]}{[t^2]}$

Momentum $\quad M = mv \quad \therefore [M] = [m] \frac{[l]}{[t]}.$

Independent Laws in Mechanics:

1. Force on a moving body $f = m \cdot a \quad \therefore [f] = [m] \frac{[l]}{[t^2]}$

2. Gravitation law $\quad f = k \frac{m_1 m_2}{r^2} \therefore [f] = [k] \cdot \frac{[m^2]}{[l^2]}$

3. Kinetic energy $\quad W = \frac{1}{2} mv^2 \therefore [W] = [m] \cdot \frac{[l^2]}{[t^2]}$

Some Dependent Relations:

Force from energy $\quad f = -\frac{\partial W}{\partial s} \therefore [f] = \frac{[W]}{[l]}$

same as (**1**) and (**3**).

Power $\quad P = \frac{dW}{dt} \therefore [P] = \frac{[W]}{[t]}$

From the three independent laws it is seen that 6 dimensions are involved, so that $\binom{6}{3} = 20$ possible *dimension systems* exist.* It would, however, be possible to use any one of the definitions (only few are listed) in order to replace a certain dimension in the laws 1, 2, 3

*Since each independent law involves 4 dimensions, this result is correct.

by a defined dimension. For example, $[l]$ could be replaced by $[v]$ $[t]$, thereby obtaining a wider choice of fundamental dimensions. However interesting the result may be, no use will be made of these possibilities at this point.

Appendix B

THE DIMENSION SYSTEMS IN HEAT

Fundamental Measurements:

Distance	$[l]$
Time	$[t]$
Temperature	$[\theta]$

Important Definitions:

Force $$f = m \cdot a \qquad \therefore [f] = [m] \cdot \frac{[l]}{[t^2]}$$

Pressure $$p = \frac{df}{dS} \qquad \therefore [p] = \frac{[f]}{[l^2]} = \frac{[m]}{[l]\,[t^2]}$$

Energy $$H = W = m\frac{v^2}{2} \therefore [W] = [m] \cdot \frac{[l^2]}{[t^2]}$$

Independent Laws in Heat Theory:

1. Boyle-Charles's law for gases

$$p \cdot V = m \cdot R \cdot \theta, \therefore \frac{[l^2]}{[t^2]} = [R] \cdot [\theta]$$

2. Conduction of heat power

$$\frac{dH}{dt} = k \cdot S \cdot \frac{d\theta}{dn} \therefore \frac{[W]}{[t]} = [k]\,[l]\,[\theta]$$

Some Dependent Relations:

Specific heat $H = s \cdot m \cdot \delta\,\theta$

$$\therefore [s] = \frac{[W]}{[m]\,[\theta]} = \frac{[l^2]}{[t^2]\,[\theta]}$$

In the two independent laws given above, 6 dimensions are involved and, therefore, 4 fundamental dimensions are required to form a dimension system in the theory of heat. The upper limit $\binom{6}{2} = 15$ dimension systems is not reached here, since the first law only involves 4 dimensions. However, it would be possible to use the definitions in order to replace $[W]$ by either $[f]$ or $[m]$, thereby again extending the choice.

Appendix C

THE DIMENSION SYSTEMS IN ELECTROMAGNETISM

Fundamental Measurements:

Distance	$[l]$
Time	$[t]$

Important Definitions:

Force $$f = m \cdot a \quad \therefore [f] = [m]\frac{[l]}{[t^2]}$$

Work $$W = \int \bar{\mathbf{f}} \cdot \overline{\mathbf{ds}} \therefore [W] = [f] \cdot [l]$$

Independent Laws in Electromagnetism:

1. Coulomb's law* (for two point charges)

$$\bar{\mathbf{f}}_e = \left(k_e \cdot \frac{Q_1}{r^2} \cdot \frac{\bar{\mathbf{r}}}{r}\right) \cdot Q_2 = \bar{\mathbf{E}}_1 \cdot Q_2 \therefore [f] = [k_e] \cdot \frac{[Q^2]}{[l^2]}$$

2. Ampere's law* (for two parallel infinitely long wires)

$$\bar{\mathbf{f}}_m = \left[\bar{\mathbf{I}}_2 \times \left(k_m \cdot \frac{\bar{\mathbf{I}}_1}{r} \times \frac{\bar{\mathbf{r}}}{r}\right)\right] \cdot l = (\bar{\mathbf{I}}_2 \times \bar{\mathbf{B}}_1) \cdot l \therefore [f] = [k_m] \cdot [I^2]$$

3. Law of conservation of electricity†

$$I = \frac{dQ}{dt}, \qquad \therefore [I] = \frac{[Q]}{[t]}$$

Some Independent Relations:

Faraday's induction law:

$$-\frac{d}{dt}\iint \bar{\mathbf{B}} \cdot \overline{\mathbf{ds}} = \beta \cdot \int \bar{\mathbf{E}} \cdot \overline{\mathbf{ds}} \therefore [k_m] \cdot [I]\frac{[l]}{[t]} = [\beta] \cdot [k_e]\frac{[Q]}{[l]}$$

so that: $[\beta] = 1$.

Again there are 3 independent laws as in mechanics, but in this case 7 dimensions are involved so that the highest number of possible dimension systems would be $\binom{7}{3} = 35$, each dimension system being composed of 4 fundamental dimensions. However, two of the laws involve only 3 dimensions, so that the number of possibilities is reduced to

$$\binom{3}{2} \cdot \binom{4}{2} = 3 \times 6 = 18.$$

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*These two laws can also be written as $f_e = k_e \dfrac{Q_1 Q_2}{r^2}$ and $f_m = k_m \dfrac{I_1 I_2}{r} \cdot l$. In this scalar form the main difference between the two fields described by the laws is not sufficiently emphasized. The electrostatic field is a scalar-potential field, produced by scalar quantities, the electric charges; the magnetostatic field is a vector potential field, produced by vector quantities, the currents. The electric field strength $\bar{\mathbf{E}}$ is a gradient, whereas the magnetic field strength $\bar{\mathbf{B}}$ is a curl. It would seem inadvisable to choose similar mathematical expressions for two physical concepts which are so extremely different.

†This law can be interpreted either as the link between electrostatic charge and the discharge current for a condenser, or as the drift of electrons in metallic conduction.

As in Appendix B, it would be possible to extend the choice of dimension systems by using the definitions given above and expressing [f] in terms of either [m] or [W].

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Performance Calculations on Induction Motors

Practical Straightforward Means for Calculating Performance of Single-Phase or Polyphase Induction Motors

C. G. VEINOTT*
Associate, A.I.E.E.

Synopsis.—Practical means for calculating the performance of both single-phase and polyphase induction motors from previously determined constants are developed in a straightforward manner. The methods are suitable for accurate calculations but can be simplified for approximate calculations. The methods given are complete and no auxiliary charts or sets of curves are required, nor need any diagram be constructed.

The single-phase calculation method is based on the cross field theory presented by H. R. West. The polyphase calculation method is based on the equivalent circuit.

I. Introduction

RECENTLY there have been presented two methods of calculating the performance of polyphase induction motors. One of these was developed by Mr. W. J. Branson[1] and is based upon an analytical solution of a circle diagram, although it might have been based upon an equivalent circuit, and requires about 100 pages of families of curves. The other method was proposed by Mr. P. L. Alger[2] and is based upon the expansion of the impedance of the equivalent circuit into the form of an infinite power series, the number of terms of which are used being dependent upon the accuracy desired. Four or five years ago, Mr. H. R. West[3] developed and proposed a symbolic method for calculating the performance of a single-phase induction motor.

In this paper are presented two calculating methods, one for the performance of single-phase induction motors and one for polyphase induction motors. Both of these methods are developed by means of complex algebra and the finished calculating sheets are quite similar in character. Neither of the methods requires an auxiliary chart, circle diagram, or set of curves, and both methods are believed to have certain distinct and unique advantages over other methods proposed to date.

The basis for the calculating method for polyphase motors proposed in this paper is the equivalent circuit of Fig. 2 and no new theories or concepts are introduced. This method can be followed to give exactly the same identical results in every respect as this equivalent circuit, for any and all values of any of the given constants. Such exactness, however, is ordinarily not necessary, and virtually all other methods which have been proposed previously as practicable contain one or more simplifying approximations, some methods making one approximation and some another.

It is unfortunate, however, that what may be safe approximations for one motor may be quite inadequate for another motor; *e. g.*, the influence of primary resistance on the primary current can usually be neglected in large motors but not in small motors. Moreover, in some cases it is desirable to make a number of approximations in order to obtain rough results quickly, and in other cases, more exact results are needed and a greater expenditure of time is justifiable.

The calculation method presented in this paper is so arranged and described that the Steinmetz equivalent circuit of Fig. 2 can be followed with absolute exactness or with varying degrees of approximation as desired, the approximations resulting in a saving in labor in calculating the constants for applying the method but not changing the general scheme of the method. This feature is felt to possess very distinct advantages also, the fact that it is neither necessary to construct a circle diagram nor to use any auxiliary charts is a not-to-be-over-looked advantage.

The single-phase calculation sheet offers, though to a more limited extent, the same general advantages as does the polyphase calculation sheet.

Since the purpose of this paper is only to present a simple, straightforward method of calculating the performance of either a single-phase or a polyphase induction motor and not to introduce any new or unfamiliar concepts, the paper itself deals only with the methods themselves, the assumptions upon which they are based, and how to make practical use of them. These methods can only be applied after the constants of the motor have been determined either from design constants, or by test, or by any other suitable means. The determination of these constants is a sufficient problem in itself and one with which this paper will not be concerned. The developments of the calculating methods are recorded in Appendixes II and III.

II. Notation

Before proceeding with an explanation of the calculation sheet, the constants appearing on these sheets will be briefly summarized and discussed.

E = impressed volts per phase
r_1 = primary resistance per phase
r_2 = secondary resistance per phase referred to the primary

*Small Motor Engg. Dept., Westinghouse Elec. & Mfg. Co., Springfield, Mass.

1. For references see Bibliography.

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r_f = resistance representing core loss when connected in parallel with X_m

r_m = resistance representing core loss when connected in series with X_m

x_1 = primary leakage reactance per phase

x_2 = secondary leakage reactance per phase referred to the primary

Xm = magnetizing reactance per phase

X = ideal short-circuit reactance of motor ($r_2 = 0$)

$= x_1 + x_2 \dfrac{Xm}{Xm + x_2}$. This is the reactance the motor would have with short-circuited secondary if the iron loss and secondary resistance were zero. This expression, however, does take into account the shunting of the secondary leakage reactance by the magnetizing reactance

Xo = $Xm + x_1$ = reactance of the primary winding with the secondary open-circuited

$Kp = \dfrac{Xm}{Xo} = \sqrt{\dfrac{Xo - X}{Xo}}$ (used only when $x_1 = x_2$)

I_1* = primary current per phase

I_2 = secondary current per phase for polyphase motors—secondary current in transformer axis for single-phase motors

I_3 = secondary current in cross field axis (single-phase motors only)

Im = total current in branch of equivalent circuit representing core loss and magnetizing current, (polyphase motors only)

Io = no-load current per phase

$i_m = \dfrac{E}{Xo}$ = primary current per phase which would flow if secondary were open-circuited, either single-phase or polyphase motors, (in polyphase motors $i_m = Io$, approximately)

$S = \dfrac{\text{actual speed}}{\text{synchronous speed}}$ (used with single-phase motors)

$s = \dfrac{\text{slip r.p.m.}}{\text{synchronous r.p.m.}}$ (used with polyphase motors)

ϕ = number of phases

Torque. The formulas for torque given in this paper are for torque in oz-ft. These formulas may be changed to give torque in other units by substituting for the quantity 112.6, the following quantities:

For lb-ft.—7.04

For kg-meters—0.974

*Note: I_1, I_2, I_3, and I_m represent currents which vary with load; I_o and i_m, for any given motor, remain constant.

III. Single-Phase Induction Motor Performance Calculations

Theoretical Basis

In the development of the calculating method for single-phase induction motors, extensive use has been made of West's[3] theory. The assumptions upon which this calculating method is based are thus similar to his in many respects. The angle of hysteretic lag between flux and m.m.f. is neglected, and the magnetomotive force or current required to produce a certain flux is assumed to be proportional to the flux just as in his paper. Sinusoidal voltages and currents, of course, are also assumed.

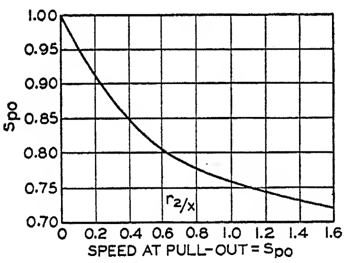
The general basis of these methods is that the voltage equations for each of the three circuits (shown in Fig. 6) *viz.*, the primary circuit, the transformer field circuit in the secondary, and the cross field secondary circuit, are set up in accordance with Kirchhoff's law. The solution of these three simultaneous equations yields expressions for the currents in each one of these circuits. From these currents and the known reactances of the motor, the fluxes, hence torque and mechanical power developed by the rotor, are obtained. This developed mechanical power is item 30 on the calculation sheet, Fig. 1.

From this point on, the treatment differs somewhat from West's method. From the power developed by the rotor is subtracted the friction and windage and the iron loss due to the cross field. (The cross field loss is assumed to be half or slightly less than half the total iron loss.) The main field iron loss is assumed to be supplied directly from the stator circuit. This is a logical procedure when the motor is considered from the standpoint of the cross field theory, though to be strictly correct, it should be remembered that the core loss is due, not to the separate effects of components of flux, but to the actually existing resultant flux.

As stated before, the main field iron loss is assumed to be supplied directly from the primary power supply, and the stator current is corrected for this additional current to supply the iron loss. This correction is item 14 on the calculating sheet. The input watts are obtained by adding the primary and secondary copper losses and main field iron loss to the developed mechanical output. Power factor and efficiency can then be figured in a conventional manner. It is highly desirable to determine the efficiency by summation of the losses as this at once tells the designer how the losses are distributed in the machine.

Since, in this method of calculation, speed is the independent variable, some means for determining the speed at maximum torque are necessary. In order to accomplish this, use was made of a Branson[4] diagram for single-phase motors as shown in Fig. 7, to determine a general analytical expression for the speed at pull-out as a function of r_2/X, upon which factor the pull-out speed principally depends. The pull-out speed has been

Constants of motor		No.				From diagram	No load	Max. torq.
		1	$S = \frac{\text{r.p.m.}}{\text{syn.}}$	0.962	0.956			0.81*
Line volts $= E$	$= 110$	2	S^2	0.925	0.913			0.655*
Total leakage $= X$	$= 8.3$	3	$(1-S^2)$	0.075	0.087			0.345*
$X_0 = 110$		4	$(1-S^2)r_1$	0.29	0.33			1.31*
r_1	$= 3.8$	5	F_1	4.98	4.98			4.98*
r_2	$= 4.65$	6	$U = (4)+(5)$	5.27	5.31			6.29*
$K_p = \sqrt{(X_0 - X)/X_0}$	$= 0.964$	7	$(1-S^2)X$	0.62	0.72			2.86*
$r_2/X = 0.56\ r_2/X_0$	$= 0.0423$	8	F_2	0.52	0.52			0.52*
$i_m = \frac{E}{X_0} = 1.00\ i_m r_2$	$= 4.65$	9	$W = (7)-(8)$	0.10	0.20			2.34*
$F_1 = (2 - K_p^2) r_2$	$= 4.98$	10	$\sqrt{U^2 + W^2}$	5.27	5.31			6.72*
$F_2 = (2\ r_1 + r_2)(r_2/X_0)$	$= 0.518$	11	$(1-S^2)E$		9.58			
$F_3 = (i_m r_2)(r_2/X_0)$	$= 0.197$	12	F_3		0.20			
$F_4 = (i_m r_2)2$	$= 9.30$	13	$M = (11)-(12)$		9.38			
$F_5 = (i_m r_2)K_p$	$= 4.48$	14	$F_9 U$		0.48			
$F_6 = [(i_m r_2)K_p]^2 r_2$	$= 93.3$	15	$N = (13)+(14)$		9.86			
$F_7 = EK_p$	$= 106.0$	16	$\sqrt{N^2 + F_4^2}$		13.54			
$F_8 = (EK_p)^2 r_2$	$= 52{,}200$	17	$I_1 = (16)/(10)$		2.55	2.49	1.865	
$F_9 = \frac{Fe \text{ loss } (m)}{E}$	$= 0.091$	18	$(1-S^2)F_7$		9.22			
		19	$\sqrt{(18)^2 + F_5^2}$		10.26			
		20	$I_2 = (19)/(10)$		1.90	1.86	0.898	
		21	SF_5		4.29			
		22	$I_3 = (21)/(10)$		0.807		0.898	
		23	$(1-S^2)F_8$	3920	4540			18,000*
		24	F_6	90	90			90*
		25	$(23)-(24)$	3830	4450			17,910*
		26	Prim. cu. loss $= I_1^2 r_1$		24.7	24	13.2	
		27	Sec. cu. loss $(m) = I_2^2 r_2$		16.8	16	3.8	
		28	Sec. cu. loss $(c) = I_3^2 r_2$		3.0	3	3.8	
		29	*Fe* loss (m)		10.0	10	10.0	
		30	$(25)\times(2)/(10)^2$	127.5	144.0		19.0	260*
		31	Input $= (26)+(27)+(28)+(29)+(30)$		198.5	195	49.8	
		32	*Fe* loss $(c) + (F$ and $W)$	19.0	19.0	19.0		19*
		33	Output $= (30)-(32)$	108.5	125.0	123		241*
		34	R.p.m. $= S \times$ syn.		1721	1721		1460*
		35	Torque $= \frac{112.6 \times (33)}{(34)}$		8.16	8.05		18.6*
		36	Eff. $= (33)/(31)$		63.0	63.2		
		37	P.F. $= (31)/EI_1$		70.8	71.0		
		38	App. eff. $= (36) \times (37)$		44.6	44.7		
		39						
		40						



*Terms to get output and torque

FIG. 1—PERFORMANCE CALCULATIONS ON A SINGLE-PHASE INDUCTION MOTOR

plotted against r_2/X on Fig. 1 and this method has been found to give generally good results as the error in pull-out speed can be quite large without materially effecting the pull-out torque.

This curve of pull-out speed against r_2/X was calculated for values of the latter up to 2.4 and approached an asymptote of 0.7, indicating that 70 per cent of synchronous speed is the lowest speed at which the pull-out torque of a single-phase induction motor can be made. It is, however, possible to lower the speed at pull-out somewhat below this value by unusual design, though it is not possible to obtain anything like the same control over it as is obtained over the pull-out speed in polyphase motors.

Simplified Calculation Sheet

On the calculating sheet, Fig. 1 it has been further assumed that the primary and secondary reactances were equal, *i. e.*, $x_1 = x_2$. This assumption is not absolutely necessary, as will be shown subsequently, but it is usually justifiable and greatly simplifies matters.

The use of this calculation sheet will be illustrated by means of an example. A 110-volt, 60-cycle, 4-pole squirrel-cage motor was picked at random and the values of E, X, Xo, r_1 and r_2 entered on the sheet. The total core loss for this motor was 19 watts which was arbitrarily distributed, 10 watts to the main field (entered in item 29) and 9 watts to the cross field, which, added to the 10 watts friction and windage, gave 19 watts for the cross field loss plus friction and windage (item 32). From the given constants, Kp etc., and the F constants were figured.

The next step is to select the speed at which the performance characteristics are desired. If the full load performance be desired, the full load slip should be estimated. Thus, it will usually be more satisfactory to attempt to estimate the quantity $1 - S^2$, which is approximately twice the slip for small slips. Just what value of $1 - S^2$ to assume is best governed by experience, as this depends generally upon the size of the motor, ratio of maximum to full load torque, etc. Ordinarily, a value somewhere between 0.05 and 0.08 may be assumed for $1 - S^2$.

In this particular case, 0.075 was assumed for $1 - S^2$, and the output for this value of $1 - S^2$ was figured by computing items 1-10, 23-25, 30 and 33. It will be observed that it is not necessary to figure all the items if only the output is desired. Those items which are necessary are marked by an asterisk in the extreme right hand column. (For a short cut to computing item 10, see Appendix I.)

Now that the output has been found for this motor for a particular value of $1 - S^2$, the full load speed or slip can be estimated with sufficient accuracy for the output can be assumed to be proportional to the quantity $S^2 (1 - S^2)$. Using this principle, the full load speed can now be approximated very closely. In the example chosen, an output of 108.5 watts corresponds to a value of $0.925 \times 0.075 = 0.0694$ for the quantity $S^2 (1 - S^2)$. The value of the latter quantity for an output of 124.3 watts (full load) is, therefore, by direct proportion, 0.0796. The corresponding value of $1 - S^2$ can be figured from any known value of $S^2 (1 - S^2)$ by means of the equation

$$1 - S^2 = \frac{1 - \sqrt{1 - 4\,[S^2\,(1 - S^2)]}}{2} \tag{41}$$

In the chosen example, this becomes

$$1 - S^2 = \frac{1 - \sqrt{1 - 4 \times 0.0796}}{2} = 0.087$$

which is the value of $1 - S^2$ corresponding to full load. (Actually $1 - S^2$ may be more readily determined from the quantity $S^2 (1 - S^2)$ on a slide rule without the use of the formula given.)

Using 0.087 as the value of $1 - S^2$, the complete performance was calculated by computing items 1 to 38 inclusive. The performance of this same motor was also calculated by means of Branson's[4] circle diagram and the results given in the third column.

The maximum torque was obtained by noting from the curve that the pull-out speed for this motor ($r_2/X = 0.56$) was approximately 0.81 and figuring the torque at this speed.

No-Load Conditions. The no-load currents can be checked approximately by taking $S = 1.0$. With this assumption, the no-load primary current becomes approximately

$$I_0 = \frac{F_4}{F_1} = \frac{2}{2 - K_p^2}\, i_m \tag{42}$$

and the secondary current, in either axis, becomes

$$I_2 = I_3 = \frac{F_5}{F_1} = \frac{K_p}{2 - K_p^2}\, i_m \tag{43}$$

The no-load input can then be estimated by adding friction and windage, iron losses, and copper losses due to the currents given by the above expressions. Calculation of no-load current and watts is illustrated in Fig. 1.

For working up constants from test, the following relations obtained from equation (42) are useful

$$\left.\begin{aligned} I_0 &= \frac{2E}{Xo + X} \\ Xo &= \frac{2E}{I_0} - X \\ Xm &= XoKp = \sqrt{Xo^2 - XXo} \end{aligned}\right\} \tag{44}$$

Performance Calculations When x_1 Differs From x_2

The procedure to be followed in this case differs only in the computation of the "F constants." The reactances which must be known are x_1, x_2, and Xm. In Table I are listed the values of the "F constants" in

TABLE I—"A CONSTANTS" AND "F CONSTANTS" FOR SINGLE-PHASE MOTORS

F constant	A constant	Expression for constant $x_1 \neq x_2$	Expression for constant $x_1 = x_2$
F_1	A_1	$r_2 \left[\frac{x_2 X_m - r_1 r_2}{(X_m + x_2)^2} + \frac{X_m + 2x_1}{X_m + x_2} \right]$	$r_2 \left[\frac{X_0 + X}{X_0} - \frac{r_1 r_2}{X_0^2} \right]$ or $r_2 \left[2 - K_p^2 - \frac{r_3 r_2}{X_0^2} \right]$
F_2	$-A_3$	$r_2 \left[\frac{r_2 (X_m + x_1)}{(X_m + x_2)^2} + \frac{2r_1}{X_m + x_2} \right]$	$(2r_1 + r_2)\ (r_2/X_0)$
F_3	$-A_5$	$E \left(\frac{r_2}{X_m + x_2} \right)^2$	$(i_m r_2)\ (r_2/X_0)$
F_4	$-A_7$	$\frac{2Er_2}{X_m + x_2}$	$(i_m r_2)\ 2$
F_5	$-A_9$	$\frac{EX_m r_2}{(X_m + x_2)^2}$	$(i_m r_2)\ K_p$
F_6	$-A_{11}$	$\frac{E^2 r_2^3 X_m^2}{(X_m + x_2)^4}$	$[(i_m r_2)\ K_p]^2\ r_2$
F_7	A_8	$\frac{EX_m}{X_m + x_2}$	EK_p
F_8	A_{10}	$\left(\frac{EX_m}{X_m + x_2} \right)^2 r_2$	$(EK_p)^2\ r_2$
F_9	A_{12}	$\frac{Fe \text{ loss } (m)}{E}$	$\frac{Fe \text{ loss } (m)}{E}$
	A_4	$x_1 + x_2 \frac{X_m}{X_m + x_2}$	X
	A_2	r_1	r_1
	A_6	E	E

erms of these reactances. (No attention need be paid o the "A constants" as these were merely used in eriving the methods.) In place of X appearing among he given data and in item 7, Fig. 1, should be entered he value of A_4, which is the short-circuit reactance.

For the sake of comparison, the reactance of the ıotor in the chosen example was assumed to be unqually distributed, 60 per cent of the reactance being n the primary. The "F constants" were computed rom the formulas given in Table I and used in the ame manner as previously. The final results were found o agree within one per cent.

General Remarks

In general, the above method gives quite satisfactory esults. This method is especially convenient when only ı part of the performance is desired. For example, if a peed torque curve be desired, only the same items used o figure the maximum torque need be figured. When iguring a speed torque curve, the cross field iron loss ınd friction should be made to decrease to zero with he speed; otherwise a following of the calculation sheet vould result in a negative torque at zero speed. For vant of better information, it is suggested when figuring torques below pull-out, that item 32, as given on the sheet, be multiplied by S^2.

Sometimes power factor or efficiency in themselves are unimportant, but the apparent efficiency or full-load current is wanted. In this case, all that need be figured in addition to the output are items 11-17 inclusive, giving the primary output at that point. The apparent efficiency, of course, is

$$\text{apparent efficiency} = \frac{\text{output watts}}{EI_1} \tag{45}$$

For rough calculations on large motors where the primary resistance is small, the effect of the latter on the primary current can easily be neglected by taking r_1 equal to zero.

IV. Polyphase Induction Motor Performance Calculations

Theoretical Basis

The generally accepted form of equivalent circuit representing the performance of a polyphase induction motor is as shown in Fig. 2, and this is the circuit proposed by Steinmetz and used by Mr. Alger in his recent

paper. In this circuit, the iron loss is represented by a resistance in parallel with the magnetizing reactance. Mr. Branson, in his recent paper,[1] represents the iron loss by a resistance, r_f, connected just inside the primary resistance.

The equivalent circuit of Fig. 2 can be reduced to the form shown in Fig. 3 which is exactly equivalent to that of Fig. 2 if the following values are chosen for r_m and X_m, where r_f and X_m' are taken from the circuit of Fig. 2

$$r_m = \frac{r_f X_m'^2}{r_f^2 + X_m'^2} = \frac{X_m'^2}{r_f} \times \frac{1}{\left[1 + \left(\frac{X_m'}{r_f}\right)^2\right]} \tag{46}$$

$$X_m = \frac{r_f^2 X_m'}{r_f^2 + X_m'^2} = X_m' \times \frac{1}{\left[1 + \left(\frac{X_m'}{r_f}\right)^2\right]} \tag{47}$$

The first approximation made in this paper is that the term in brackets in equations (46) and (47) is assumed equal to unity. With ordinary motors, this

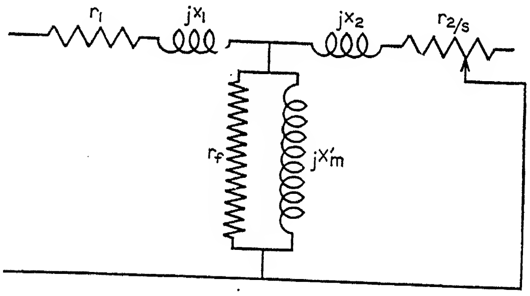


Fig. 2—Steinmetz Equivalent Circuit of a Polyphase Induction Motor

assumption merely means a difference of one or two per cent in reactance only of the core loss and magnetizing branch of the equivalent circuit, but means an appreciable saving in labor when calculating performance.

Usually the constants of a motor corresponding to the circuit of Fig. 2, are not known with sufficient accuracy to make a distinction between X_m' and X_m. If desired, however, and r_f and X_m' are known, r_m and X_m can be calculated by means of equations (46) and (47) and applied directly. In this paper, however, r_m is calculated by the simpler formula

$$r_m = \frac{\text{Core loss}}{\phi \times i_m^2} \tag{48}$$

The method of deriving the calculation method was briefly as follows. Expressions for the primary and secondary currents in terms of the constants of the motor and slip were obtained by ordinary circuit analysis. The mechanical power developed by the secondary, or the secondary output is

$$I_2^2 r_2 \phi \times \frac{1 - s}{s} \tag{49}$$

This appears as item 20 on the calculation sheet, Fig. 5. Subtraction of the friction and windage from the secondary output gives the output of the motor. The input is obtained by adding the core loss, the secondary and primary copper losses to the secondary output. The core loss is assumed constant so that this summation gives a slightly higher input than would be ob-

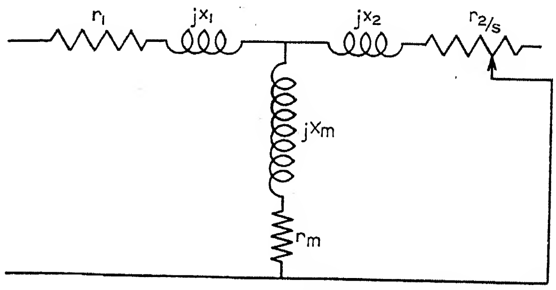


Fig. 3—Another Form of Circuit of Fig. 2

tained by strictly following the equivalent circuit. To follow this equivalent circuit exactly, the copper loss in the resistance r_m can be calculated for each load point and used in place of the core loss. To do this, it is necessary to calculate I_m for each load point. For any load point, I_m is given by

$$I_m = -\frac{\sqrt{(12)^2 + A_{10}^2}}{(10)^*} \tag{50}$$

where A_{10} has the value given in Table II. I_m could thus be figured and for each individual load point, and the quantity $\phi I_m^2 r_m$ entered in item 19 instead of the core

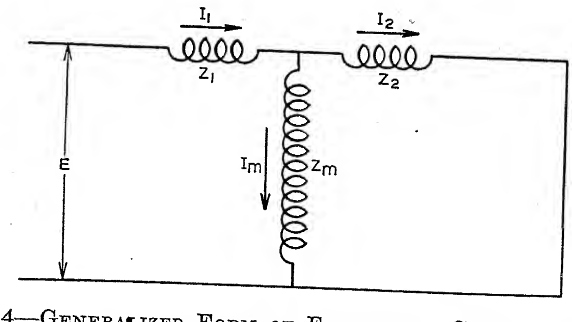


Fig. 4—Generalized Form of Equivalent Circuit of the Polyphase Induction Motor

loss, and thus the input obtained would be exactly the same as the power input to the equivalent circuit (real component of $\phi \times \check{E} \times$ conjugate of $\check{I}_1$).

Simplified Calculation Sheet of Fig. 5

In deriving the simplified calculation method of Fig. 5, a further assumption has been made—namely, that the primary reactance equals the secondary reactance. The calculation sheet is virtually self-ex-

*(12) and (10) refer to items 12 and 10 in Fig. 5.

TABLE II—"A CONSTANTS" AND "F CONSTANTS" FOR POLYPHASE MOTORS

F constant	A constant	Expression for constant $x_1 \neq x_2$	Expression for constant $x_1 = x_2$
F_1	$-A_1$	$x_1 + x_2 \dfrac{X_m}{X_m + x_2} - \dfrac{r_1 r_m}{X_m + x_2}$	$X - \dfrac{r_m}{X_0} r_1$
F_2	A_4	$\dfrac{(X_m + x_1)\, r_2}{(X_m + x_2)}$	r_2
F_3	A_2	$\dfrac{(r_1 + r_m)\, r_2}{X_m + x_2}$	$r_2 \left(\dfrac{r_1 + r_m}{X_0}\right)$
F_4	A_3	$\dfrac{r_m (x_1 + x_2)}{X_m + x_2} + r_1$	$r_1 + \dfrac{2}{1 + K_p} \dfrac{r_m X}{X_0}$ or $r_1 + \dfrac{r_m}{X_0} X$
F_5	A_5	$\dfrac{E r_m}{X_m + x_2}$	$i_m r_m$
F_6	A_6	$\dfrac{E r_2}{X_m + x_2}$	$i_m r_2$
F_7	A_9	$\dfrac{E \sqrt{r_m^2 + X_m^2}}{X_m + x_2}$	$E \sqrt{K_p^2 + \left(\dfrac{r_m}{X_0}\right)^2}$ or EK_p
	A_7	E	E
	A_8	$\dfrac{E X_m}{X_m + x_2}$	EK_p
	A_{10}	$\dfrac{E x_2}{X_m + x_2}$	$\dfrac{i_m X}{1 + K_p}$

planatory. The known constants of the motor, namely, E, X, X_o, K_p, r_1, r_2, friction and windage and core loss are entered at the left side of the sheet in the spaces provided. The F constants are then computed from these given constants. The slip at which the performance is desired is assumed in item 1. In order to get the output, it is unnecessary to figure each item. Only those items marked by an asterisk are necessary for determination of the output corresponding to the assumed slip. If the first value of slip assumed gives nearly full load output, the next value of slip can be found by assuming the output proportional to the slip. (A useful short cut for figuring items 10 and 14 is described in Appendix I.)

The slip at maximum torque can be found from the expression

$$s_m = \frac{r_2}{\sqrt{r_1^2 + X^2}} \tag{40}$$

Maximum torque is, of course, the torque figured at this slip and starting torque is the torque figured for a slip of unity. The formula given in item 26 can not be used for starting torque since both the output and speed are zero, but starting torque is readily found by putting $s = 1$ in equation (35), Appendix III, obtaining

$$\text{starting torque} = \frac{112.6}{\text{syn. r.p.m.}} I_2^2 r_2 \phi$$

$$= \frac{112.6}{\text{syn. r.p.m.}} \times (18)\, s = 1 \tag{51}$$

The use of this calculation is illustrated by means of a numerical example.

Performance Calculation When x_1 Differs From x_2

If x_1, x_2 and X_m are known, the "F constants" should be calculated in terms of the latter by using the suitable expressions from Table II. It can then be forgotten how the "F constants" were obtained and the performance calculated exactly as shown in Fig. 5.

If it is desired, an equivalent circuit of the form of Fig. 2 can be used as a basis for the calculation of the performance by first reducing it to a circuit of the form of Fig. 3 by means of equations (46) and (47). Moreover, the core loss can be assumed to vary with load exactly as does the copper loss in the resistance representing core loss in the equivalent circuit by calculating A_{10} and following the process just described.

Approximate Calculations

In addition to the approximations mentioned, other assumptions can be made to simplify calculations for rough work. The calculation sheet is so arranged, for example, that either the primary resistance or iron loss can be neglected by putting r_1 or r_m or both equal to zero.

V. Conclusions

In this paper, practical methods for calculating the performance of both single-phase and polyphase motors have been developed by straightforward means. Either calculating method consists of the filling in of the blank spaces of a calculation sheet especially prepared for this purpose. The performance can easily be calculated roughly or with practically any desired degree of accuracy by the expenditure of a trifle more labor, depending upon the accuracy with which the constants of the motor are known. The methods are thus suited for either rough design calculations or for more refined calculations, the same form being used for either, the use of the same form for either type of calculation being a distinct advantage.

Moreover, neither of the methods presented in this paper requires the constructing of a circle diagram nor any auxiliary charts or sets of families of curves.

The author wishes gratefully to acknowledge helpful suggestions from Messrs. C. R. Boothby, E. P. Codling, and H. E. Ellis.

Appendix I

There is a method, the origin of which is somewhat uncertain, of evaluating the square root of the sum or difference of two squares which so simplifies this operation for slide rule calculations as to make it well worth a description here since this operation is performed several times in the calculating methods described in this paper.

E = volts per phase	= 127
ϕ = no. of phases	= 3
X = s. c. reactance (per ph.)	= 5.0
X_0 = o. c. reactance (per ph.)	= 98.0
$K_p = \sqrt{\frac{X_0 - X}{X_0}}$	= 0.970
Friction and windage (watts)	= 20
i_m = no load amps = E/X_0	= 1.297
Core loss (total)	= 37
$r_m = \frac{\text{core loss}}{\phi \times i_m^2}$	= 7.32
r_1 = pri. res. per phase	= 2.40
$F_2 = r_2$	= 1.55
$F_3 = r_2 \frac{r_1 + r_m}{X_0}$	= 0.1537
$F_1 = X - \frac{r_m}{X_0} r_1$	= 5.82
$F_4 = r_1 + \frac{r_m}{X_0} X$	= 2.85
$F_5 = i_m r_m$	= 9.49
$F_6 = i_m r_2$	= 2.01
$F_7 = E K_p$	= 123.2
Stg. torque = $\frac{112.6}{\text{syn. r.p.m.}} \times (18)_{s=1}$ (in oz-ft.)	
Slip at max. torque = $s_m = \frac{r_2}{\sqrt{r_1^2 + x^2}}$	= 0.240

*Terms to get output and torque

Item		Full load	Stg. T.	Max. T.
1	Slip = s (assume)	0.0297	1.00	0.24*
2	$1/s$	33.7	1.00	4.17*
3	$1 - s$	0.970	0.00	0.76*
4	F_1	5.82	5.82	5.82*
5	$F_3 \times (2)$	5.17	0.15	0.64*
6	U = (5) − (4)	−0.65	−5.67	−5.18*
7	F_4 =	2.85	2.85	2.85*
8	$F_2 \times (2)$	52.3	1.55	6.46*
9	W = (7) + (8)	55.2	4.40	9.31*
10	$\sqrt{U^2 + W^2}$	55.2	7.17	10.66*
11	F_5	9.49		
12	$F_6 \times (2)$	67.75		
13	(11) + (12)	77.24		
14	$\sqrt{(13)^2 + E^2}$	148.6		
15	I_1 = (14) ÷ (10)	2.69		
16	$I_2 = F_7$ ÷ (10)	2.235	17.20	11.56*
17	Pri. loss = $I_1^2 \times r_1 \times \phi$	52		
18	Sec. loss = $I_2^2 \times r_2 \times \phi$	23.4	1375	620*
19	Core loss	37		
20	Sec. output = (18) × (2) × (3)	766		1961*
21	Input = (17)+(18)+(19)+(20)	878		
22	Friction and windage	20		15*
23	Output = (20) − (22)	746		1946*
24	R.p.m. = (3) × syn. r.p.m.	1746		1369*
25	Torque = 112.6 × (23) ÷ (24)	48.1	86	160*
26	Eff. = (23) ÷ (21) × 100	84.9		
27	P.f. = (21) × 100/$EI_1 . \phi$	85.7		
28	% full load	100		

Fig 5—Performance Calculations on a Polyphase Induction Motor

Assume that we wish to evaluate $\sqrt{a^2 + b^2}$. By simple algebra

$$\sqrt{a^2 + b^2} = a\sqrt{1 + \left(\frac{b}{a}\right)^2}$$

Either b or a may be the larger, but it will generally be easier to divide the larger number by the smaller one.

To illustrate, let us evaluate $\sqrt{2.34^2 + 6.29^2}$. Divide 6.29 by 2.34 using the C and D scales. The square of the quotient, $\left(\frac{b}{a}\right)$, is located under the left index of the B scale which reads 7.22. Add one mentally, obtaining 8.22. Move the slide until the left index of the B scale is opposite 8.22. Now, move the runner to the smaller number or 2.34 on the C scale and read 6.72, which is the desired result, on the D scale. The

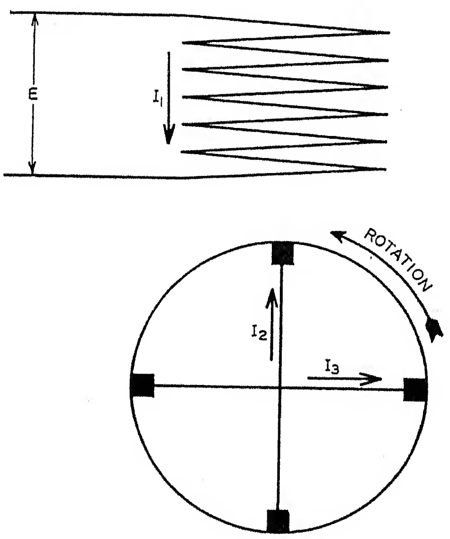


FIG. 6—THE SINGLE-PHASE INDUCTION MOTOR VIEWED FROM THE CROSS FIELD BASIS

method is simpler to apply than explain. Similarly, the square root of the difference between two squares can be determined by subtracting one mentally from the square of the quotient of the larger number by the smaller one, extracting the square root and multiplying this root by the smaller number.

Appendix II

THEORY OF THE SINGLE-PHASE MOTOR

Determination of the Currents. A single-phase motor can be represented diagrammatically as shown in Fig. 6.

Using Kirchhoff's law that the applied voltage must equal the impedance drops, Mr. West writes, for the primary,

$$E = r_1 I_1 + jx_1 I_1 + jX_m (I_1 - I_2) \qquad (1)$$

Similarly, in the rotor, the sum of all the induced voltages in each circuit must independently be equal to zero, giving the following two equations:

$$-jX_m (I_1 - I_2) - S(X_m + x_2) I_3 + r_2 I_2 + jx_2 I_2 = 0 \qquad (2)$$

$$j(X_m + x_2) I_3 - SX_m (I_1 - I_2) + Sx_2 I_2 + r_2 I_3 = 0 \qquad (3)$$

These three equations can be re-arranged and solutions readily obtained for I_1, I_2, and I_3 by means of determinants. The determinant for the common denominator reduces to

$$U_1 + jW_1$$

Where

$$U_1 = -r_1 r_2^2 + 2r_2 x_1 (X_m + x_2) + r_2 X_m (X_m + 2x_2) + (1 - S^2) r_1 (X_m + x_2)^2 \qquad (4)$$

$$W_1 = -r_2^2 x_1 - 2r_1 r_2 (X_m + x_2) - r_2^2 X_m + (1 - S^2)[x_1 (X_m + x_2)^2 + x_2 X_m (X_m + x_2)] \qquad (5)$$

The numerator of I_1 is

$$E[-r_2^2 + (1 - S^2)(X_m + x_2)^2 - j2r_2 (X_m + x_2)] \qquad (6)$$

The numerator of I_2 is

$$EX_m [(1 - S^2)(X_m + x_2) - jr_2] \qquad (7)$$

The numerator of I_3 is

$$-SEX_m r_2 \qquad (8)$$

To make calculations on different motors more nearly alike, it is best as Mr. West points out, to divide equations (4) to (8) by $(X_m + x_2)^2$ obtaining the following expressions for the three currents

$$I_1 = \frac{A_5 + (1 - S^2) A_6 + jA_7}{U + jW} \qquad (9)$$

$$I_2 = \frac{A_8 (1 - S^2) + jA_9}{U + jW} \qquad (10)$$

$$I_3 = \frac{A_9 S}{U + jW} \qquad (11)$$

Where

$$U = A_1 + A_2 (1 - S^2)$$
$$W = A_3 + A_4 (1 - S^2) \qquad (12)$$

and A_1, A_2, etc., have the values given in Table I. Equations (10) and (11) are the basis for items 20 and 22 on the calculation sheet. The primary current, however, is obtained from equation (18).

Output. Mr. West shows that the net torque in synchronous watts developed by the rotor is, in the symbols of this paper

Torque (in synchronous watts)

$$= \frac{E^2 X_m^2 r_2 S[(1 - S^2)(X_m + x_2)^2 - r_2^2]}{U_1^2 + W_1^2} \qquad (13)$$

$$= \frac{S(1 - S^2) A_{10} + SA_{11}}{U^2 + W^2} \qquad (14)$$

Where A_{10} and A_{11} have the values given in Table I. The mechanical power developed by the rotor, ex-

clusive of core loss and friction and windage is, therefore:

$$\text{power developed in watts} = \frac{(1 - S^2)\, A_{10} + A_{11}}{U^2 + W^2} \times S^2 \tag{16}$$

This equation is the basis for item 30 on the single-phase calculation sheet, Fig. 1A. From this quantity is subtracted the cross field iron loss and friction and windage to obtain the output. The input is obtained by adding to the former quantity the main field iron loss and the primary and secondary copper losses.

Correction of Primary Current for Main Field Core Loss. In order to correct the primary current for the main field core loss, it is assumed that slightly more than half the core loss is supplied directly from the stator power supply. The method of accounting for the core loss cannot be illustrated directly either by a diagram or by an equivalent circuit. The iron loss is considered as being supplied by an in-phase component of current which flows through the primary circuit. This in-phase component of current which supplies the main field core loss is

$$A_{12} = \frac{\text{main field iron loss}}{E} \tag{17}$$

Adding this current to I_1 as given in equation (9), gives for the primary current corrected for iron loss

$$I_1 = \frac{A_5 + (1 - S^2)\, A_6 + A_{12}U + j\,(A_7 + A_{12}W)}{U + jW} \tag{18}$$

It will be found, however, that in normal motors, W is usually small compared with U up to well above full load, so that for all practical purposes, the effect of the term A_{12} on I_1 cannot be found in ordinary slide rule computations, and it is therefore justifiable to neglect it. On the calculation sheet, the primary current is figured from equation (18), neglecting the term $A_{12}W$.

Simplification of the Constants. If the leakage reactance be assumed to be equally divided between the primary and secondary, the constants A_1 to A_{12} inclusive can be reduced to expressions very much easier to calculate numerically, and the final results will be little different than if account were taken of the actual distribution of leakage. In the majority of cases, the amount of leakage chargeable to the primary and that to the secondary is not accurately known since no method has yet been devised to separate these leakages by an actual test. It is therefore justifiable, for ordinary purposes, to assume the $x_1 = x_2$. The reactance of the primary with the secondary open-circuited will be, therefore

$$X_m + x_1 = X_m + x_2 = X_0 \tag{19}$$

The short-circuit reactance of the motor, taking into account the shunting of the secondary leakage by the magnetizing reactance is

$$x_1 + x_2 \frac{X_m}{X_m + x_2} = x_2 + x_1 \frac{X_m}{X_m + x_1} = X \tag{20}$$

From the above relations, a further interesting relation can be shown to be exactly true when $x_1 = x_2$

$$\frac{X_m}{X_0} = \sqrt{\frac{X_0 - X}{X_0}} = K_p \tag{21}$$

The primary current with the secondary open is

$$i_m = \frac{E}{X_0} \tag{22}$$

Using equations (19) to (22) the values obtained for the A constants can be much simplified as shown in Table I. In expressions in this table, no approximations have been made although on the calculation sheet the term $-\dfrac{r_1 r_2}{X_0}$ has been dropped from F_1 as it is insignificant here.

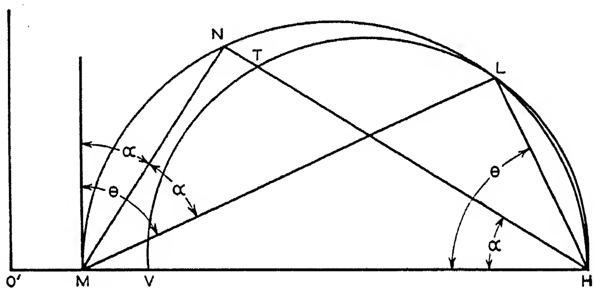


FIG. 7—BRANSON CIRCLE DIAGRAM OF A SINGLE-PHASE INDUCTION MOTOR, LOCKED AND PULL-OUT POINTS

MAXIMUM OR PULL-OUT TORQUE

Perhaps the most logical way to determine the maximum torque would be to write for the developed or internal torque, the following expression from (16)

$$\text{Internal torque} = \frac{112.6 \times S}{\text{syn. r.p.m.}} \times \frac{(1 - S^2)\, A_{10} + A_{11}}{U^2 + W^2} \tag{23}$$

differentiate it with respect to S and equate to zero to obtain the speed at maximum torque. Such a procedure, however, leads to a sixth degree equation (really a third degree equation as only the even powers of S exist) which is too cumbersome for practical use. Since it is not necessary to know the speed at pull-out with a high degree of accuracy in order to obtain the maximum torque sufficiently close, the following alternative method was used to obtain the speed at pull-out.

Fig. 7 shows a typical Branson circle diagram of a single-phase motor. $O'M = i_m$, $O'V$ no-load current and $O'H$ the locked current, also $< LMH = \tan^{-1} \dfrac{r_2}{X}$. For any load point T, the speed, as a

fraction of synchronous speed is, from Branson

$$S = \sqrt{\frac{TH - MNr_2/X}{TH}} = \sqrt{1 - \frac{MNr_2/X}{TH}} \qquad (24)$$

The point T for maximum torque is taken so that the line NH bisects the arc VL. This amounts to the same thing as bisecting the arc ML. We can now calculate MN and NH as follows for

$$\theta = \cot^{-1}\frac{r_2}{X} \text{ (complement of } < LMH)$$

$$\alpha = \frac{\theta}{2} = \frac{1}{2}\cot^{-1}\frac{r_2}{X} \qquad (25)$$

and

$$MN = MH \sin\alpha$$

$$NH = MH \cos\alpha$$

but TH can be taken, in most cases, as approximately 95 per cent of NH, so the speed at pull-out becomes

$$S_{p0} = \sqrt{1 - \frac{(MH \sin\alpha)\, r_2/X}{0.95\, MH \cos\alpha}}$$

$$= \sqrt{1 - 1.05\frac{r_2}{X}\tan\alpha} \qquad (26)$$

where α has the value given in equation (**25**). Thus, the speed at pull-out depends principally upon r_2/X. The value of Spo was calculated for different values of r_2/X and plotted on the curve given on the calculation sheet, Fig. 1.

No-Load Conditions. The various relations given in equations (**42**), (**43**), and (**44**) were deduced by simple algebra from the values of the F constants and need not be repeated here.

Appendix III

Theoretical Basis of Polyphase Calculation Sheet

Fig. 4 shows the more general form of equivalent circuit for either Fig. 2 or Fig. 3. The currents, in terms of the impedances, are:

$$I_1 = \frac{E\,(Z_m + Z_2)}{Z_1 Z_m + Z_1 Z_2 + Z_2 Z_m} \qquad (27)$$

$$I_2 = \frac{E Z_m}{Z_1 Z_m + Z_1 Z_2 + Z_2 Z_m} \qquad (28)$$

$$I_m = \frac{E Z_2}{Z_1 Z_m + Z_1 Z_2 + Z_2 Z_m} \qquad (29)$$

It is now a simple matter to substitute the individual resistances and reactances in the above equations. For the sake of convenience in numerical computations, it will be found highly desirable to divide numerators and denominators by the quantity $(X_m + x_2)$. This done, the following equations are obtained

$$I_1 = \frac{A_5 + A_6/s + jA_7}{U + jW} = \frac{F_5 + F_6/s + jE}{U + jW} \qquad (30)$$

$$I_2 = \frac{A_5 + jA_8}{U + jW} = \frac{F_7}{\sqrt{U^2 + W^2}} \text{ (numerically)} \qquad (31)$$

$$I_m = \frac{A_6/s + jA_{10}}{U + jW} \qquad (32)$$

where

$$U = A_1 + A_2/s = F_3/s - F_1 \qquad (33)$$

$$W = A_3 + A_4/s = F_4 + F_2/s \qquad (34)$$

The values of A_1, A_2 etc., and F_1, F_2, etc., are given in Table II. All of the A constants were not used on the final calculation sheet (especially when $x_1 = x_2$) and so those that were used were rearranged and the symbols F_1, F_2, etc., were assigned to those that were used.

Simplification of the Constants

If the primary and secondary reactances be assumed equal, the relations of equations (**19**) to (**22**) apply directly. The values of the various expressions in Table II were determined for $x_1 = x_2$ and are listed in this Table. No approximations have been made and each one of these expressions is exact except the second value given for F_4 and F_7; the first value is exact and it can be seen that each approximation is insignificant.

The procedure to be followed from this point in obtaining the complete performance is amply described in part IV of the paper with the exception of how the slip at maximum torque was obtained, a matter which is developed in the following paragraphs.

Slip at Maximum Torque

The torque developed by a polyphase motor, neglecting friction and windage, is

$$T = \frac{112.6}{\text{syn r.p.m.}} \times \frac{I_2{}^2 r_2 \times \phi}{s} \qquad (35)$$

$$= \frac{112.6}{\text{syn r.p.m.}} \times \frac{F_7{}^2 \times r_2 \times \phi}{\left[\left(\frac{F_3}{s} - F_1\right)^2 + \left(\frac{F_2}{s} + F_4\right)^2\right] s}$$

The torque will be a maximum when the expression

$$\left[\left(\frac{F_3}{s} - F_1\right)^2 + \left(\frac{F_2}{s} + F_4\right)^2\right] s$$

is a minimum, a condition which can be found by differentiating the latter and setting equal to zero, whence the slip at maximum torque is found to be

$$s_m = \sqrt{\frac{F_2{}^2 + F_3{}^2}{F_1{}^2 + F_4{}^2}} \qquad (37)$$

In terms of the fundamental constants of the motor, the above expression becomes

$$s_m = r_2 \sqrt{\frac{1 + \left(\frac{r_1 + r_m}{X_0}\right)^2}{\left(X - \frac{r_m}{X_0} r_1\right)^2 + \left(r_1 + \frac{2}{1+K_p} \times \frac{r_m}{X_0} X\right)^2}} \quad (38)$$

If iron loss be neglected, $r_m = 0$ and (38) becomes

$$s_m = \frac{r_2}{\sqrt{r_1^2 + X^2}} \times \sqrt{1 + \left(\frac{r_1}{X_0}\right)^2} \quad (39)$$

For most practical cases, the quantity under the second radical is so nearly unity that it may be neglected and thus

$$s_m = \frac{r_2}{\sqrt{r_1^2 + X^2}} \text{ (approx.)} \quad (40)$$

Maximum Torque. The maximum torque in terms of the F constants can be found by substituting equation (37) in equation (36) obtaining

$$\text{maximum torque} = \frac{112.6}{\text{syn. r.p.m.}}$$

$$\times \frac{F_7^2 \times r_2 \times \phi}{2\,[\sqrt{(F_3^2 + F_2^2)\,(F_1^2 + F_4^2)} + F_2F_4 - F_1F_3]} \quad (41)$$

If the values of the F's in Table II be put into the above expression and r_m be assumed equal to zero, equation (41) reduces to

$$\text{maximum torque} = \frac{112.6}{\text{syn. r.p.m.}}$$

$$\times \frac{(EK_p^2)\,\phi}{2\,[\sqrt{r_1^2 + X^2} + K_p^2 r_1]} \quad (42)$$

This latter equation is quite useful in itself for figuring the maximum internal torque of a polyphase induction motor. It is also valuable as a check on the method of Fig. 5 to determine if the F constants are numerically correct.

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Discussion

H. R. West: It seems that many designers object to the use of the general method for calculating the performance of a machine as described by Mr. Veinott because it does not enable them to follow the processes with physical interpretation. The answer is, of course, that the purpose of a method of calculation is to enable the designer to predetermine machine performance with the desired accuracy in the least time. Such methods as the circle diagram or the equivalent circuit are, of course, invaluable to anyone who has much to do with motors, but not necessarily for purposes of calculation. That method of calculation should be used which gives the desired accuracy with the least labor, and I believe that the results of algebraic analyses, arranged in systematic form such as Mr. Veinott is using, can profitably be used more than they are at present.

In connection with the correction for the core loss component of primary current in the single-phase motor; Mr. Veinott considers that this component, in calculating the primary copper loss, causes a voltage drop which is not taken into account in figuring the input to the rotor. This means that the calculated motor output is too large by a very slight amount. The error is certainly very small. Has this error been found to be negligibly small in comparison to the error that would be made if the total core loss were subtracted from the output and no correction were made for the primary copper loss?

R. E. Hellmund: Those who designed induction motors many years ago and obtained at that time a rather close agreement between calculations and load tests undoubtedly wonder why new and improved calculating methods such as that presented by Mr. Veinott and others (see bibliography given in paper) should still be an active subject of discussion at Institute meetings. At the same time, it may be a surprise to many to learn that the discrepancy between the results obtained from load tests and those obtained by calculating methods from no-load values and other motor constants has been greater during recent years than it was, say, 20 to 25 years ago. There are no doubt various reasons for this and a brief discussion of a few of them is given in the following.

A comparison of present-day designs and those made about 20 years ago indicates a marked decrease in motor size. This change has taken place gradually, so that many of the effects of it have not been evident at any one time, and, furthermore, they probably have been obscured by various individual changes in the numerous variables entering into the design. Considering, however, that now, as then, the torque of the motor is proportional to the field and the ampere conductors, one cannot escape the conclusion that generally speaking there must be with the smaller dimensions higher flux densities and also more ampere conductors within a unit of space in the up-to-date motors than was found in the older ones; most likely this also means in general more ampere conductors per slot than in the older designs. This means that the zigzag and slot leakage fluxes increase, which will tend toward greater saturation at load, especially in the tooth tips of partially-closed-slot machines; to this must be added the fact that the main flux densities in the same parts have also increased. If we consider now that all theories deriving motor performance from the motor constants neglect the effect of saturation, it is not surprising that in the modern designs there should be a greater discrepancy between such calculated results and tests made under actual load conditions.

Let us, for the sake of argument, assume the extreme case of complete saturation of the tooth tips under load condition by the combined effect of main and leakage fluxes, while at no-load the main flux by itself does not bring about an appreciable saturation of these same parts. Under these assumptions, many of the conditions under load will approach those obtained with an open-slot machine, while at no-load there would be the full benefit of the partially-closed-slot construction. It follows then that the magnetizing current of the main flux under load would be larger, so that the power factor is undoubtedly different from that derived by calculations based on no-load conditions. It follows further that the flux distribution under load in the air-gap will approach that of an open-slot machine and thus cause extra core losses in the surface and in the teeth of both members

similar to those that would be obtained if both members were constructed with open slots, and which, as every designer knows, will appreciably differ from those obtained with partially-closed slots. In other words, it is evident that the full-load core losses will be different from the no-load losses which are made the basis of the usual performance calculations. Naturally these extreme assumptions do not exist in actual practise, but it is quite evident that a closer approach to them is generally obtained with the smaller up-to-date motors than with the very much larger motors of the past.

Other changes in design might be cited which would further explain certain differences between the old and the present-day designs. In pointing out these differences, it should not be inferred that this is meant as a criticism of the newer designs. On the contrary, the designers and manufacturers have rendered very valuable service not only by making appreciable reductions in the cost of the motor, but also by greatly reducing the motor dimensions with resultant greater ease of application of motors for all purposes, all of which, of course, is of benefit to the user. This discussion is merely intended to indicate that by the continuous improvement and refinement of motors, the designer has created new problems for himself and perhaps brought about the necessity for a different point of view. Rather than priding himself upon the close agreement between his calculated results and those obtained from actual load tests, he will have to continue the refinement and applicability of his calculating methods such as has been done in Mr. Veinott's paper, as well as of his test methods under load, for the purpose of permitting a more correct determination and analysis of the difference between the two. It is only through such efforts that the effect of changed conditions upon the various constants can be correctly analyzed and so pave the way for still further improvement in motor design.

P. H. Trickey: I have used Mr. Veinott's method for over a year for both single-phase and polyphase motors and for ratings between 2 hp. and 3 watts output, and I have had very satisfactory results.

P. C. Smith: Anyone spending the entire day in the construction of circle diagrams would, no doubt, welcome the easier, faster method presented by Mr. Veinott. The analytical method lacks the visual picture given by the diagram, but it is less tedious, which goes a long way towards off-setting the advantage of the visual picture. The analytical method published by Mr. G. T. Smith in the *Electrical Journal*, December 1922, has been in use since that time on larger polyphase motors, but it was devised particularly for large motors and approximations were made, for simplicity's sake, which make it unsuitable for fractional horsepower sizes. Mr. Veinott's method is more universal, permitting quick approximations or accurate calculations as desired.

The paper goes into great detail in dealing with the current through the shunt circuit, showing the influence of core loss but apparently neglects friction and windage. Do not friction and windage influence this current? The paper also indicates that core loss decreases with the load. This is not always the case.

Item 18 of the polyphase calculation makes use of the secondary resistance in determining the secondary loss. In cases where the actual tested slip is available, it might be advisable to substitute this for the secondary resistance.

C. G. Veinott: Mr. West has raised a question about the relative correctness of the method of correcting the primary current for core loss as presented in this paper with merely subtracting the total core loss from the output. The latter mentioned method assumes all the core loss power to be transferred across the air gap and hence, the power developed by the rotor at full load is slightly too large. The method presented in this paper has been found to give satisfactory results. Indeed, the results are so nearly correct, that it would be difficult to determine if Mr. West's suggestion would be more accurate or not.

Mr. Smith asks whether or not friction and windage influence the current through the shunt circuit. Friction and windage are a mechanical load on the motor and do not influence the current in the shunt circuit.

I am not certain that I understand Mr. Smith's point about secondary resistance. The secondary resistance can be corrected in cases where the actual tested slip is available by multiplying the calculated secondary resistance by the ratio of tested slip to calculated slip. If this is Mr. Smith's idea, it is a very good suggestion.

Measurement of Stray Load Loss
in Polyphase Induction Motors

BY C. J. KOCH*
Associate, A.I.E.E.

Synopsis.—*The purpose of this paper is to present the results of experiments carried out to establish a practical method of measuring stray load loss in induction motors. The nature of stray load losses is discussed, methods of measuring these losses are described, and proposals for the revision of the Institute Standards to include stray load losses are made.*

Definition of Stray Load Losses

As the name indicates, the stray load losses are those losses in addition to the I^2R losses in the motor windings, which occur as a result of the load currents. These losses occur in all types of machines, and their inclusion in efficiency tests on synchronous machines has long been required by the A.I.E.E. Standards.

For d-c. machines, however, only an arbitrary allowance of 1 per cent, and for induction motors no allowance, for these losses is made in the Standards for lack of a satisfactory method of measurement in either case. Were this difficulty removed the desirability of including the stray load losses in efficiency determinations could not be questioned.

The magnitude of stray load losses in polyphase induction motors is not well known at present, but in general it is of the order of from 2 to 5 per cent, in squirrel-cage motors of from 1 to 5 hp. and it decreases gradually with increase in motor rating until it is less than 1 per cent for motors of several hundred horsepower. The "conventional efficiency" of polyphase induction motors as determined by the A.I.E.E. Standards is thus always higher than the actual motor efficiency by the amount of the stray load losses.

This has resulted in awkward situations at times, particularly in the case of motors used to drive apparatus such as pumps or generators. The overall efficiency of the pumping or generating unit being known by measurement and the motor efficiency being determined by the manufacturer on the basis of conventional efficiency, the existence of stray load loss in the motor operates to lower the apparent efficiency of the pump or generator.

Besides removing these inconveniences, the development of any easy method of measuring stray load losses should permit the losses themselves to be studied more intensively and perhaps ultimately to be greatly reduced. The experiments on polyphase induction motor stray load loss determination here described were undertaken with these objects in view.

Possible Sources of Stray Load Loss

The losses included in the calculation of conventional efficiency are, primary copper losses as determined by the load current and the measured d-c. resistance of the primary, core loss as determined under running light conditions, friction and windage usually also determined from the running light or excitation tests, and the secondary copper loss determined from the measured slip under load.

These leave as a possible source of stray load loss (a) additional primary copper loss due to eddy currents in the primary conductors, (b) eddy losses induced in the end shields, clamping fingers, etc., by the leakage fields of the primary coil ends, etc., (c) increase in the iron or core loss attendant upon the increase of the stator current from its running-light to its full-load value, and the similar change in the rotor current.

As this paper is principally concerned with stray load losses in moderate size polyphase motors, (c) is the principal item with which we are concerned. Additional losses under item (a) are now so thoroughly understood by designers that they are not permitted to reach large values, and even though corrective measures to reduce them are not undertaken, methods of calculating them are well known and may be relied upon to determine their magnitude. Losses under heading (b) are very difficult to calculate. These losses may be appreciable, especially in high-speed motors where the end leakage reactance is high.

Change of Core Loss with Load

The running-light core loss of any induction motor is usually several times as large as the "fundamental frequency" loss calculated from the volumes of the stator core and teeth and the magnetic densities existing in these parts. Part of this difference may be due to increased fundamental frequency loss as the result of burrs or punching strains, but the major portion is the tooth-frequency core loss of the motor. It is the tooth frequency core loss which changes with load and forms the principal seat of load loss in moderate size induction motors. The fundamental and tooth-frequency elements of the core loss may be experimentally separated by measurement of the running-light slip of an induction motor.

Although it is difficult to calculate the tooth-frequency core loss in an induction motor under load or for that matter in a motor running light, it is not difficult to show that the tooth-frequency core loss is a function of the load and that its increase with load constitutes a real reason for the observed stray load losses.

*General Electric Company, Schenectady, N. Y.

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Consider for example the losses in the rotor surface and teeth of a slip-ring induction motor. These are induced by the stator tooth harmonic flux. This flux may be considered as consisting of two component fluxes, the two component fields having the same number of poles. The first of these is independent of load and is the harmonic flux produced by the action of the fundamental m.m.f. on the stator slot openings. The second varies with load and is due to the tooth harmonic magnetomotive force produced by the concentration of the stator current in slots. It is the increase of this tooth harmonic magnetomotive force with load which accounts for the change of the tooth-frequency core loss with load.

In a squirrel-cage motor, all of the tooth-frequency core loss need not necessarily occur in the laminations. Inasmuch as each rotor tooth of a squirrel-cage motor is embraced by a low-resistance electric circuit, little pulsation of flux can occur in a rotor tooth. Instead, a high-frequency current circulates in the rotor bars and the flux is forced to pulsate in the stator teeth even when the motor is running light. The loss due to this high frequency current in the rotor bars is as much a part of the tooth-frequency core loss of the motor as the eddy and hysteresis losses in the laminations themselves.

Indirect Methods of Measuring Stray Load Loss

All methods of measuring stray load losses may be classified as either direct or indirect. Indirect methods aim to measure the total loss in the motor, subtraction of the conventional losses yielding the stray load loss. Some of these methods are:

a. *The input-output method.* In this method the motor output is measured directly on a dynamometer and the input by wattmeters. The difference is the total motor loss. If it could be carried out accurately the input-output method would of course be the one infallible method of determining stray load loss. Actually in even moderate size motors, both the input and the output have become such large numbers compared to their difference that even a small error in either the one or the other results in very poor accuracy, by the time the necessary subtractions have been made to obtain the stray load loss.

Some of the results presented later in this paper were obtained by the input-output method. The results were obtained with good accuracy only by virtue of the highly developed technique and very accurate equipment, resulting from study and experience in making input-output tests over a period of several years. The equipment was refined to a high degree, the dynamometers being provided with oscillating bearings and the dynamometer errors being accurately determined by running each of three dynamometers against the other two.

b. In order to avoid the necessity of making mechanical measurements of the output an identical motor may be used as a generator, the output then being measured electrically. The load losses are then divided equally between the two machines, after allowing for their different frequencies.

c. *Calorimeter methods.* These consist in measuring the amount of heat carried away from the motor by the cooling air. They are only applicable to large motors and those in which the cooling air is forced to flow in definite passages and even here they yield only fair accuracy.

Other ingenious methods of this nature may be devised such as running the motor in a box with a known amount of air passing through, etc., but these methods are hardly applicable to ordinary shop conditions.

Direct Methods of Measuring Stray Load Loss

Direct methods of measuring stray load loss are based on a procedure by which either the rotor or stator winding is excited with direct current, the unexcited winding being short-circuited. The rotor is revolved at synchronous speed and the amount of power required to do this is measured. This is usually accomplished by coupling the motor to a dynamometer or by driving it from a calibrated direct-current motor. For accurate results, the rating of the driving motor should not be more than 10 per cent of the induction motor rating.

Under these conditions a fundamental frequency current is induced in the conductors of the unexcited member. This current will be of such magnitude that its m.m.f. will very closely equal and oppose the m.m.f. of the direct-current excitation. The only fluxes existing in the motor then are the tooth harmonic fluxes which produce the tooth-pulsation and surface losses the motor experiences under load. In order to find the amount of these losses, it is necessary to subtract from the amount of power required to drive the rotor, the friction and windage loss and also an amount of power equal to the I^2R loss produced by the fundamental frequency current in the unexcited member. The remainder is the surface and tooth-pulsation loss caused by the opposing currents and is approximately equal to the actual stray load loss.

These methods have the advantage that they are usually easy to carry out and the inaccuracies in test are not greatly increased by the subsequent calculations as is the case with the input-output method. Their adaptability to shop conditions makes these methods suitable for approval by the A.I.E.E. if their validity can be established.

Accepting the results of these tests as the true stray load loss of the motor is equivalent to assuming that the tooth-frequency core loss due to the load current m.m.f. may be separately determined as in the direct method, and added to the running-light core loss to obtain the total core loss under load. Whether or not this is a justifiable assumption must be proved experimentally, although it has been found approximately true for synchronous machines. The actual tooth harmonic fluxes which exist in a motor under load are difficult to express analytically and there is little possibility of

proving the validity of the direct method by analytical means. Recourse must then be made to comparative tests by direct methods and dynamometer tests. These comparative tests have been made on a number of motors. The results are given in Table I.

TABLE I—LOAD LOSS TESTS

		Input-output method	Method A	Direct methods: Methods B and C	Direct methods: Method D
Motor No. 1	Full load	306	200	315	
	25% overload	498	270	459	
Motor No. 2	Full load	165	87	174	
	25% overload	234	145	281	
Motor No. 3	Full load	109	120	122	
	25% overload	190	165	197	
Motor No. 4	Full load	321	360	914	
	25% overload				
Motor No. 5	Full load	153	100	230	
	25% overload	228	160	369	
Motor No. 6	Full load	315		246	
	25% overload	505		586	
Motor No. 7	Full load	487		680	500
	25% overload	835		1,015	860
Motor No. 8	Full load	324	240	420	
	25% overload	619	380	720	
Motor No. 9	Full load	606	210	530	
	25% overload	884	410	940	
Motor No. 10	Full load	526	165	462	
	25% overload	957		732	
Motor No. 11	Full load			210	146
	25% overload			396	287

Direct Methods Applied to Slip-Ring Motors

In making measurements on slip-ring motors, it is customary to excite the rotor with direct current, the stator terminals being short-circuited. An ammeter is included in one of the stator phases. With the rotor revolving at synchronous speed, the rotor excitation is regulated to give a fundamental frequency current in the stator corresponding to the particular load for which the load loss measurement is being made. The amount of loss in the stator winding is then known from the direct current resistance of the stator winding.

It should be noted that when the rotor is excited with direct current, fundamental frequency current flows in the stator windings and the stray load loss which is measured includes, in addition to the tooth pulsation and surface losses, those additional losses mentioned under sources (a) and (b). As these are legitimately a portion of the motor stray load loss, it is preferable, wherever possible, to excite the rotor or secondary winding with direct current rather than the stator. When the stator is excited with direct current the (a) and (b) losses are not present and the indicated stray load loss will on this account be less than actual.

The above method of measuring stray load loss in slip-ring motors will be recognized as nothing more than the accepted method of measuring short-circuit core loss in synchronous motors and generators. A slight difference exists in the case of induction motors, however, and this must be corrected for in testing squirrel-cage motors as well as slip-ring motors.

Direct methods of measuring stray load loss will, of course, give zero load loss for zero stator current. Actually the motor stray load loss is, by definition, zero when the stator current has its running-light or excitation value. This discrepancy may be corrected for by assuming that the exciting current causes a small amount of tooth frequency core loss which is already included in the running-light core loss. The amount of this loss may be found by making measurements with the stator current adjusted to its running-light value. The value of the stray load loss for this value of current being already included in the running-light core loss, should be subtracted from the stray load loss for all other values of current.

Although this method of correcting for the effect of the magnetizing current may not be strictly accurate, the actual amount of the correction is usually small, except for very low-speed motors.

Direct Methods Applied to Squirrel-Cage Motors

In testing squirrel-cage motors, the stator winding is, of course, the only member which can be excited with direct current.

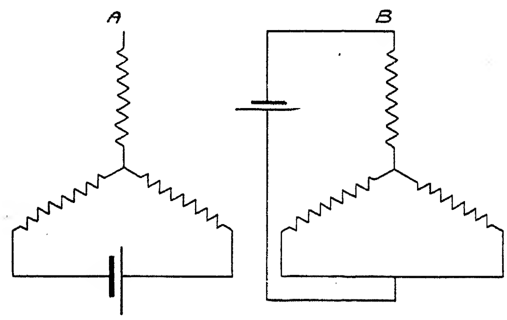

FIG. 1

It is necessary, in order to carry out this method, to determine what value of direct-current excitation corresponds to any particular value of stator load current and also the value of the loss in the rotor due to the fundamental frequency current in the rotor bars. This latter loss must be subtracted from the power required to rotate the rotor or the "rotational watts" in the same way that the stator I^2R loss is subtracted in the case of the slip-ring motor.

The necessary relation between d-c. excitation and r.m.s. load current depends on the way in which the stator terminals are connected. In either case direct-current and alternating-current values are equivalent when they produce the same magnetomotive force in the air-gap. If the connections are as in Fig. 1A

$$I_{\text{D-C.}} = I_{\text{A-C.}} \sqrt{2} \times \frac{\sqrt{3}}{2} = 1.22\, I_{\text{A-C.}} \qquad (1)$$

or if the connections are as in Fig. 1B

$$I_{\text{D-C.}} = I_{\text{A-C.}} \sqrt{2} = 1.414 \text{ A-C.} \qquad (2)$$

It is immaterial which connection is used as long as the proper equation above is used.

In order to determine the loss in the rotor due to the fundamental-frequency current circulating in the rotor bars, four methods are available. Three of these were described in an article by H. E. Linck which was published in the *Archiv fur Elektrotechnik*, Nov. 5, 1929. The methods described in this article seemed promising and furnished one of the incentives for these experiments.

Method A. This makes use of data obtained from a blocked rotor, or impedance test. Simultaneous values of stator current and input watts are taken, with the rotor blocked at standstill and balanced alternating voltages of rated frequency applied to the terminals. If from the input watts under these conditions the stator I^2R loss is subtracted, the remainder is principally the rotor loss. This should be identical with the rotor loss

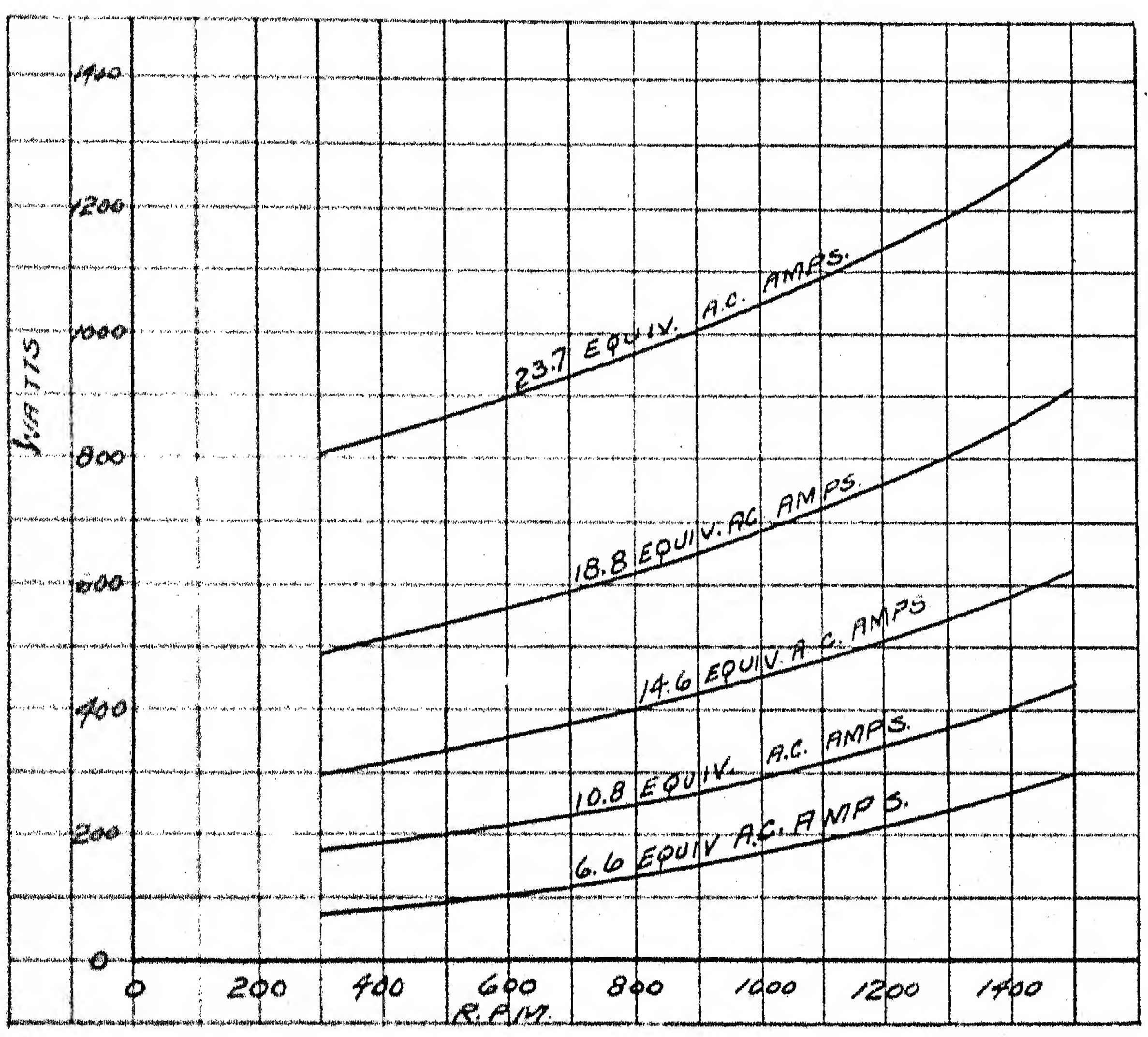

FIG. 2

during the stray load loss test if the impedance watts and rotational watts are based on equivalent values of stator current.

The rotor loss determined in this way is too large, however, on three accounts. In the first place, if the air-gap is small the squirrel-cage currents will not be uniform but will have a tooth harmonic pulsation in their space distribution. The rotor copper loss will thus be larger than that due to the fundamental rotor current which alone determines the full-load slip. Also, since there are actually stator losses of types (a) and (b), and likewise a certain amount of core loss due to the stator leakage flux which exists both under impedance test and under actual load conditions, the rotor loss determined by this method is too large by their amount also. These errors, making the rotor loss too large, make the difference between the total rotational loss and the rotor loss too small. Thus the stray load loss determined in this way is always less than the true stray load loss occurring under short-circuit conditions. As the effect of superposition of load and no-load fluxes is generally to reduce to total loss below the sum of the open- and short-circuit core losses, due to saturation, the error in the loss determination is partially corrected for.

The stator losses of types (a) and (b) and the core losses due to the stator leakage flux above mentioned can be separately determined by repeating the impedance test on the stator with the rotor removed. These losses are usually so small in comparison with the total stray load loss however, that this additional test is not warranted except possibly for very large motors.

This method A has an advantage over method B, to be described, in that it gives more correct results for double squirrel-cage types of motors.

Method B. This method of determining the rotor loss consists in taking data for a series of curves as shown in Fig. 2. These are curves of rotational watts plotted against speed for various values of stator excitation. For each value of stator excitation the intersection of the rotational watts curve with the axis of zero speed gives the value of rotor I^2R loss.

If the rotor bar resistance varies with frequency an error is introduced and the method becomes useless unless a calculated correction for the change in rotor resistance is made. If this correction is not made this method gives too low a rotor loss and the load loss so determined is correspondingly high. This correction becomes very large when the rotor bars are of the deep bar and double squirrel-cage type.

Since in using this method the stator is excited with direct current, stray load losses of types (a) and (b) will not appear in the result. These are small however, in motors of moderate size.

Method C. This method requires that only one rotational watts curve at synchronous speed be determined by test. The rotor loss is assumed to be equal to the slip loss under load, which is available from the measurements of watts input and slip made when the motor is tested under load. The slip loss should be measured at about 25 per cent overload in order to minimize the effect of the magnetizing current, and the rotor loss is assumed to vary as the square of the exciting current.

It is obvious that the method is practically the equivalent of method B, so that the remarks pertaining to double squirrel-cage rotors and losses of types (a) and (b) apply equally here.

The slip loss of the motor under load for any value of stator current is found with sufficient accuracy from the usual formula,

Slip loss = (input—core loss—primary copper loss) slip.

Method D. This, like method A, makes use of data obtained from a blocked rotor test. The actual torque developed by the motor at standstill, with balanced alternating voltages of fundamental frequency applied to the stator, is measured by means of a brake arm, or wheel and pulley.

The rotor loss is then equal to the corresponding value of synchronous kilowatts, or to the product of the measured standstill torque in foot pounds by 0.142 times synchronous speed in r.p.m. The value so obtained is less, and more nearly correct, than that given by method *A*, as it does not include the stator losses of types (a) and (b). There is still a slight error due to the net torque produced by the harmonics in the rotor current distribution, but this is normally small.

The standstill torque should preferably be measured at a current of 100 to 150 per cent of the full-load value, and for other loads it may be assumed to vary as the square of the current. The torque at full-load current is normally only 2 to 10 per cent of full-load torque, or it is of the same order as the static friction torque. For this reason, very careful torque readings with the rotor turned both with and against the torque must be taken to average out the friction and the irregularities of the torque position curve.

Results

A summary of the results of measurements of the stray load loss of a number of induction motors using both input-output and direct methods is contained in Table I.

A review of these results indicates that for squirrel-cage motors the best results are obtained using method *C*, or preferably method *D*. Method *C* is easily carried out, and gives accurate results for all small motors which have no appreciable eddy current effect in the rotor bars at starting. When double squirrel-cage rotors are used the method is inaccurate. Motor No. 4 was built with a double squirrel-cage rotor.

Using method *D*, the only error in the determination of the true stray load loss is the omission of the stator losses of types (a) and (b). This error is normally small and may be balanced by the consideration that the stray load loss under actual load conditions is probably somewhat less than that at short circuit because of saturation.

For slip-ring motors the most satisfactory method appears to be to measure the rotational loss with the rotor excited with direct current and to subtract the stator I^2R loss as determined from the measured stator current and the d-c. resistance.

Detailed outlines of these preferred tests are given in Appendix A. It is proposed that after confirmation or modification by other investigators, these methods be taken as the basis for a revision of the Institute Standards to require the inclusion of induction motor stray load losses in efficiency determinations.

Appendix A

Proposed Method of Determining Stray Load Loss in Induction Motors

Slip-Ring Motors

1. Measure the power required to drive the motor at synchronous speed with the primary winding short-circuited and sufficient d-c. excitation applied to the secondary winding to cause full-load current to circulate in the primary; the power required is designated W. From the d-c. resistance (making any corrections necessary for differences in temperature) determine the primary I^2R loss and designate this as T.

2. Repeat the test just outlined with the primary current equal to the magnetizing current; and determine the rotational power required and the I^2R loss; these latter are designated W_o and T_o.

3. The stray load loss is then equal to $W - W_o - T + T_o$.

Squirrel Cage Motors

1. Apply d-c. excitation to the stator using two of the stator terminals and leaving the third open; the amount of direct-current required is 1.22 times the full-load alternating-current per terminal. Under these conditions determine the power required to rotate the rotor at synchronous speed; this is designated W.

2. With the rotor at standstill apply to the stator balanced polyphase voltages of normal frequency of sufficient value to produce full-load current per terminal, and measure the starting torque thus produced. A check on the correctness of the torque obtained is given by the fact that its value expressed in synchronous watts, must be somewhat less (usually from 5 to 20 per cent) that the stator watts input less the stator copper loss as calculated from the d-c. resistance. Then T = synchronous watts torque = 0.142 × syn. speed (r.p.m.) × torque (lb. ft.).

3. Repeat the test just described, determining W_o with d-c. excitation equivalent to normal a-c. magnetizing current and calculate

$$T_o = T_3 \left(\frac{\text{Magnetizing current}}{\text{Full-load current}} \right)^2$$

4. The stray load loss is then $W - W_o - T + T_o$.

In practise, the starting torque varies closely as the square of the primary current; it is desirable therefore to measure the value of T at only one value of current, selected to obtain the greatest accuracy of test, and to obtain the values of T at other points by proportionality. The best value of current to select for this test will depend somewhat upon the type and size of motor. If the current is too large, overheating takes place and errors arise due to incorrect conductor temperature determinations. On the other hand, if too low values of current are used, the small torque produced in comparison with the static friction makes it difficult to obtain accurate values. It is recommended that full-load current normally be used for squirrel-cage and ball-bearing motors, while values of 150 per cent full load current are recommended for normal use on sleeve-bearing motors of less than 25 hp.

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Discussion

Orrin C. Rutledge: Since Mr. Koch's tests were made on machines of different sizes and types a more comparative idea of the results can be obtained by placing them on a percentage basis. Taking the values in Table I obtained by input-output method as 100 per cent in each case we obtain the following figures for the direct methods.

Method	% Load	Number of machines	Average	Minimum	Maximum
A	100	8	68.2%	31.4%	112.2%
	125	6	63.5	46.4	86.8
B and *C*	100	9	110.4	78.1	150.5
	125	9	112.6	76.5	161.0
D	100	1	103.0		
	125	1	103.0		

Method *A* gives low values and Mr. Koch has explained the reason for this.

Method *D* is the one proposed by Mr. Koch for adoptions in the standards but from Table I it is only possible to compare this with the input-output method on one machine.

Methods *B* and *C* give permissable average values but as may be seen there is a considerable spread from minimum to maximum.

I have been making some tests on a 10-hp. wound rotor machine and have endeavored to determine stray load loss by both input-output and direct methods, computing copper losses by d-c. resistance and measured currents in all cases. Plotting loss against line current the input-output method gives the curve (1) Fig. 1, which varies approximately as the cube of load component of line current.

The direct method gives approximately the same results whether the excitation is placed on rotor or stator, using either connection *A* or *B* in Mr. Koch's Fig. 1. This is shown in this discussion as curve 2, Fig. 1. It will be noted that the curve passes through the origin and has a value of about 60 watts when the equivalent current has a value equal to the no-load magnetizing current. It is considered that this value corresponds to the no-load pulsation loss but another test, proposed by Mr. Alger in 1920, showed a no-load pulsation loss of about twice this value.

If this is a true value of no-load pulsation loss it should be subtracted from all subsequent points moving the curve vertically downward into the position shown as curve 3, since by definition there is no "load loss" at no-load.

I would suggest consideration of the possibility that the a-c. equivalent of the d-c. current is comparable to the load component of line current under normal operation rather than the total line current. In that case we would add exciting component vectorially for each point to get it in terms of line current. This would have the effect of moving the curve nearly horizontally to the right instead of vertically downward, and of increasing the slope somewhat, (see curve 4).

The same process would be applied to Mr. Koch's method *D* for determining copper loss of the unexcited member. This loss would then also be zero when the line current equals the exciting current rather than when the line current is zero.

The curves indicate that in the vicinity of full load the loss by direct measurement is of the same order of magnitude as that by input-output determination. This checks with Mr. Koch's results. However, the shape of the curve would still leave open the question as to whether or not the pulsation loss by the d-c. method is a correct representation of the true pulsation loss occurring under load.

O. C. Schoenfeld and S. F. Henderson: We have been studying the subject of stray load losses with the object of determining their value in our designs and are glad that it has been brought up for discussion. We, also, have been testing motors by the input-output method for a considerable period, but found, as did Mr. Koch, that many refinements were necessary to obtain results of the degree of accuracy desired for a study of the stray load losses.

The method outlined by Mr. Koch has some merits, but it must be admitted that it is not theoretically correct, nor could we check it experimentally. We believe that proper consideration has not been given to the effect of saturation. This factor alone can have considerable influence and as a result the d-c. short-circuit method gives values of losses in excess of those obtained under normal load conditions. A theoretical discussion as

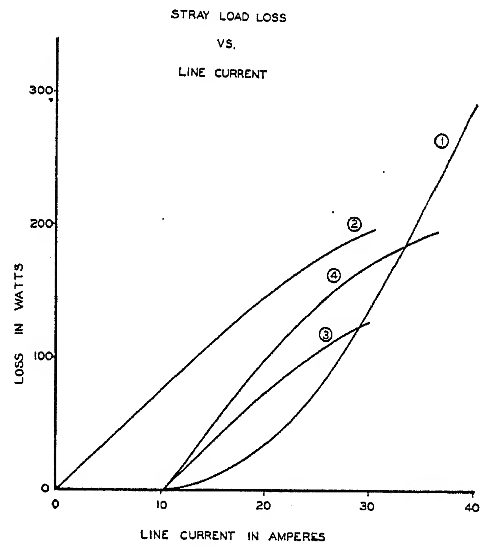

FIG. 1

well as an experimental check of this method and the effect of saturation is given in an article by L. Dreyfuss in the July 7, 1928 issue of *Archiv für Elektrotechnik.* Making use of a paper entitled "Iron Losses for Simultaneous Magnetization at Two Different Frequencies" written by F. Schroeter and published in the 1924 issue of the same magazine, Linckh, in the article mentioned by Mr. Koch, explains this influence and proves it experimentally by tests on a series of 10 motors. The tests which we made check the observations of these authors.

Linckh found that correction factors were necessary and states, "It is probable that for practical purposes the accuracy of the constants is sufficient. However, the values have been found by readings on a comparatively small number of motors and it should be desirable that others should make similar investigations, especially on machines of larger ratings." The correction factors proposed are

$$\text{slip-ring motors } 0.5\,\frac{I^2 - I_o^2}{I^2}$$

$$\text{squirrel-cage motors } \frac{I^2 - I_o^2}{I^2 DC}$$

where the currents are stator currents. The method is, otherwise, essentially the same as that proposed by Mr. Koch and it will be noted that the corrections are of such magnitude that they cannot be neglected.

The following is a brief tabulation of our tests.

Full load efficiencies and stray load losses

Motor No.		Brake values	Linckh B	Linckh C	Koch B	Koch C	Koch D
1	Stray loss watts	115.0	140.0	165.0	225.0	265.0	431.0
	% Efficiency	86.5	86.1	85.7	85.0	84.4	82.4
	% Conventional eff	88.4					
2	Stray loss watts	140.0	216.0		347.0		
	% Efficiency	86.5	85.3		83.8		
	% Conventional eff	88.7					
3	Stray loss watts	402.0	410.0		643.0		
	% Efficiency	82.3	82.2		79.5		
	% Conventional eff	88.6					
4	Stray loss in watts	157.0	170.0		270.0		372.0
	% Efficiency	85.9	85.5		84.3		83.0
	% Conventional eff	88.3					
5	Stray loss in watts	87.0	77.0		130.0		220.0
	% Efficiency	86.2	86.4		85.3		83.6
	% Conventional eff	88.3					
6	Stray loss watts	112.0	145.0		225.0		362.0
	% Efficiency	86.2	85.8		84.8		83.0
	% Conventional eff	88.1					

These results indicate that the method proposed does not give sufficiently accurate results. In several cases the conventional efficiency gave results closer to the input-output value, although the error is in the opposite direction. In general the correction factors proposed by Linckh were satisfactory, but based on the few tests available they should not be accepted as sufficiently correct without further checking.

The investigation of this method appears to have been limited to ratings below 30 hp. and according to our tests is not sufficiently accurate, particularly for the smaller ratings. The input-output test is subject to the criticism that it requires accurate test facilities and any errors in readings show up directly in the efficiency. The method proposed neither simplifies the facilities nor shortens the time required for testing, and the accuracy is not any greater. We do, however, believe that it is a step in the right direction and this paper and the resulting discussion may pave the way for a new or modified procedure. The method has not, to our knowledge, been checked in larger ratings and it may prove to be sufficiently accurate in this range where a dynamometer is not available. This can be determined only by experimental tests, and it appears that further investigation is warranted.

L. E. Hildebrand: The present conventional efficiency ignoring load loss does not give correct efficiency.

The theoretically correct input-output test requires a highly trained crew, highly developed equipment and highly developed technique to obtain satisfactory results and even then, it is expensive and subject to question.

In all practical work a conventional efficiency test *including* a conventional load loss is certainly superior to ignoring load loss. Also in the hands of an average crew with average equipment it should be superior to the impractical input-output test providing a method is used for determining a conventional load loss which gives consistently repeatable results approximately equal to the true load loss. We may as well face the fact that there has been developed no method for accurately determining the real load loss. A conventionally determined load loss should average close to the true load loss if proper account is taken of the factors which might vitiate the results.

Mr. Koch has pointed out those connected with the determination of the subtractable true secondary I^2R loss. His method for slip-ring motors is certainly essentially sound. His methods *B*, *C* and *D* all seem equally sound for single-cage motors and *C* is more convenient than *B*, so we should use either *C* or *D*. *C* is unsound for double cage and other rotors with deep bar effect. His method *D* would seem the best in general if it were not for two factors, first, the determination of the true starting torque at full load current and operating temperature is not quite as simple as it sounds due to the friction and heating factors. Second, method *C* sometimes gives closer to the true facts with single-cage motors.

It would seem that a method for determining a conventional efficiency including a conventional load loss should be legalized. Mr. Koch's proposed method *D* (with *C* as a permissible alternative for single-cage motors) seems practical for a tentative permitted method.

H. L. Barnholdt: The subject treated in Mr. Koch's paper is a very timely one and a satisfactory solution to this difficult problem is much to be desired.

That the problem of directly measuring stray load losses in induction motors is a difficult one is evidenced by the wide discrepancies shown by the test results. For squirrel-cage induction motors it is stated that method *D* is preferred, but only one test is indicated comparing it with the input-output test and while this checks quite closely, it is entirely possible that if a number of tests was made according to method *D* they would show as wide discrepancies as methods *A*, *B*, and *C*. Since the several methods admittedly are based on experimental data only, it is believed that further tests and greater consistency of test results are necessary before attempting standardization.

The method proposed for squirrel-cage motors does not take into account additional primary copper losses due to eddy currents in the primary conductors or the eddy losses induced in the end shields, etc., by the leakage fields of the primary coil ends, although the latter is admitted to be appreciable, especially in high-speed motors. Hence, even after making all these tests, the validity of the test results would still be open to question by the pump builders, particularly since high-speed squirrel-cage motors are used mainly by them.

For these reasons, it would probably be better, on smaller motors, to determine the efficiency by the input-output dynamometer method which, if properly carried out, could not be open to question. This would take care of the bulk of the motor production and would cover the class of machines where the stray load losses are the greatest.

In the case of larger induction motors, it is known that the stray load losses are relatively of less importance than on smaller machines. For example, some years ago the writer had occasion to make input-output efficiency tests of two 7,000-hp. induction motors and the efficiency by this method checked to within one-quarter of one per cent with the efficiency computed from the separate losses determined in the usual manner. Since it is impracticable to apply the dynamometer test regularly to larger motors and since here the stray load losses are relatively small, an arbitrary allowance say ¾ per cent could be made to cover stray load losses on large induction motors. Input-output tests could be made as opportunities presented themselves, with the idea of revising the allowance as soon as more test data would become available. This would have the advantage of simplicity, would inflict no hardship on anyone and would probably give results fully as accurate as those obtained by the proposed method.

C. G. Veinott: Mr. Koch is to be highly commended for his endeavor to measure the stray load loss in polyphase induction motors, however inaccurate his method may be. That such losses do actually exist there can be no doubt and it is essential for the designer to have some idea of their magnitude before they can be successfully reduced.

Suppose that the stray load losses were known, how would Mr. Koch suggest account be taken of these in calculating the performance of the motor by the equivalent circuit?

Mr. Koch recommends that either his method or some other

method for determining stray load losses be added to A.I.E.E. "conventional" efficiency calculations. At present the Institute recognizes directly measured efficiency and conventional efficiency but rule 9-302 states that "Unless otherwise specified the conventional efficiency shall be employed." The Institute thus makes no distinction between large and small motors. I think Mr. Koch's paper brings out reasons why the Institute should make a distinction between large and small motors and should, in my opinion, recommend the directly measured efficiency (input-output) for the latter. This opinion is stated on the grounds that, for small motors, stray load losses are most important, the input-output method is easier, cheaper and more reliable to make. Just where the dividing line should be put is perhaps doubtful, but it certainly should be above 3 hp.

P. L. Alger: It is very desirable to establish a standard method for determining induction motor load losses, and while Mr. Koch's proposals are fundamentally sound, there is much to be done in the way of study and tests before any method can be adopted as a standard.

Measurement of the rotational losses by driving the motor at normal speed with direct current in the stator winding is an essential element in any direct measurement of stray load loss. A difficulty arises, however, in the determination of the correct secondary I^2R loss to be subtracted from the total rotational losses. If the rotor I^2R loss is determined on the basis of the resistance corresponding to normal load slip, the value obtained is too small, since it does not include the full frequency rotor iron and copper losses. These errors are very large on deep bar or double squirrel-cage motors, and for this reason methods B and C are regarded as undesirable for general use.

While determination of the rotor losses by measurement of the standstill torque at full load current gives theoretically almost correct results, it is very difficult to carry out in practise on small motors, because of the irregularity of the torque position curve and the errors due to bearing friction.

Determination of the rotor losses by measuring the impedance watts input and subtracting from it the primary I^2R is convenient and approximately correct in the case of overhung slot motors, but it gives a value of rotor loss which is slightly too large, and therefore gives a value of stray load loss somewhat too small. This error is somewhat offset by the existence of saturation in the flux leakage paths under normal load conditions. It seems practical, therefore, to get accurate results by this method, if a simple correction factor of perhaps 10 per cent is applied to the primary copper loss, to be subtracted from the impedance watts input.

In conclusion method A should form the basis of standards for stray load loss measurement in the case of overhung primary slot motors of 1 hp. and larger, with the addition of a multiplying factor of perhaps 1.1 for the primary copper loss on the standstill impedance test.

Method D should form the basis of standards for stray load loss measurement in the case of open primary slot motors (which are generally built in ratings of 20 hp. and larger).

Direct input-output tests should remain standard for fractional hp. motors and should remain optional for larger motors.

The Induction Motor Subcommittee of the Committee on Electrical Machinery is now reviewing this problem, and plans to submit some definite recommendations on the subject to the Standards Committee in the near future.

C. J. Koch: We are fortunate in having discussion contributed by gentlemen who have had the opportunity to test in their factories the procedures described in this paper. The result of their experiences leaves in doubt the desirability of adopting the proposals of this paper at least for motors of moderate size. It is perhaps true that for motors below 50 hp. a direct input-output test on a dynamometer will give the true efficiency with sufficient accuracy and with less expense than the no-load test required to find the conventional efficiency and the separate determination of the stray load loss. For large motors the dynamometer is quite impracticable however and it is believed the proposed methods will be of real value.

A few specific questions have been asked. It is true that using any of the proposed methods for squirrel-cage motors, stray load loss due to additional eddy current losses in the stator conductors, clamping fingers, etc., will not be included as the stator is excited with direct current. These losses in modern designs are usually small but if a question arises regarding their magnitude they may be easily checked by exciting the stator with alternating current and measuring the watts loss with the rotor removed.

There is no simple way of including the stray load loss in the equivalent circuit. For any value of load it should be subtracted from the gross output of the motor along with the friction and windage losses. This is based on the assumption that the stray load loss is all increase in tooth frequency core loss. (See Alger and Eksergian.) This procedure although correct would not be really an inclusion of the stray load loss in the equivalent circuit as for other loads the equivalent circuit would not tell the magnitude of the stray load loss. This would be found by other means.

Torque-Angle Characteristics of Synchronous Machines Following System Disturbances

BY S. B. CRARY*
Associate, A.I.E.E.

and

M. L. WARING*
Associate, A.I.E.E.

Synopsis.—A method of predetermining the torque-angle characteristic of a synchronous machine following a system disturbance is of importance in calculating the transient stability limit and in evaluating the effect on this limit of additional rotor circuits or of a change in excitation. In the first part of this paper equations are derived which are of sufficient generality to evaluate such factors. Two types of system disturbances are considered:

1. Switching in or out of a connecting line,

2. Occurrence of a balanced three-phase system fault.

In the second part, an actual case of switching out a connecting line is calculated and the results compared with field tests taken on the New York Power and Light Corporation's system.

No attempt has been made to draw any general conclusions as to the effect of amortisseurs or changes in excitation. A subsequent paper will present results which have been obtained from the application of the method presented in this paper.

Introduction

THE determination of the behavior of synchronous machines during transient conditions subsequent to a system disturbance is becoming increasingly interesting and important. In determining this behavior it is essential to know the torque-angle characteristic of the given machine under conditions resulting from disturbances of various sorts. The most important of such disturbances are variations in (1) mechanical torque, (2) excitation voltage, (3) amount of external reactance, (4) system voltage. System voltage may vary either in magnitude or in angular position.

The cases of cyclic variation of impressed torque, sudden angular displacement and synchronizing out of phase have already been treated by Doherty and Nickle.[4] Equations for the damping and synchronizing components of pulsating torque caused by a given small angular pulsation of the rotor for any number of rotor circuits have been derived by Park.[2]

A problem of particular interest to operators is that of the variation in torque caused by a change in external reactance due either to the switching in or out of a line, or due to the occurrence of a system fault. This problem has been discussed in some detail by various authors[5,6,7,8] under the subject of system stability. The purpose of this paper is to establish general equations from which may be derived specific equations applying to any of the particular conditions mentioned above. Starting with the fundamental relations developed by Park,[2] general equations are derived for the positive phase sequence torque of a synchronous machine subsequent to the following system disturbances:

1. Switching in or out of a connecting line.
2. Occurrence of a balanced three-phase system fault.

The evaluation of the torque characteristics following such disturbances is of particular importance in the determination of transient stability limits. A step-by-step method of calculation is given so that the electrical torque of a machine having any number of rotor circuits can be determined at any time of its swing. The formulas and step-by-step method presented make it possible to predetermine such phenomena as that produced by amortisseur windings, high-speed excitation, etc.

Assumptions

In order to simplify the mathematical work and to permit the use of operational solutions certain assumptions which appear to be reasonable and practical have been made. These are:

1. The machine is connected to a relatively large system; *i. e.*, it is assumed that the machine terminals are connected to an infinite bus through external impedance.
2. The machine is ideal as defined in reference 1.
3. Armature and line resistance are negligible.
4. The effect of the direct-current components of transient armature current, and all induced currents resulting therefrom, may be neglected in computing the torque.
5. The component of transient armature voltage due directly to the rate of change of magnitude of the rotor flux may be neglected in comparison with the voltage due to the rotation of this flux with respect to the armature. This assumption is justifiable for all ordinary machines.
6. The variations in rotor speed due to its oscillation with respect to the connected system may be neglected in the calculation of induced voltages.

I—Mathematical Analysis

Fundamental Relations

Using the two-reaction method of analysis it is possible to determine the characteristics of a synchronous machine from a consideration of two circuits. One of these, the direct axis circuit, represents those circuits which are symmetrical about the axis of the poles and the other, the quadrature axis circuit, represents those circuits which are symmetrical about the axis between the poles.

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*Both of the Central Station Engineering Dept., General Electric Company, Schenectady, New York.

4. For references see Bibliography.

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The fundamental relations upon which are based the equations developed in this paper were derived by Park[2] and are as follows:

$$\left.\begin{aligned}\Psi_d &= G(p)E - x_d(p)i_d \\ \Psi_q &= -\,x_q(p)i_q\end{aligned}\right\} \qquad (1)$$

$$T = i_q\Psi_d - i_d\Psi_q \qquad (2)$$

$$\left.\begin{aligned} i_{do} &= \frac{E_{do} - \Psi_{do}}{x_d} \\ i_{qo} &= -\,\frac{\Psi_{qo}}{x_q}\end{aligned}\right\} \qquad (3)$$

$$\left.\begin{aligned}\Psi_d &= e\cos\delta \\ \Psi_q &= -\,e\sin\delta\end{aligned}\right\} \qquad (4)$$

$$\left.\begin{aligned}e_d &= p\Psi_d - ri_d - \Psi_q p\theta \\ e_q &= p\Psi_q - ri_q + \Psi_d p\theta\end{aligned}\right\} \qquad (5)$$

The operational impedance of the direct-axis circuit met by the current resulting from a unit function change of terminal linkages in the direct axis is called $x_d(p)$. Similarly $x_q(p)$ is the operational impedance of the quadrature axis circuit. $G(p)$ is an operator which relates the field voltage with the current and linkages in the direct axis. A method of deriving these operators from the equivalent circuits of the direct and quadrature axes has been presented.[13]

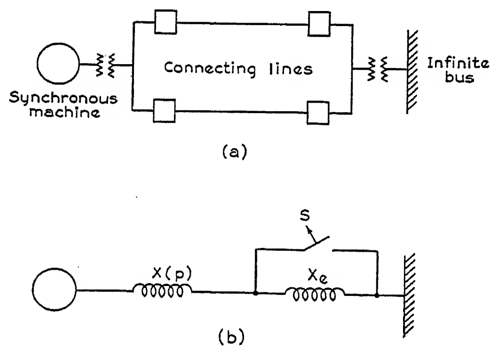

FIG. 1—SYSTEM DIAGRAM AND EQUIVALENT CIRCUIT FOR SWITCHING OPERATION

Case 1. Switching Out a Connecting Line

Consider the case of a synchronous machine connected to a very large system through two or more parallel transmission lines as shown in Fig. 1a. The machine is operating with a steady excitation voltage E_{do} and displacement angle δ_o before $t = 0$. If, at the instant $t = 0$, one of the lines is suddenly opened, there will be an instantaneous change in the amount of reactance between the machine and the infinite bus. An equivalent circuit representing this condition is shown in Fig. 1b where a reactance x_e is suddenly inserted by opening the switch S; $x(p)$ represents $x_d(p)$ for the direct and $x_q(p)$ for the quadrature axes; $x(p)$ includes the machine reactance plus the external reactance between the machine terminals and the infinite bus before the switch S is opened.

The effect of opening a switch which is carrying a current i' may be simulated by leaving the switch closed and impressing a current equal and opposite to i' through it so that by the superposition of the two, the resultant current through the switch is zero. Since no current was flowing through x_e before opening the switch the component of the impressed current which flows through x_e is the resultant current flowing between the machine and the infinite bus. We have, then:

$$i = \frac{x(p)}{x_e + x(p)}\, i' \qquad (6)$$

where

i = the value of current after opening the switch

and

i' = the value of current that would flow through the switch S if it were not opened.

The components of i' are:

$$\begin{aligned} i_d' &= i_{do} + \Delta i_d \\ i_q' &= i_{qo} + \Delta i_q \end{aligned} \qquad (7)$$

where from equation (1):

$$\begin{aligned}\Delta i_d &= -\,\frac{1}{x_d(p)}\,\Delta\Psi_d + \frac{G(p)}{x_d\,(p)}\,\Delta E \\ \Delta i_q &= -\,\frac{1}{x_q\,(p)}\,\Delta\Psi_q\end{aligned} \qquad (8)$$

Substituting equations (3) and (8) in equations (7):

$$\begin{aligned} i_d' &= \frac{E_{do} - \Psi_{do}}{x_d} - \frac{1}{x_d(p)}\,\Delta\Psi_d + \frac{G(p)}{x_d(p)}\,\Delta E \\ i_q' &= -\,\frac{\Psi_{qo}}{x_q} - \frac{1}{x_q(p)}\,\Delta\Psi_q\end{aligned} \qquad (9)$$

Equations (9) may now be substituted in equation (6) giving the direct and quadrature components of transient current of the machine subsequent to the switching operation. After re-arrangement and substitution of equations (4) in (9):

$$\begin{aligned} i_d &= \frac{E_{do} - e\cos\delta_o}{x_d}\;\frac{x_d(p)}{x_e + x_d(p)} \\ &\quad - \frac{e}{x_e + x_d(p)}\,(\cos\delta - \cos\delta_o) + \frac{G(p)}{x_e + x_d(p)}\,\Delta E \qquad (10) \\ i_q &= \frac{e\sin\delta_o}{x_q}\;\frac{x_q(p)}{x_e + x_q(p)} + \frac{e}{x_e + x_q(p)}\,(\sin\delta - \sin\delta_o)\end{aligned}$$

The operators contained in equations (10) may be expanded by Heaviside's expansion theorem when operating on a unit function. However, the second terms of both equations contain an operator which is operating on a function of the machine angle which may vary with time. If the relationship between the angle and time is known it would be a simple matter to expand

the expression by means of Duhamel's integral. However, since this relation is not generally known it is usually necessary to obtain the solution by means of a step-by-step calculation.

One form of Heaviside's expansion theorem is:

$$\frac{Y(p)}{Z(p)} = \frac{Y(o)}{Z(o)} + \left[\frac{Y(\infty)}{Z(\infty)} - \frac{Y(o)}{Z(o)}\right] \sum a_n \epsilon^{p_n t} \quad (11)$$

where

$$a_n = \frac{Z(\infty)Z(o)}{Y(\infty)Z(o) - Z(\infty)Y(o)} \cdot \frac{Y(p_n)}{p_n Z'(p_n)}$$

p_n = root of equation $[Z(p) = 0]$

and $\Sigma a_n = 1.0$.

Applying this theorem to the operators in equations (10):

$$i_d = \frac{E_{do}}{x_e + x_d} - \frac{e\cos\delta}{x_e + x_d} - \frac{E_{do} - e\cos\delta_o}{x_d} \frac{x_e(x_d - x_d'')}{(x_e + x_d)(x_e + x_d'')} \sum a_{dn}\epsilon^{-\alpha_{dn}t} - \frac{e(x_d - x_d'')}{(x_e + x_d)(x_e + x_d'')} \sum b_{dn}\epsilon^{-\alpha_{dn}t}(\cos\delta - \cos\delta_o) + \frac{G(p)}{x_e + x_d(p)} \Delta E_d$$

$$i_q = \frac{e\sin\delta}{x_e + x_q} - \frac{e\sin\delta_o}{x_q} \cdot \frac{x_e(x_q - x_q'')}{(x_e + x_q)(x_e + x_q'')} \sum a_{qn}\epsilon^{-\alpha_{qn}t} + \frac{e(x_q - x_q'')}{(x_e + x_q)(x_e + x_q'')} \sum b_{qn}\epsilon^{-\alpha_{qn}t}(\sin\delta - \sin\delta_o) \quad (12)$$

Substitution of equations (4) and (12) in equation (2) gives:

$$T = \frac{E_d e \sin\delta}{x_e + x_d} + e^2 \frac{x_d - x_q}{2(x_e + x_d)(x_e + x_q)} \sin 2\delta - \frac{E_{do}e - e^2\cos\delta_o}{x_d} \sin\delta \frac{x_e(x_d - x_d'')}{(x_e + x_d)(x_e + x_d'')} \sum a_{dn}\epsilon^{-\alpha_{dn}t}$$
$$- e^2\sin\delta \frac{x_d - x_d''}{(x_e + x_d)(x_e + x_d'')} \sum b_{dn}\epsilon^{-\alpha_{dn}t}(\cos\delta - \cos\delta_o) - \frac{e^2\sin\delta_o}{x_q}\cos\delta \frac{x_e(x_q - x_q'')}{(x_e + x_q)(x_e + x_q'')} \sum a_{qn}\epsilon^{-\alpha_{qn}t}$$
$$+ e^2\cos\delta \frac{(x_q - x_q'')}{(x_e + x_q)(x_e + x_q'')} \sum b_{qn}\epsilon^{-\alpha_{qn}t}(\sin\delta - \sin\delta_o) - \frac{e\sin\delta}{x_e + x_d} \sum c_{dn}\epsilon^{-\alpha_{dn}t} \Delta E \quad (13)$$

The above expression is the equation for the torque developed by a synchronous machine when an external reactance is suddenly switched in, accompanied by a change in angle from δ_o to δ and a change of excitation voltage ΔE. This expression for torque is general, within the limitations of the original assumptions, in that it includes the effect of any number of rotor circuits as well as any change in excitation voltage.

If the variations of δ and E as functions of time are known, Duhamel's integral may be applied and the electrical torque determined directly at any time after the disturbance. However, if they are not known, as is the more usual case, the step-by-step method of calculation presented in Appendix A may be used to determine the angular swing of the machine.

For one rotor circuit in each axis equation (13) becomes:

$$T = \frac{E_d e}{x_e + x_d} \sin\delta + e^2 \frac{x_d - x_q}{2(x_e + x_d)(x_e + x_q)} \sin 2\delta - \frac{E_{do}e - e^2\cos\delta_o}{x_d} \sin\delta \frac{x_e(x_d - x_d')}{(x_e + x_d)(x_e + x_d')} \epsilon^{-\frac{x_d + x_e}{(x_d' + x_e)T_{do}'}t}$$
$$- e^2\sin\delta \frac{x_d - x_d'}{(x_e + x_d)(x_e + x_d')} \epsilon^{-\frac{x_d + x_e}{(x_d' + x_e)T_{do}'}t}(\cos\delta - \cos\delta_o) - \frac{e^2\sin\delta_o}{x_q}\cos\delta \frac{x_e(x_q - x_q')}{(x_e + x_q)(x_e + x_q')} \epsilon^{-\frac{x_q + x_e}{(x_q' + x_e)T_{qo}'}t}$$
$$+ e^2\cos\delta \frac{(x_q - x_q')}{(x_e + x_q)(x_e + x_q')} \epsilon^{-\frac{x_q + x_e}{(x_q' + x_e)T_{qo}'}t}(\sin\delta - \sin\delta_o) \quad (14)$$

Case 1a. Constant Flux Linkages Maintained in Both Axes

In order to simplify calculations the approximations are frequently made that constant flux linkages are maintained in the predominant rotor circuit of each axis at any instant after the occurence of the disturbance and that the effect of additional rotor circuits is negligible. Such an assumption simply specifies that the predominant rotor circuit of each axis has an infinite time constant whereas all additional rotor circuits in both axes have zero time constants.

If such an assumption is made equation (13) may be greatly simplified as all of the exponentials will become either unity or zero depending upon whether the time constant of the particular circuit it represents is infinite or zero. Also $x_d'' = x_d'$; $x_q'' = x_q'$. Equation (13) becomes:

$$T = \frac{x_d' E_{do}e}{x_d(x_d' + x_e)} \sin\delta + \frac{e^2(x_d' - x_q')}{2(x_d' + x_e)(x_q' + x_e)} \sin 2\delta + \frac{e^2(x_d - x_d')}{x_d(x_d' + x_e)} \cos\delta_o \sin\delta - \frac{e^2(x_q - x_q')}{x_q(x_q' + x_e)} \sin\delta_o \cos\delta \quad (15)$$

If x_e is zero equation (**15**) corresponds to equation (**46**) of reference 4 for sudden angular displacement. Also, making $x_e = 0$, $E_{do} = e$ and $\delta_o = 0$, (the last two relations being the conditions for a machine initially open-circuited) equation (**15**) reduces to equation (**61**) of reference 4 for synchronizing out of phase.

In making stability studies involving several machines the assumptions are commonly made that $x_d' = x_q'$ and $x_d = x_q$. Equation (**15**) becomes:

$$T = \frac{x_d' \, E_{do} e}{x_d \, (x_d' + x_e)} \sin \delta + \frac{e^2 \, (x_d - x_d')}{x_d \, (x_d' + x_e)} \sin (\delta - \delta_o) \qquad \textbf{(16)}$$

By trigonometric transformations equation (**16**) may be readily shown to be equal to the more common form:

$$T = \frac{E' e}{x_d' + x_e} \sin \delta' \qquad \textbf{(17)}$$

Where E' = the voltage back of transient reactance.
δ' = the angle between E' and e.

Case 1b. Switching in a Connecting Line

If a connecting line is switched in, the equivalent one line circuit of Fig. 1b may be used by giving x_e a negative value. The above equations may therefore be applied for switching in a connecting line, as well as for switching out a connecting line, providing x_e is given the proper sign.

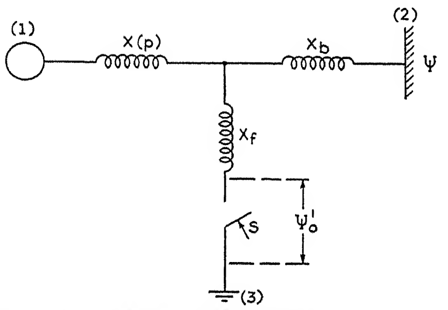

FIG. 2—EQUIVALENT CIRCUIT REPRESENTING OCCURRENCE OF BALANCED FAULT

Case 2. Occurrence of a Balanced Three-Phase System Fault

The calculation of the transient stability limit frequently involves the determination of the relation between torque, machine angle and excitation following the occurrence of a balanced three-phase system fault. Faults on a system connecting one machine and an infinite bus can be reduced to an equivalent circuit of the form shown in Fig. 2. The occurrence of the fault is represented by closing the switch, S. $x(p)$ is the machine reactance plus the reactance of that part of the equivalent Y-circuit which extends from the machine terminals to the junction point of the Y.

In keeping with assumptions 3, 4, 5, and 6, equations (**5**) reduce to the following simple relations:

$$e_d = - \Psi_q$$

$$e_q = \Psi_d$$

Switching operations may then be made by suddenly applying flux linkages equal in magnitude to the corresponding voltages. In the network of Fig. 2, the effect of closing the switch may, therefore, be simulated by leaving it open and, at the instant $t = 0$, impressing across it a value of flux linkages, Ψ_o', equal and opposite to the value that existed before $t = 0$. At $t = 0$, δ and E may also change, depending upon the conditions of the problem.

Before $t = 0$, the current flowing through x_b to the infinite bus is

$$i_{do} = \frac{E_{do} - \Psi_{do}}{x_b + x_d}.$$

$$i_{qo} = - \frac{\Psi_{qo}}{x_b + x_q} \qquad \textbf{(18)}$$

After $t = 0$, the component of current in x_b due to closing the switch is

$$i_{d1} = - \frac{\Psi_{do}'}{x_{d23}(p)} = - \frac{i_{do} \, x_b + \Psi_{do}}{x_{d23} \, (p)}{}^*$$

$$i_{q1} = - \frac{\Psi_{qo}'}{x_{q23}(p)} = - \frac{i_{qo} x_b + \Psi_{qo}}{x_{q23}(p)} \qquad \textbf{(19)}$$

The component of current in x_b due to the change in δ is:

$$i_{d2} = - \frac{\Delta \Psi_d}{x_{d22}(p)}$$

$$i_{q2} = - \frac{\Delta \Psi_q}{x_{q22}(p)} \qquad \textbf{(20)}$$

The component of current in x_b due to a change in excitation voltage is:

$$i_{d3} = \frac{G(p) \Delta E_d}{x_{d12}(p)}$$

$$i_{q3} = 0 \qquad \textbf{(21)}$$

By superposition, the resultant current in x_b, after $t = 0$, is the algebraic sum of the components given by equations (**18**) to (**21**). Then

$$i_d = \frac{E_{do} - \Psi_{do}}{x_b + x_d} - \frac{i_{do} x_b + \Psi_{do}}{x_{d23}(p)} - \frac{\Delta \Psi_d}{x_{d22}(p)} + \frac{G(p) \Delta E_d}{x_{d12}(p)}$$

$$i_q = - \frac{\Psi_{qo}}{x_b + x_q} - \frac{i_{qo} x_b + \Psi_{qo}}{x_{q23}(p)} - \frac{\Delta \Psi_q}{x_{q22}(p)} \qquad \textbf{(22)}$$

Substituting equations (**22**) and (**4**) in (**2**) and simplifying

$$T = \frac{E_{do} e \sin \delta}{x_b + x_d} \left[\frac{x_b + x_d(p)}{x_{d12}(p)} \right]$$

---

*$x_{12}(p)$, $x_{13}(p)$, and $x_{23}(p)$ used hereafter are transfer impedances in operational form. $x_{22}(p)$ is a driving point impedance. See Appendix III of reference 6.

$$- \frac{e^2 \sin\delta \cos\delta_o}{x_b + x_d} \left[1 + \frac{x_d}{x_{d23}(p)} \right]$$

$$- \frac{e^2 \sin\delta}{x_{d22}(p)} (\cos\delta - \cos\delta_o)$$

$$+ \frac{e^2 \cos\delta \sin\delta_o}{x_b + x_q} \left[1 + \frac{x_q}{x_{q23}(p)} \right]$$

$$+ \frac{e^2 \cos\delta}{x_{q22}(p)} (\sin\delta - \sin\delta_o) + e \sin\delta \frac{G(p)}{x_{d12}(p)} \Delta E_d \qquad (23)$$

If the operators in the above expression are expanded by theorem (11), the equation for transient torque becomes

$$T = \frac{E_d e \sin\delta}{x_{d12}} + e^2 \frac{x_d - x_q}{2\, x_{d12} x_{q12}} \sin 2\delta$$

$$+ \frac{E_{do} e \sin\delta}{x_b + x_d} \frac{x_b (x_d - x_d'')}{x_{d12} x_{d13}''} \sum a_{dn} \epsilon^{-\alpha_{dn} t}$$

$$+ \frac{e^2 \sin\delta \cos\delta_o}{x_b + x_d} \frac{x_d (x_d - x_d'')}{x_{d12} x_{d13}''} \sum b_{dn} \epsilon^{-\alpha_{dn} t}$$

$$- e^2 \sin\delta \frac{x_d - x_d''}{x_{d12} x_{d12}''} \sum c_{dn} \epsilon^{-\alpha_{dn} t} (\cos\delta - \cos\delta_o)$$

$$- \frac{e^2 \cos\delta \sin\delta_o}{x_b + x_q} \frac{x_q (x_q - x_q'')}{x_{q12} x_{q13}''} \sum a_{qn} \epsilon^{-\alpha_{qn} t}$$

$$+ e^2 \cos\delta \frac{x_q - x_q''}{x_{q12} x_{q12}''} \sum b_{qn} \epsilon^{-\alpha_{qn} t} (\sin\delta - \sin\delta_o)$$

$$- \frac{e \sin\delta}{x_{d12}} \sum d_{dn} \epsilon^{-\alpha_{dn} t} \Delta E_d \qquad (24)$$

The above expression is the equation for the electrical torque developed by a synchronous machine subsequent to the occurrence of a balanced fault on the system accompanied by a change in angle from δ_o to δ and in excitation of ΔE. As in *Case 1* the electrical torque can be determined at any time of the rotor swing either by use of the step-by-step process or by the application of Duhamel's integral.

When x_f includes the equivalent reactance for unbalanced faults determined by the method of symmetrical components,[6,8] equation (25) will give the electrical torque if the additional assumption is made that the negative and zero phase sequence losses are negligible. If these losses are appreciable their effect may be included by the method given in Appendix B.

Case 2a. Constant Flux Linkages Maintained in Both Axes

As in *Case 1a* calculations may be greatly simplified, when justifiable, by making the assumption that the predominant rotor circuit of each axis has an infinite time constant and that all additional rotor circuits have zero time constants. If this assumption is made equation (24) may be simplified to the following expression:

$$T = \frac{E_{do} e}{x_{d12}'} \frac{x_b + x_d'}{x_b + x_d} \sin\delta + \frac{e^2}{2} \frac{x_d' - x_q'}{x_{d12}' x_{q12}'} \sin 2\delta$$

$$+ \frac{e^2}{x_{d12}'} \frac{x_d - x_d'}{x_b + x_d} \sin\delta \cos\delta_o$$

$$- \frac{e^2}{x_{q12}'} \frac{x_q - x_q'}{x_b + x_q} \cos\delta \sin\delta_o \qquad (25)$$

If $x_b = 0$ and $x_f = \infty$ equation (25) corresponds to equation (46) of reference 4 for sudden angular displacement.

II—Application of Mathematical Analysis

In the latter part of 1929, field tests* were made to determine the effect of amortisseur windings on the transient behavior of waterwheel generators following

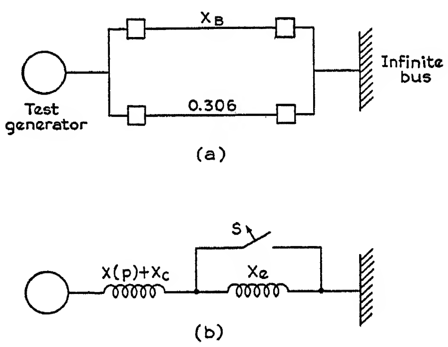

FIG. 3—TEST ARRANGEMENT AND EQUIVALENT CIRCUIT FOR SWITCHING OPERATION

switching disturbances. Two machines were available for testing purposes; one had no additional rotor circuits other than a top field collar, and the other was provided with an incomplete copper amortisseur. The machines were otherwise identical. The results of these tests provide a means of checking the mathematical analysis given in Part I.

The machine being tested was delivering power to a relatively large 110-kv. system through several circuits. A disturbance was created by opening all the circuits but one, which is a condition similar to that of *Case 1* in the first part of this paper.

In order to make use of equation (13) two assumptions as to the test set-up were necessary. These are:

a. The point of interconnection with the 110-kv. system can be considered as an infinite bus. Actually the infinite bus is farther back in the system. To

*A companion paper, *Field Tests to Determine Damping Characteristics of Synchronous Generators*, by F. A. Hamilton, Jr., see page 775 of this issue, describes the test machines, set-up, and results.

TABLE I—SWING CURVE CALCULATION OF MACHINE WITH AMORTISSEUR

	Before 0	After 0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$15.13\ T_a$	0	0	2.098	2.602	0.4418	−1.636	−2.840	−2.763	−1.610	0.0290	1.439	2.135
$\Delta\delta$	0	0	2.098	4.700	5.142	3.506	0.666	−2.097	−3.707	−3.678	−2.239	−0.104
δ	19.62	19.62	21.718	26.418	31.560	35.066	35.732	33.635	29.928	26.250	24.011	23.907
2δ	39.24	39.24	43.44	52.84	63.120	70.13	71.46	67.27	59.86	52.50	48.02	47.81
$\sin\delta$	0.3358	0.3358	0.3701	0.4450	0.5234	0.5746	0.5840	0.5540	0.4989	0.4423	0.4069	0.4053
$\sin 2\delta$	0.6326	0.6326	0.6876	0.7970	0.8920	0.9405	0.9481	0.9223	0.8648	0.7939	0.7434	0.7409
$\cos\delta$	0.9419	0.9419	0.9290	0.8956	0.8521	0.8185	0.8118	0.8325	0.8666	0.8969	0.9135	0.9142
$\cos\delta_n - \cos\delta_{n-1}$	0	0	−0.0129	−0.3334	−0.0435	−0.0336	−0.0067	0.0208	0.0341	0.0302	0.0166	0.0007
$B = -A(\cos\delta_n - \cos\delta_{n-1})$	0	0	0.0207	0.0154	0.0138	0.0129	0.0123	0.0118	0.0113	0.0108	0.0104	0.0100
$A_n = 1.599$				0.0535	0.0397	0.0356	0.0333	0.0317	0.0304	0.0292	0.0280	0.0269
$A_{n+1} = 1.187$					0.0695	0.0516	0.0463	0.0433	0.0412	0.0395	0.0379	0.0364
$A_{n+2} = 1.064$						0.0538	0.0399	0.0358	0.0335	0.0319	0.0306	0.0293
$A_{n+3} = 0.9959$							0.0107	0.0079	0.0071	0.0066	0.0063	0.0061
$A_{n+4} = 0.9475$								−0.0332	−0.0246	−0.0221	−0.0207	−0.0197
$A_{n+5} = 0.9082$									−0.0545	−0.0405	−0.0363	−0.0340
$A_{n+6} = 0.8722$										−0.0483	−0.0359	−0.0322
$A_{n+7} = 0.8374$											−0.0265	−0.0197
$A_{n+8} = 0.8048$												−0.0011
C	1.317	0.9597	0.9802	0.9863	0.9896	0.9920	0.9940	0.9957	0.9973	0.9989	1.000	1.002
$B + C$	1.317	0.9597	1.0009	1.0552	1.1126	1.1459	1.1365	1.0930	1.0417	1.0060	0.9938	1.004
$a = (B + C)\sin\delta$	0.4422	0.3222	0.3704	0.4695	0.5823	0.6584	0.6637	0.6055	0.5197	0.4449	0.4044	0.4069
$b = 0.2149 \sin 2\delta$	0.2496	0.1359	0.1478	0.1713	0.1917	0.2021	0.2037	0.1982	0.1858	0.1706	0.1598	0.1592
$\sin\delta_n - \sin\delta_{n-1}$	0	0	0.0343	0.0749	0.0784	0.0512	0.0094	−0.0300	−0.0550	−0.0567	−0.0354	−0.0016
$E = D(\sin\delta_n - \sin\delta_{n-1})$	0	0	0.0107	0.0022	0.0005	0.0001	0.0000	0	0	0	0	0
$D_n = 0.3135$				0.0235	0.0049	0.0011	0.0003	0.0001	0	0	0	0
$D_{n+1} = 0.0654$					0.0246	0.0051	0.0012	0.0003	0.0001	0	0	0
$D_{n+2} = 0.0153$						0.0160	0.0033	0.0008	0.0002	0	0	0
$D_{n+3} = 0.0036$							0.0029	0.0006	0.0001	0	0	0
$D_{n+4} = 0.0009$								−0.0094	−0.0020	−0.0005	−0.0001	−0.0
$D_{n+5} = 0.0002$									−0.0173	−0.0036	−0.0008	−0.0002
$D_{n+6} = 0.0000$										−0.0178	−0.0037	−0.0009
											−0.0111	−0.0023
												−0.0050
F	0	−0.0471	−0.0098	−0.0022	−0.0005	−0.0001	−0.0001	0.0000	0	0	0	0
$E + F$	0	−0.0471	0.0009	0.0235	0.0295	0.0222	0.0076	−0.0076	−0.0189	−0.0219	−0.0157	−0.0084
$c = (E + F)\cos\delta$	0	−0.0444	0.0008	0.0210	0.0251	0.0182	0.0062	−0.0063	−0.0164	−0.0196	−0.0143	−0.0077
$T_e = a + b + c$	0.6918	0.4137	0.5190	0.6618	0.7991	0.8787	0.8736	0.7974	0.6891	0.5959	0.5499	0.5584
$T_a = 0.691 - T_e$	0	0.2773	0.1720	0.0292	−0.1081	−0.1877	−0.1826	−0.1064	0.0019	0.0951	0.1411	0.1326

simplify calculations the finiteness of the bus was neglected with the knowledge that the value used for x_e would be too small.

b. The circuits opened during the test can be considered and treated as one equivalent circuit. These circuits actually consisted of a system network with other connected machines. The reactance of this equivalent circuit was then determined by the division of power. Before switching, the power factors of the two circuits were practically the same, so that this assumption is reasonable.

These assumptions made it possible to reduce the system to the one-line diagram in Fig. 3a which may be represented by the equivalent circuit in Fig. 3b. The rating of the test machines (9,000 kva.) was selected as the base per unit kva.

Check of Power Output of Machine Equipped with Incomplete Copper Amortisseur

The following operational impedances of the machine equipped with an amortisseur were determined from the design data:

$$x_d(p) = 0.91 - \frac{1339p^5 + 778.2p^4 + 148.2p^3 + 10.98p^2 + 0.2545p}{1730p^5 + 1030p^4 + 202p^3 + 15.6p^2 + 0.396p + 0.000361}$$

$$x_q(p) = 0.524 - \frac{122.3p^3 + 32.4p^2 + 2.134p}{783.7p^3 + 263.2p^2 + 28.09p + 0.9202}$$

It was not necessary to determine $G(p)$ as the tests were made with constant field voltage ($\Delta E_d = 0$).

The inertia constant H was calculated from the design data and is equal to 1.79. Also $E_d = 1.19$, $e = 1.06$, $x_b = 0.058$, $x_c = 0.049$, $x_e = 0.257$ and initially $\delta_o = 19.62°$ and $T = 0.691$.

Substituting the machine and system impedances and the initial quantities in equation (**13**) the expression for torque output was obtained. The swing curve work sheet was set up (Table I) and the step-by-step calculations carried out according to Appendix A.

Fig. 4 presents a comparison between the calculated and test curves of electrical power output. The calculated initial drop in power was not quite as great as the test drop in power. The calculated oscillations are damped out somewhat faster than the test oscillations. Both of these errors can be largely accounted for by the fact that the assumed infinite bus is actually finite, and that the armature circuit resistance has been neglected.

Fig. 5 shows the calculated torque angle ellipse which describes the oscillation of the rotor for the first two swings. The marked points on the ellipse are points of equal time intervals.

Check of Power Output of Machine with No Additional Rotor Circuits Other Than a Top Field Collar

From the design data of the second generator the following operational impedances were determined:

$$x_d(p) = 0.91 - \frac{0.7329p^2 + 0.1298p}{p^2 + 0.1994p + 0.0001940}$$

$$x_q(p) = 0.524$$

Also $H = 1.79$, $E_d = 1.19$, $e = 1.06$, $x_b = 0.046$, $x_c = 0.04$, $x_e = 0.266$ and initially $\delta_o = 19.36°$ and $T = 0.693$.

In a manner similar to that described in the preceding case, the swing of the rotor and the output of the machine were calculated. See Fig. 6 for a comparison between the test and calculated values of power output. Fig. 7 is the torque angle characteristic of this machine for the first two oscillations.

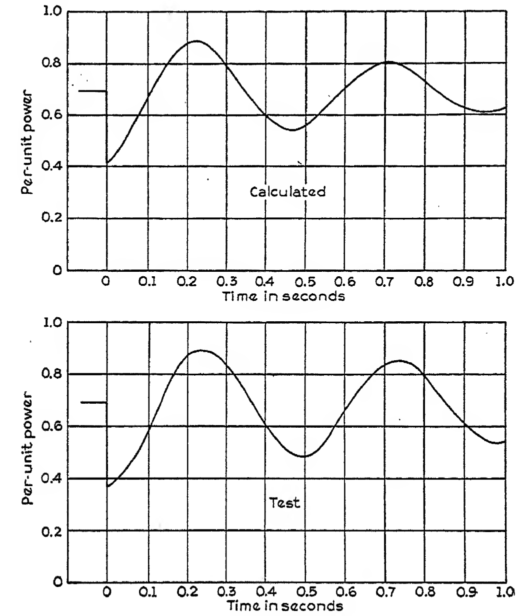

FIG. 4—COMPARISON OF CALCULATED AND TEST VALUES OF TRANSIENT POWER (WITH AMORTISSEUR)

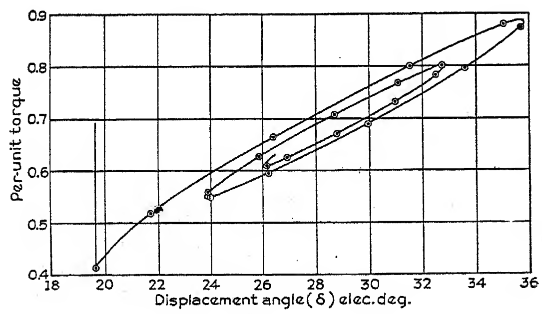

FIG. 5—CALCULATED TORQUE-ANGLE CHARACTERISTIC (WITH AMORTISSEUR)

CONCLUSION

Equations have been derived in this paper by which the transient behavior of a synchronous machine following the common types of system disturbances may be calculated. They are of sufficient generality that the effect of amortisseur windings or of excitation may be evaluated. Only such assumptions have been made in their derivation as are necessary to reduce them to a practical basis. If additional assumptions are made the equations may be reduced to the less general but more familiar equations of electrical torque derived by other authors. A comparison of the calculated and test results shows good agreement. The general equations may also be used as a basis for the derivation of more specific equations applying to particular conditions.

Appendix A

STEP-BY-STEP METHOD

Step-by-step methods have been developed and explained in several other papers.[5,6,7] However a method well adapted for use with the general torque equations given in Part I is obtained from Duhamel's theorem* expressed as a summation rather than as an integral.

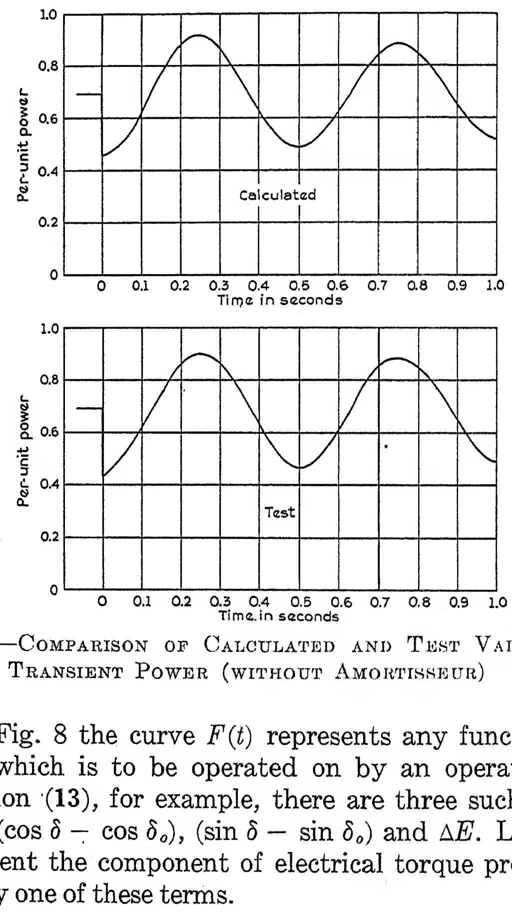

FIG. 6—COMPARISON OF CALCULATED AND TEST VALUES OF TRANSIENT POWER (WITHOUT AMORTISSEUR)

In Fig. 8 the curve $F(t)$ represents any function of time which is to be operated on by an operator. In equation (**13**), for example, there are three such functions ($\cos \delta - \cos \delta_o$), ($\sin \delta - \sin \delta_o$) and ΔE. Let $T(t)$ represent the component of electrical torque produced by any one of these terms.

Then

$$T(t) = f(p)F(t) \qquad \textbf{(1a)}$$

where $f(p)$ = an operator

Let

$$F(\Delta t)_n = F(n\Delta t) - F[(n-1)\Delta t] \qquad \textbf{(2a)}$$

The curve $\delta(t)$ may be approached by assuming that at $t = 0$ a unit function $\delta(o)$ is suddenly impressed. After a time interval (Δt) a second unit function

*See Chapter XIV of "Heaviside's Operational Calculus," by E. J. Berg.

$\delta(\Delta t)_1$ is added. At the end of a second time interval a third unit function $\delta(\Delta t)_2$ is added, etc. The value of the function $T(t)$ at any time t is then equal to the sum of the torques due to $\delta(o)$ beginning at $t = 0$, $\delta(\Delta t)_1$ beginning at $t = \Delta t$, $\delta(\Delta t)_2$ beginning at $t = 2\Delta t$, etc. The torque due to $\delta(o)$ is $\delta(o) \cdot \phi(t)$ where $\phi(t)$ is the solution with unit function impressed at $t = 0$. The torque due to $\delta(\Delta t)_1$ is $\delta(\Delta t)_1 \cdot \phi(t - \Delta t)$. The torque due to $\delta(\Delta t)_2$ is $\delta(\Delta t)_2 \cdot \phi(t - 2\Delta t)$, etc.

$$\therefore T(t) = \delta(o) \cdot \phi(t) + \sum_{n=1}^{n=\infty} \delta(\Delta t)_n \cdot \phi(t - n\Delta t) \qquad \textbf{(3a)}$$

Equation (**3a**) provides a step-by-step method for obtaining the expansion of any terms containing operators operating on functions of the machine angle. All other terms containing operators operating on unit function alone may be expanded directly by equation (**11**). The procedure from this point on is similar to the

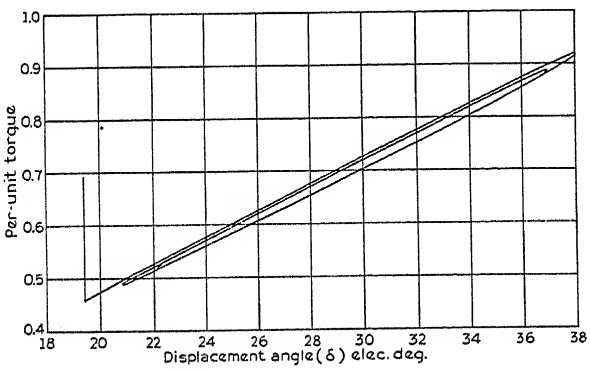

FIG. 7—CALCULATED TORQUE-ANGLE CHARACTERISTIC (WITHOUT AMORTISSEUR)

ordinary swing curve calculation using the following equations given in Appendix IV of reference 6.

$$T_a = T_m - T_E$$

$$\Delta\delta_1 = \frac{kT_o}{2}$$

$$\delta_1 = \delta_o + \Delta\delta_1$$

$$\Delta\delta_2 = \Delta\delta_1 + kT_1$$

$$\delta_2 = \delta_1 + \Delta\delta_2, \text{ etc.}$$

The details of the calculation may best be understood by reference to the sample swing curve calculation given in Part II.

Appendix B

METHOD FOR DETERMINING THE TORQUE CHARACTERISTICS WHEN RESISTANCE IS INCLUDED

It may be desirable to determine the influence of line and armature resistances upon the torque characteristic. Cases also arise in which the negative phase and zero phase sequence losses may be appreciable; as, for example, in the case of a machine having a high resistance amortisseur, or of a transformer having its neutral grounded through a resistor. The following method of analysis may be used to evaluate such factors.

If assumptions 4, 5, and 6 are made, equation (**5**) becomes

$$e_d = -ri_d - \Psi_q$$
$$e_q = -ri_q + \Psi_d \qquad \textbf{(1b)}$$

The effect of armature and line resistances may then be included in equivalent circuits for the direct and quadrature axes as shown in Fig. 9. Since an induced voltage and its corresponding flux linkages are in time

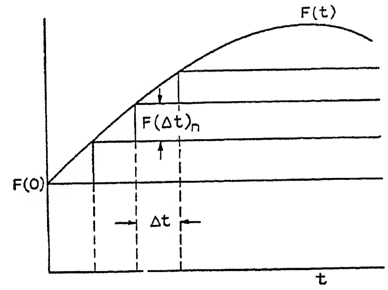

FIG. 8—ILLUSTRATION OF STEP-BY-STEP PROCESS

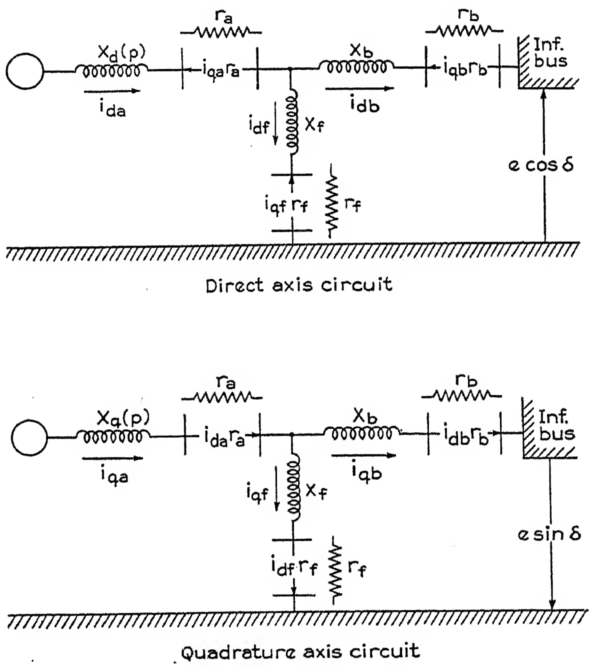

FIG. 9—EQUIVALENT FLUX LINKAGE CIRCUITS INCLUDING EFFECT OF ARMATURE CIRCUIT RESISTANCE

quadrature with each other, current flowing through a resistance in one axis causes an equivalent flux linkage drop in the other axis. The directions in which these equivalent flux linkage drops are acting are indicated by arrows.

As a simple example assume that $x_f = \infty$, $x_b = 0$, $r_b = 0$, constant flux linkages are maintained in the main field winding and there are no additional rotor circuits. Then

$$x_d(p) = x_d'$$
$$x_q(p) = x_q' = x_q$$

From Fig. 9 the following relations can be written

$$\begin{aligned} \Psi_d' - e\cos\delta &= i_q r + i_d x_d' \\ e\sin\delta &= i_q x_q - i_d r \end{aligned} \qquad \textbf{(2b)}$$

where

Ψ_d' = constant value of flux linkages in the main field.

Solving equations (**2b**) simultaneously

$$i_d = \frac{\Psi_d' x_q - e x_q \cos\delta - er\sin\delta}{x_d' x_q + r^2}$$

$$i_q = \frac{\Psi_d' r - er\cos\delta + e x_d' \sin\delta}{x_d' x_q + r^2} \qquad \textbf{(3b)}$$

The electrical torque received at the infinite bus may be obtained by substitution of equations (**3b**) and (**4**) in (**2**). The electrical torque output of the machine can be determined by adding this result to the resistance loss $r\ (i_d^2 + i_q^2)$, since per unit torque equals per unit power in accordance with assumption 6.

When r_f includes the machine negative phase sequence resistance (r_2), half of the negative phase resistance loss in the rotor is supplied mechanically by the machine shaft and half electrically by the system. The net mechanical torque input to the machine is then equal to the total shaft torque minus one-half the total negative phase sequence loss in the rotor of the machine. The net accelerating torque on the rotor of the machine is, then, the difference between the net mechanical torque input and the positive phase sequence electrical torque output.

The zero phase sequence loss can be determined by including in r_f the proper components of zero phase sequence resistance and by evaluating the zero phase components of i_{df} and i_{qf} that flow through them. Then

$$P_o = r_o\ (i_{dfo}^2 + i_{qfo}^2) \qquad \textbf{(4b)}$$

Appendix C

Determination of Damping Torque from Area of Torque-Angle Ellipse for Small Oscillations

From reference 2 for small oscillations:

$$\Delta T = T_s \Delta\delta + T_d p \Delta\delta \qquad \textbf{(1c)}$$

Since the oscillation of one machine with respect to a system is nearly harmonic, the angle of oscillation about the mean may be approximately expressed by

$$\Delta\delta = -\ [\Delta\delta]\cos st \qquad \textbf{(2c)}$$

where $[\Delta\delta]$ is the maximum deviation and $s/2\pi$ is the frequency of oscillation.

Substituting equation (**2c**) in (**1c**)

$$\Delta T = -\ T_s[\Delta\delta]\cos st + sT_d[\Delta\delta]\sin st \qquad \textbf{(3c)}$$

The area of the ellipse is:

$$A = \int_{\theta=0}^{\theta=2\pi} \Delta T d(\Delta\delta) \qquad \textbf{(4c)}$$

where $\theta = st.$

Substituting (**3c**) in (**4c**):

$$A = \int_0^{2\pi} \{ -\ T_s[\Delta\delta]\cos\theta + sT_d[\Delta\delta]\sin\theta \}\ [\Delta\delta]\sin\theta\, d\,\theta \qquad \textbf{(5c)}$$

which reduces to $A = \pi s T_d [\Delta\delta]^2$

giving

$$T_d = \frac{A}{\pi s\ [\Delta\delta]^2} \qquad \textbf{(6c)}^*$$

Nomenclature

All quantities are expressed as a fraction of their normal values which are per unit values as defined in reference 3. The subscripts d and q refer to the direct and quadrature axes respectively. The subscript o specifies initial steady state conditions. The coefficient Δ denotes the increment change of the quantity which it precedes. An attempt has been made to follow established nomenclature as closely as possible.

e = infinite bus voltage.
E = nominal or excitation voltage.
Ψ = flux linkages of infinite bus.
i = armature current.
$x_d(p)$ = operational impedance of the direct axis circuit met by the current resulting from a unit function change of terminal linkages in the direct axis.
$x_q(p)$ = operational impedance of quadrature axis circuit.
$G(p)$ = an operator which relates field voltage with current and linkages in the direct axis.
p = an operator d/dt.
δ = displacement angle of the nominal voltage from the infinite bus voltage expressed in electrical radians. It is the actual space phase lag or lead of the rotor.
s = per unit frequency of rotor oscillations.
θ = angle in electrical degrees between the axis of the rotor and a stationary axis in space.
t = time expressed as a fraction of the time to pass one electrical radian at fundamental frequency. Unit time is thus $1/2\pi f$.
n = a subscript denoting any field circuit, also denoting any time interval.
r = armature circuit resistance.
T_o' = open circuit time constant in radians.
x† = synchronous reactance.
x'† = transient reactance.
x''† = sub-transient reactance.

*This expression was derived by I. H. Summers.

†Note: These values of reactance include certain reactances external to the machine terminals which differ depending upon the conditions of the case considered.

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Discussion

J. W. Butler: Messrs. Crary and Waring have done a creditable piece of work in making available equations for determining the transient characteristics of synchronous machines, giving in particular, due consideration to every rotor circuit. The power oscillations were checked accurately by a step-by-step solution using the general torque equation (13) of the text. The solution was perfectly general within the assumptions, without dividing the torque into its components, generally spoken of as synchronizing and damping torques.

However, if it is desired to calculate the damping torque for comparatively large sustained oscillations, it can be obtained from the general equation (13) as follows:

Let the change in the angle δ, be of the form

$$\Delta\delta = [\Delta\delta] \sin st$$

Then, by use of the superposition theorem, the response of the system can be found to this variation in angle. In order to integrate the resultant expression, represent the sine and cosine terms, by the first two terms of the series, such as

$$\cos\,[(\Delta\delta)\sin st] = 1 - \frac{[(\Delta\delta)\sin st]^2}{2\,!}$$

$$\sin\,[(\Delta\delta)\sin st] = (\Delta\delta)\sin st - \frac{[(\Delta\delta)\sin st]^3}{3\,!}$$

Using these forms in the transient terms of equation (13), due to the unit function change in δ, and applying the superposition theorem there is obtained an expression for torque, for a sinusoidal change in angle. Due to the fact that a sustained oscillatory condition is being considered only the sustained terms are to be retained in this expression. Integrating the sustained terms, with respect to st, from o to 2π, the area of the torque angle ellipse is obtained. Dividing the resultant expression for area by $\pi s\,(\Delta\delta)^2$ as shown in Appendix C, the expression for the damping torque coefficient is obtained. The above procedure was carried out and is too bulky to include here but the final result is

$$T_d = e^2\,\frac{x_d - x_d''}{x_d x_d''}\sum b_{dn}\alpha_{dn}\left\{\frac{\sin^2\delta_o'}{\alpha_{dn}^2+s^2}\left[1-\frac{(\Delta\delta)^2}{8}\right]\left[1-\frac{(\Delta\delta)^3}{6\,(\alpha_{dn}^2+9s^2)}\left(\frac{3\alpha_{dn}^2+3s^2}{4}+6s^2\right)\right] + \frac{(\Delta\delta)^2}{4}\,\frac{\cos^2\delta_o'}{\alpha_{dn}^2+4s^2}\left[1-\frac{(\Delta\delta)^2}{12}\right]\right\}$$
$$+\,e^2\,\frac{(x_q - x_q'')}{x_q x_q''}\sum b_{qn}\alpha_{qn}\left\{\frac{\cos^2\delta_o'}{\alpha_{qn}^2+s^2}\left[1-\frac{(\Delta\delta)^2}{8}\right]\left[1-\frac{(\Delta\delta)^3}{6(\alpha_{qn}^2+9s^2)}\left(\frac{3\alpha_{qn}^2+3s^2}{4}+6s^2\right)\right] + \frac{(\Delta\delta)^2}{4}\,\frac{\sin^2\delta_o'}{\alpha_{qn}^2+s^2}\left[1-\frac{(\Delta\delta)^2}{12}\right]\right\}$$

It is of interest to note that this expression, letting $[\Delta\delta] = o$, checks Park's results in reference 2, derived in a different manner.

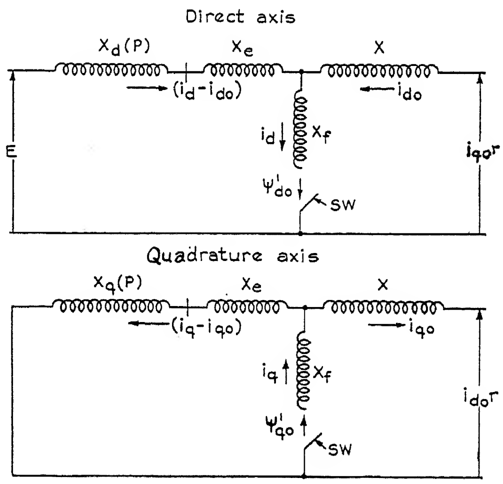

FIG. 1—EQUIVALENT CIRCUIT OF A GENERATOR FEEDING AN IMPEDANCE LOAD

Closing switch *SW* simulates a three-phase fault on feeder

Another thing which I feel should be emphasized is the equivalent circuit shown in Fig. 9. It can be used to calculate any sort of a transient in a machine, including resistance, within the assumptions made. One particular use I have made of the circuit is in calculating decrement curves, for a fault on a generator feeding an impedance load, considering the effect of the load resistance in the time constants and magnitudes of the current. For this application the circuit reduces to that shown in Fig. 1.

Relative to Fig. 1, the following linkage equations may be written.

$$\Psi_{do}' + i_{qo}r = xi_{do} + i_d x_f$$
$$\Psi_{do}' = (i_d - i_{do})\,[x_e + x_d\,(p)] + i_d x_f$$
$$\Psi_{qo}' - i_{do}r = i_q x_f + i_{qo}x$$
$$\Psi_{qo}' = i_q x_f + (i_q - i_{qo})\,[x_e + x_q\,(p)]$$

where

i_d = fault current direct axis
i_{do} = load current direct axis
i_q = fault current quadrature axis
i_{qo} = load current quadrature axis

x = load reactance
r = load resistance
x_f = fault reactance
x_e = external reactance
$x(p)$ = operational impedance of generator
Ψ' = linkage at point of fault, before fault occurred.

Solving these, there is obtained for the fault currents

$$i_d = \frac{\Psi_{do}' \left[x\,(x+a)\,x_{q31}\,(p) + r^2\,(x_f+b)\right] + \Psi_{qo}' rab}{D(p)}$$

$$i_q = \frac{\Psi_{qo}' \left[x\,(x+b)\,x_{d31}\,(p) + r^2\,(x_f+a)\right] - \Psi_{do}' rab}{D(p)}$$

where

$$a = x_e + x_d\,(p)$$
$$b = x_e + x_q\,(p)$$
$$D(p) = x^2 x_{d31}\,(p)\,x_{q31}\,(p) + r^2\,(x_f+a)\,(x_f+b)$$

Solving these operational equations by the expansion theorem there is obtained

$$i_d = I_d + (I_d'' - I_d) \sum b_{dn}\,\epsilon^{-\alpha_n t}$$
$$i_q = I_q + (I_q'' - I_q) \sum b_{qn}\,\epsilon^{-\alpha_n t}$$
$$\alpha_n = \text{roots of } D(p) = o$$
$$b_n = \frac{1}{(I'' - I)}\;\frac{Y\,(\alpha_n)}{\alpha_n\,Z'\,(\alpha_n)}$$
$$\sum b_n = 1.0$$

From these equations the effect of the resistance in the load, which is generally neglected, can be readily determined.

S. H. Wright: A statement of particular importance to the utility engineer is that in the synopsis of the article by Messrs. Crary and Waring, namely, "No attempt has been made to draw any general conclusions as to the effect of amortisseurs . . . . A subsequent paper will present results which have been obtained from the application of the method presented in this paper." This rather suggested that the future paper may discuss the more general problem of determining the most suitable type of damper winding for specific applications. In this regard, the following discussion is thought to be pertinent.

Certain of the prime advantages of low and high resistance dampers are as follows:

1. Low-resistance dampers tend rapidly to damp out swings which occur during symmetrical circuit conditions (that is where current magnitudes are equal in the three phases).

2. For two-machine stability problems (or multi-machine problems, where there are no serious compound oscillations) high-resistance dampers give higher transient stability limits, especially for the longer clearing times, such as greater than 16 cycles where $f = 60$ cycles per sec. This was brought out by C. F. Wagner, (see item 11 of bibliography of Crary and Waring paper, particularly Fig. 3 of this reference). However, this gain is apparently negligible for fault durations of the order of 8 cycles or less, and 8 cycles clearing time is being successfully obtained with modern high-speed circuit breakers and relays. The higher the speed of clearing any type of line fault, the more the entire occurrence approaches merely the condition of switching out a circuit.

In years to come it is likely that the average high-voltage system will reduce its clearing time to 8 cycles or less and it is possible that at the same time, interconnections and other factors will make compound system oscillations more important; in such a case the greater damping ability, under symmetrical circuit conditions, of low-resistance dampers would become of paramount importance.

S. B. Crary: Mr. Butler has made an interesting application of the general torque equations. His solution is limited to a forced oscillation such as is obtained in a compressor drive and is interesting in that it can be applied to large angular deviations.

Mr. Wright's conclusions concerning the influence of decreased switching times on the gains to be realized from high and low resistance amortisseur windings correspond with our opinions. When the switching times are very short, a high resistance amortisseur is of slight benefit in improving stability limits. On the other hand, the low resistance amortisseur is of aid in damping out all angular oscillations and is generally beneficial.

Field Tests to Determine the Damping Characteristics of Synchronous Generators

BY F. A. HAMILTON, Jr.*

Associate, A.I.E.E.

Synopsis.—Practical system tests of the damping action of generators with and without low resistance damper windings, but otherwise identical, have been made. The effects of normal variations of generator field circuit resistance, field current, and loading, were investigated. Data for checking calculated machine constants also were obtained. The tests results are valuable from a quantitative, as well as from a qualitative standpoint.

* * * * *

Introduction

In the School Street Station of the New York Power and Light Corporation there are two waterwheel-driven generators which, originally, were identical 40-cycle units. Early in 1927 one of these machines was rebuilt for 60-cycle operation. Later in the same year the rebuilding of the companion unit was undertaken.

Meanwhile arguments for and against the use of damper windings were advanced; it was becoming apparent that comparative tests on an actual system

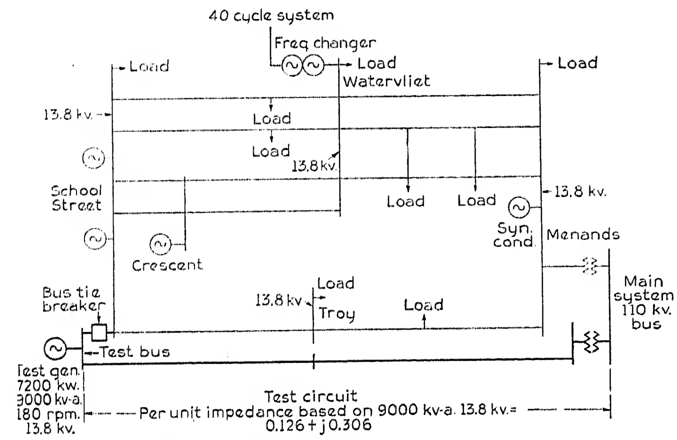

Fig. 1—Arrangement of Circuits and Equipment

might help to determine their effect upon transient power limits.

The idea of adding damper windings to the second machine to be rebuilt for 60-cycle operation, and of comparative tests on this and on the machine already rebuilt without them, was broached to the New York Power and Light Corporation. As a result, low-resistance damper windings were installed in the second machine to be rebuilt, and plans for testing were developed.

Force of circumstances ultimately prevented extensive tests to establish relative power limits for various types of short circuits of different durations. But less severe tests to check the validity of existing methods of analysis, and to further the development of damping theory, were finally completed late in 1929.

This paper describes these tests, and includes a resumé of the data obtained. Evidence of the accuracy of available methods of calculating machine constants is presented. The effectiveness of damper windings in rapidly smoothing out balanced disturbances is shown. In a companion paper, a method of predicting generator behavior during disturbances is developed, and calculations based on this method verified by comparison with the test data.[1]

Description of Tests

The circuits, and the major equipment employed, were arranged as in Fig. 1.

Both test machines were rated 7,200 kw., 9,000 kva., 180 r.p.m., 13,800 volts, 60-cycles. Both machines had

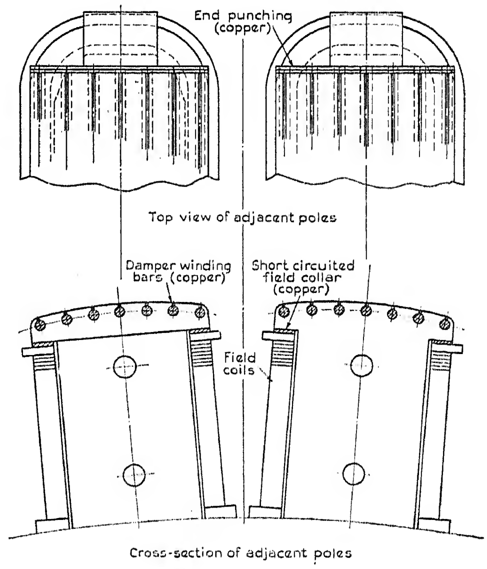

Fig. 2—Type of Damper Windings Employed

top field collars. One had no damper windings. The other was equipped with low-resistance damper windings of the type shown in Fig. 2.

Each test machine was subjected to a series of tests, those made on one machine being duplicated as nearly as possible on the other.

The first item in Table I refers to tests which were made to obtain data for checking calculated machine

*Central Station Engg. Dept., General Electric Co., Schenectady, N. Y.

Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

1. For references see Bibliography.

TABLE I—SUMMARY OF TESTS

Item	Type of test	Damper windings	Initial and final load kw.	*Initial and final field current amps.	Final power factor	Field circuit resistance ohms	T_d (see Appendix)
1	Machine constant	Yes	0	15	0	0.8	
		No	0	15	0	0.8	
2	Damping	Yes	6,300	285	0.985 lag	0.8	5.78
		No	6,300	290	0.991 lag	0.8	1.69
3	Damping	Yes	6,300	260	0.995 lag	0.8	5.34
		No	6,300	265	0.996 lag	0.8	2.02
4	Damping	Yes	6,300	240	1.0	0.8	5.34
		No	6,300	255	1.0	0.8	1.95
5	Damping	Yes	6,300	225	0.995 lead	0.8	6.1
		No	6,300	230	0.995 lead	0.8	2.09
6	Damping	Yes	6,300	200	0.980 lead	0.8	5.22
		No	6,300	210	0.980 lead	0.8	2.14
7	Damping	Yes	6,300	185	0.965 lead	0.8	5.22
		No	6,300	195	0.965 lead	0.8	1.71
8	Damping	Yes	6,300	225	0.994 lead	1.0	6.1
		No	6,300	230	0.994 lead	1.0	1.73
9	Damping	Yes	6,300	225	0.994 lead	1.2	6.3
		No	6,300	230	0.994 lead	1.2	1.59
10	Damping	Yes	0+	180		0.94	5.63
		No	0+	200		0.92	1.12

*Field current required to produce rated open-circuit voltage is approximately 180 amperes.

constants. For these tests the procedure was to (1) connect the test generator to the system, (2) operate it at no-load in an under-excited condition and, 3) suddenly disconnect it from the system. Fig. 3 comprises oscillographic records of the ensuing phenomena, the field current curves being of particular interest.

The remaining items in Table I refer to tests which were made to determine the relative damping characteristics of the two machines. For these tests the procedure was to connect the test generator to the test bus, load it and the test circuits in predetermined fashion, and trip the bus tie breaker thus initiating a balanced disturbance with the test machine remaining connected to the main system by means of a single circuit. Figs. 4 and 5 comprise typical oscillographic records of such tests, the oscillatory power curves being of particular interest.

The same exciter and excitation circuits were used for all tests. Voltage regulators were not used. Prior to each test the generator field rheostat was adjusted to give the desired value of generator field circuit resistance. The desired value of generator field current was then obtained by manual manipulation of the exciter field rheostat.

The voltage of the main system bus varied but little during the testing period, and during each individual test remained practically unchanged.

Machine Constants

The constants of each test machine were calculated from design data in the manner described by Linville.[2] Using these values and an operational method outlined by Park,[3] the action of each machine's field current was calculated for the conditions existing while the records of Fig. 3 were being obtained. In Fig. 6 the calculated curves are compared with those taken from the oscillographic records of Fig. 3.

In developing the calculated field current curves, direct axis constants were used; quadrature axis con-

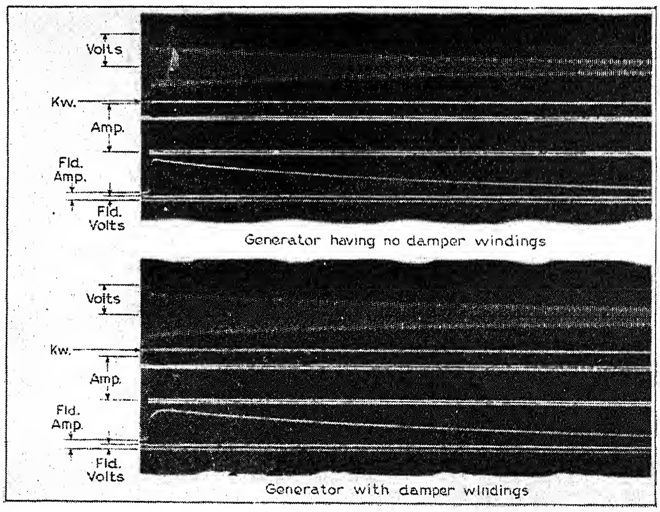

Fig. 3—Oscillographic Records of Machine Constant Tests

stants were not used because the tests to be checked were made at no-load. The excellent agreement between the calculated and test curves indicates the accuracy of the method used to derive the direct axis constants. The same method was used to calculate the quadrature axis constants. Thus, the reasonable-

ness of the calculated machine constants, used by the authors of the companion paper[1] in theoretically checking the power swing records of Fig. 4, is indicated.

The more important constants of the two test machines are given in Table II.

Damping Torque Coefficients

The time required for the damping of a balanced disturbance to $\frac{1}{\epsilon}$ of its initial value is inversely proportional to the damping torque coefficient, T_d, which can be determined from test records by the method given in the Appendix. In this way the damping torque coefficient was determined for each of the tests referred to in items 2 to 10 of Table I.

These results are plotted in Figs. 7 and 8, both of which show the relative damping action of machines with and without low resistance damper windings of the type shown in Fig. 2. The damping torque coefficients for the damping winding machine are, approximately, three times those of the other machine. The major portions of power oscillations involving the damper winding machine were damped out in about one-third of the time required for the damping out of like oscillations involving the machine not equipped with these windings.

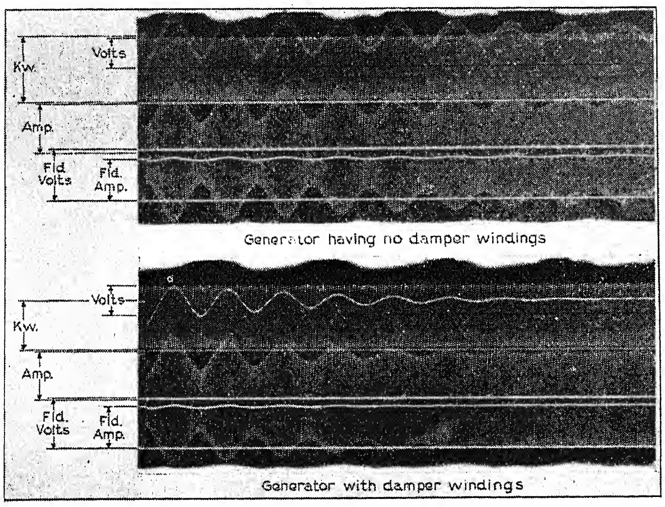

Fig. 4—Oscillographic Records of Damping Tests—Loaded Conditions

TABLE II—CALCULATED MACHINE CONSTANTS

Constant	With damper windings	No damper windings
x_d	0.91	0.91
x_d'	0.26	0.26
x_d''	0.14	0.177
x_q	0.524	0.524
x_q'	0.524	0.524
x_q''	0.368	0.524
T_{do}'	2.86 (sec.)	2.86 (sec.)
T_{do}''	0.055 (sec.)	0.0136 (sec.)
T_{qo}''	0.044 (sec.)	
H	1.78 (sec.)	1.78 (sec.)

A comparison of the power swings shown in Fig. 4, and a comparison of those shown in Fig. 5, provide a more graphic illustration of the increased rapidity with which balanced disturbances are damped out when machines are equipped with low-resistance damper windings.

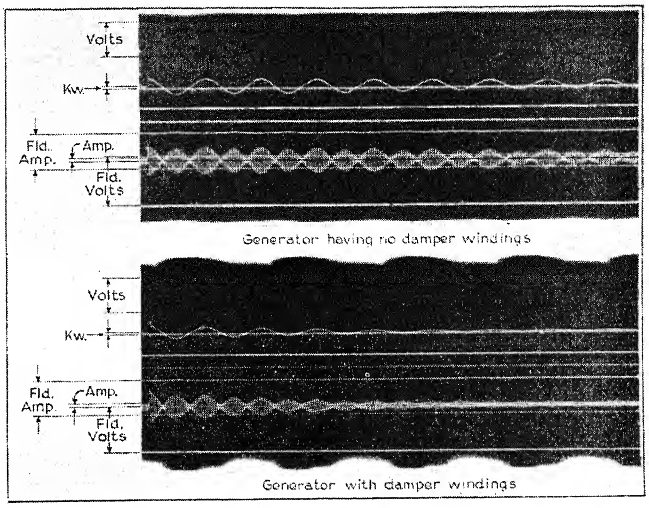

Fig. 5—Oscillographic Records of Damping Tests—Lightly Loaded Conditions

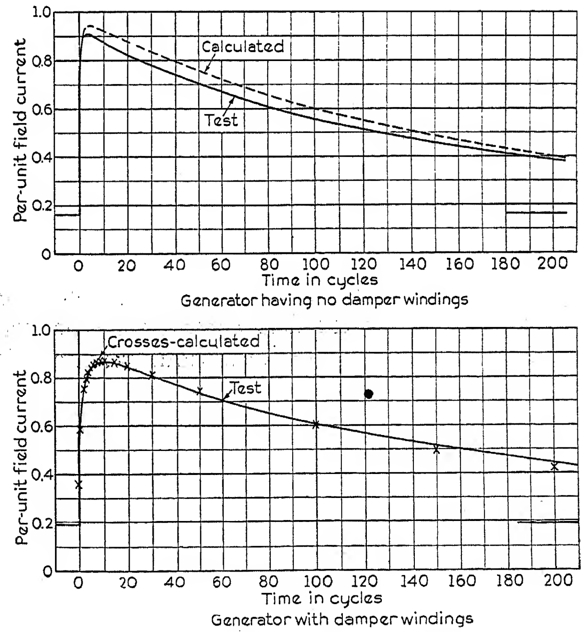

Fig. 6—Machine Constant Tests—Comparison of Calculated and Test Results

Fig. 7 also shows that damping varies but little with normal values of generator field circuit resistance. Fig. 8 also shows that damping varies inappreciably with values of generator field current within normal operating limits.

The effect of generator loading can be judged by comparing the values of T_d given in item 10 of Table I

with values of T_d tabulated for other tests. For the machine with damper windings T_d was approximately the same, regardless of the average load. For the machine having no damper windings, T_d during no-load operation was only about one-half as great as with the machine loaded.

Conclusions

Field tests are useful and important because they provide graphic evidence concerning phenomena not well understood, and data with which to verify available methods of analysis, or extensions of existing theory. Both purposes were fulfilled by the tests described.

Low-resistance damper windings quite obviously assist the rapid smoothing out of balanced disturbances such as occur owing to (1) the switching in or out of lines, (2) synchronizing, (3) load changes or the tripping of loaded generators, (4) three-phase short circuits, and (5) the occurrence and subsequent clearing of other types of faults.

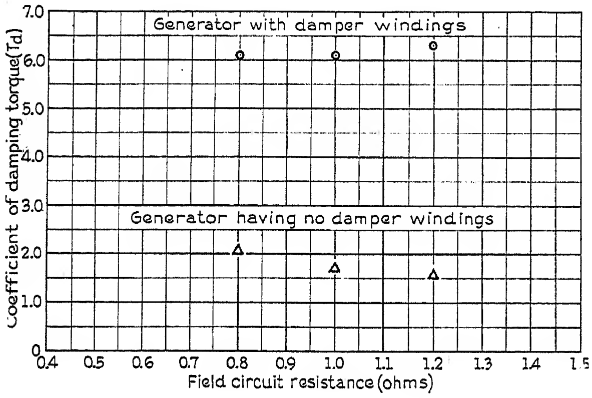

Fig. 7—Damping Torque Coefficient vs. Generator Field Circuit Resistance

The test results indicate that generator field circuit resistance, and generator field current may be varied within normal operating limits without greatly affecting the generator damping characteristics.

The damping torque coefficients given in Table I may be considered representative for machines of the type used during the tests when connected to a large system through an impedance approximating that of the test circuit of Fig. 1 and operated at any load up to full load.

The value of field tests as a source of data for checking available methods of calculating machine constants has been shown. The importance of field tests as a source of data for verifying the theory employed to predetermine the action of machines during system disturbances is shown by the companion paper.[1]

The New York Power and Light Corporation is deserving of much credit for the results of these tests, for which it made available a portion of its system, and in the carrying out of which Mr. Powell, Mr. Bromeson, and other members of its staff provided material ssistance.

Appendix

Determination from Test Records of Damping Torque Coefficients

The electrical torque of a synchronous machine during an oscillation may, in an approximate manner, be separated into two components. These are the damping and the synchronizing torques. If the oscilla-

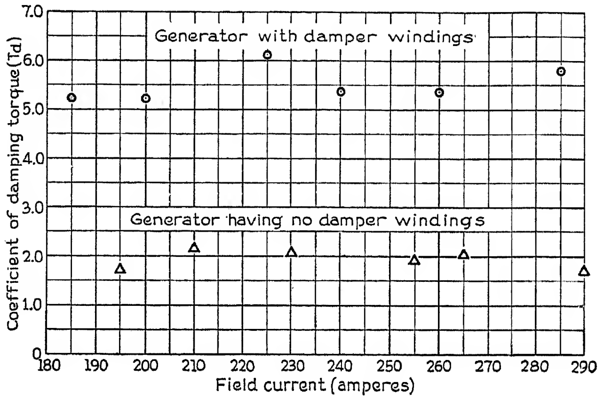

Fig. 8—Damping Torque Coefficient vs. Generator Field Current

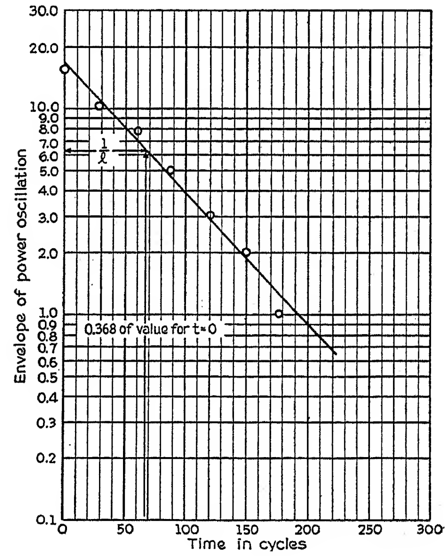

Fig. 9—Determination of Time Constant of Power Oscillation Decrement

Multiply cycles by 2π to obtain $\frac{1}{l}$ in radians before substituting in equation (3)

tions are small the synchronizing torque at any instant is very nearly proportional to the angular displacement of the generator rotor. Under such conditions this, torque may be represented by a constant, T_s, times the angular displacement. The damping torque depends upon the rate of change of angle and may be represented by a constant, T_d, times the rate of change of angular displacement.

The differential equation for the motion of a generator rotor with constant mechanical torque input, and small angular displacement, is approximately

$$4\pi f H \frac{d^2\delta'}{dt^2} = -T_s\delta' - T_d\frac{d\delta'}{dt} \qquad (1)$$

where

f = system frequency in cycles per second.

H = inertia constant of the machine, or

$$\frac{0.231\ WR^2\left[\frac{\text{r.p.m.}}{1{,}000}\right]^2}{\text{Base kw.}} \text{ (Sec.).}$$

δ' = angle in electrical radians of the rotor from the final angle of equilibrium.

T_s = coefficient of synchronizing torque, *i. e.*, per unit torque per radian of angular displacement.

T_d = coefficient of damping torque, *i. e.*, per unit torque per radian of angular displacement per radian time.

t = time measured in radians, *i. e.*, unit time is the time required for the rotor to pass one electrical radian at fundamental frequency (f).

The solution of equation (1), when $\frac{d\delta'}{dt} = 0$, and $\delta' = -\delta_0'$, at $t = 0$, is

$$\delta' = -\delta_0'\,\epsilon^{-lt}\cos st \qquad (2^*)$$

*Page, "Introduction to Theoretical Physics," p. 63.

where

δ_0' = the initial angular displacement of the rotor from the final angle of equilibrium,

$$l = \frac{T_d}{8\pi f H} \qquad (3)$$

$$s = \sqrt{\frac{T_s}{4\pi f H} - \left[\frac{T_d}{8\pi f H}\right]^2} \qquad (4)$$

Since the maximum and minimum values of displacement angle will occur approximately simultaneously with the maximum and minimum values of electrical power, the time constant of the decrement of the power envelope equals $\frac{1}{l}$ which is determined from a plot on semi-log paper of the power oscillation as shown by Fig. 9. Therefore, T_d can be obtained directly from equation (3) and the test oscillographic record of electrical power.

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Design of Capacitor Motors for Balanced Operation

BY P. H. TRICKEY*

Associate, A.I.E.E.

INTRODUCTION

THE present methods of calculating the full load performance of capacitor motors are quite long and tedious. This paper presents a simple, convenient method of calculating full-load conditions.

It is generally agreed that a capacitor motor should be designed to operate under nearly balanced conditions at full load, meaning by this that there shall be no transfer of power from one phase to the other and balanced currents in the rotor. With such a design, each phase produces equal flux and the rotor operates as if in a two-phase stator.

METHOD OF OBTAINING BALANCED CONDITIONS

An attempt is sometimes made to approximate balanced conditions by making the voltages applied to each phase 90 electrical degrees apart and proportional to the turns in the windings. However, it is obvious that in order to obtain true balanced conditions, the field produced by each phase must be of the same magnitude and 90 degrees apart. If this is true, the rotor currents will also be of the same value and displaced 90 degrees. Under such conditions, the induced voltage, and not the applied voltage must be balanced. The magnetizing current being proportional to the induced voltage, it will, therefore, also be balanced and the primary currents as well as the secondary currents will be 90 degrees displaced and inversely proportional to the effective turns. Fig. 1 shows the vector diagram for a capacitor motor.

As a general rule the main winding has been determined by the necessary maximum torque of the motor. This in turn determines the primary resistance, and the current in the main phase. It remains, therefore, to determine the number of turns in the capacitor phase, the size of wire and the impedance of the capacitor itself. Since the resistance of the capacitor is controlled by the available space for it, and the allowable loss, the designer can usually estimate fairly closely as to its value. It will be treated hereafter as a known constant and if necessary the motor may be redesigned when the resistance of the capacitor differs from the assumed value. Thus, there are three variables to be determined, number of turns, size of wire, and capacitive reactance.

SOLUTION OF THE VECTOR DIAGRAM

Under the first necessary condition, the two fluxes must be alike. If this is the case the induced voltages in each phase must be 90 deg. apart, and their magnitudes must be in the ratio of the effective conductors in each phase.

$$e_{2S} = jKe_{2m} \tag{1}$$

where

e_{2S} = induced voltage in the capacitor phase

e_{2m} = induced voltage in the main phase

$$K = \frac{\text{effective conductors in capacitor phase}}{\text{effective conductors in main phase}}$$

j = operator to revolve vector 90 deg.

Under the second necessary condition, the two primary currents must be 90 deg. apart and their magnitudes must be in the inverse ratio of the effective conductors.

$$I_{iS} = \frac{j}{K}\, i_{1m} \tag{2}$$

Where

i_{1S} = primary current in capacitor phase

i_{1m} = primary current in main phase

$$E_S = e_{2S} + i_{1S}Z_{1S} \tag{3}$$

Where

E_S = capacitor phase voltage

Z_{1S} = primary impedance of capacitor phase

But

$$E_S = E_M - i_{1S}Z_c \tag{4}$$

Where

E_m = main phase and line voltage

Z_c = impedance of capacitor

(a) *Solution of equation* (3)

$$E_S = jKe_{2m} + \left(\frac{j}{K}\, i_{1m}\right)(Z_{1S})$$

But

$$Z_{1S} = r_{1S} + jX_{1S}$$

Where

r_{1S} = primary resistance of capacitor phase

X_{1S} = primary reactance of capacitor phase

If the winding distribution is the same in each phase

$$X_{1s} = K^2X_{1m}$$

$$r_{1s} = KK_a r_{1m}$$

Where

$$K_a = \frac{\text{area of conductor in main phase}}{\text{area of conductor in capacitor phase}}$$

*Small Motor Engg. Dept., Westinghouse Electric & Mfg. Co., Springfield, Mass.

Presented at the Northeastern District Meeting of the A.I.E.E., Providence, R. I., May 4-7, 1932.

r_{1m} = primary resistance of main phase
X_{1m} = primary reactance of main phase

$$E_s = jKe_{2m} + \left(\frac{j}{K}\, i_{1m}\right)(KK_a r_{1m} + jK^2 X_{1m})$$

$$= jKe_{2m} - KX_{1m} i_{1m} + jK_a r_{1m} i_{1m}$$

But

$$e_{2m} = E_m - i_{1m} Z_{1m}$$

$$E_s = jKE_m - jKi_{1m}(r_{1m} + jX_{1m}) - KX_{1m}i_{1m} + jK_a r_{1m} i_{1m}$$

$$= jKE_m - jKi_{1m}r_{1m} + jK_a r_{1m} i_{1m}$$

It will be noted that X_{1m} drops out at this point eliminating the question of dividing the leakage reactance into primary and secondary components.

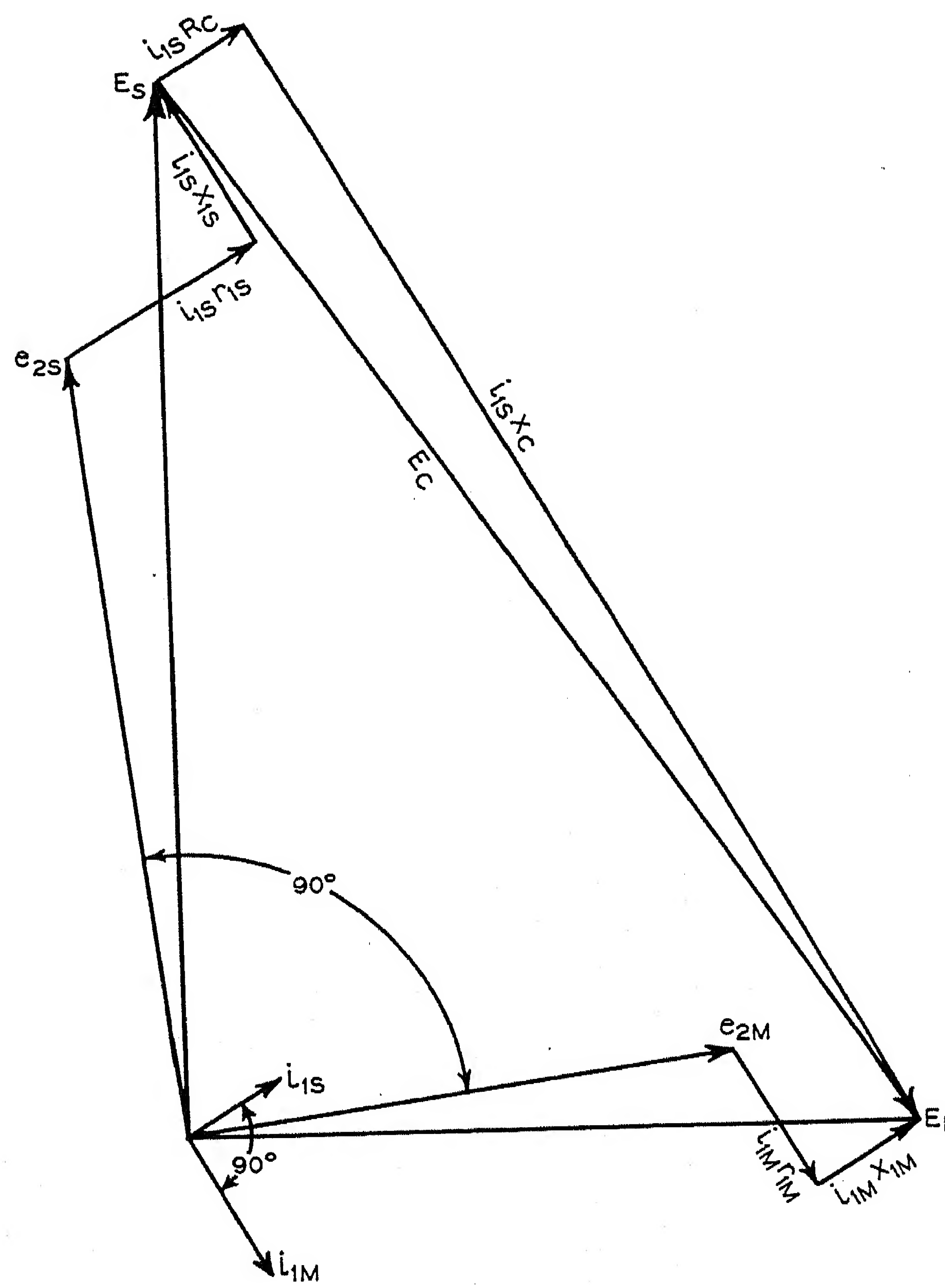

Fig. 1—Vector Diagram of Capacitor Motor Under Balanced Operating Conditions

For simplification, let

$$i_{1m} = A - jB$$

Where

A = power component of main phase primary current $= I_m$ times power factor

B = reactive component of main phase primary current $= \sqrt{I_m^2 - A^2}$

I_m is the phase current as determined by calculating the motor as a two-phase motor with each phase identical to the main winding of the capacitor.

$$E_s = K_a B r_{1m} - KBr_{1m} + j(KE_m + K_a A r_{1m} - KAr_{1m}) \tag{5}$$

(b) *Solution of equation* (4)

$$E_s = E_m - i_{1s} Z_c$$
$$Z_c = R_c - jX_c$$

Where

R_c = effective resistance of capacitor

X_c = effective capacitive reactance of capacitor (positive)

$$E_s = E_m - \frac{j}{K}(A - jB)(R_c - jX_c)$$

$$= E_m - BR_c/K - AX_c/K + j(BX_c/K - AR_c/K) \tag{6}$$

Since these two equations for E_s must be identical, the components must be equal.

Horizontal Components

$$E_m - BR_c/K - AX_c/K = K_a B r_{1m} - KBr_{1m}$$
$$KE_m - BR_c - AX_c = KK_a B r_{1m} - K^2 B r_{1m} \tag{7}$$

Vertical Components

$$BX_c/K - AR_c/K = KE_m + K_a A r_{1m} - KAr_{1m}$$
$$BX_c - AR_c = K^2 E_m + KK_a A r_{1m} - K^2 A r_{1m} \tag{8}$$

Multiplying by B and A respectively in equations (7) and (8) and eliminating X_c.

$$K^2[AE_m - r_{1m}(A^2 + B^2)] - K[BE_m - K_a r_{1m}(A^2 + B^2)] + (A^2 + B^2) R_c = 0 \tag{9}$$

Since this quadratic equation gives a difficult solution to use, a cut and try method is followed.

$$K[AE_m - r_{1m}(A^2 + B^2)] = [BE_m - K_a r_{1m}(A^2 + B^2)] - (A^2 + B^2) R_c/K$$

$$K = \frac{BE_m - K_a r_{1m}(A^2 + B^2) - (A^2 + B^2) R_c/K}{AE_m - r_{1m}(A^2 + B^2)} \tag{10}$$

But $A^2 + B^2 = I_m^2$ where I_m is the magnitude of i_{1m}

$$K = \frac{BE_m - K_a r_{1m} I_m^2 - (R_c/K) I_m^2}{AE_m - r_{1m} I_m^2} \tag{11}$$

In the above equation everything has been determined by the design of the main winding except K and K_a. The method of using this equation is as follows: K_a must, of course, be a function of $\sqrt[3]{2}$ since wire sizes vary in this definite ratio. The usual motor will have one, two, or three wire sizes smaller than the main winding, and thus K_a will be 1.26, 1.59, or 2.00. One of these is assumed, and since, as will be shown later, K should be somewhere near equal to K_a, the value of K on the right-hand side of equation (11) is taken equal to the assumed value of K_a. The equation is then solved for K. The value of K obtained is substituted for the assumed value and a second solution is made. Since the third term is usually small, it is seldom necessary to make more than two solutions. For every value of K_a, there will be a definite value of K, or in other words, there are as many balanced motors as there are wire

sizes. However, the change in K for different values of K_a is very small. Fig. 2 shows the effect of changing K_a upon the constants and performance of a typical motor. It will be noted that the only values which change in any great degree are the weight of wire and the current density, and that whatever gain is made in reducing capacitor phase copper is made at the expense of a higher current density in the capacitor winding. It would seem desirable, therefore, to choose K as nearly equal to K_a as wire sizes will permit.

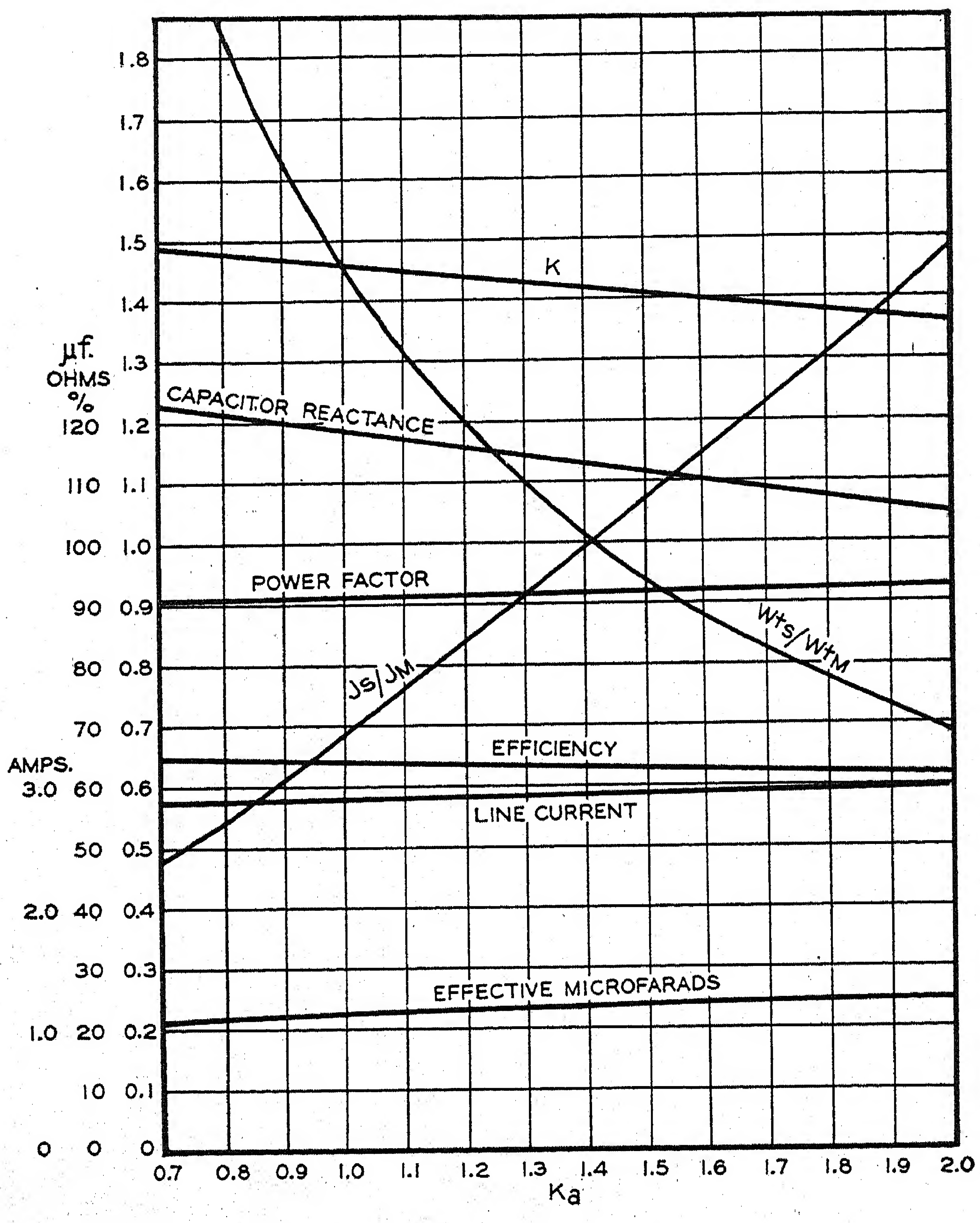

FIG. 2—EFFECT OF Ka ON CAPACITOR MOTOR PERFORMANCE

$$Ka = \frac{\text{area of conductor in main phase}}{\text{area of conductor in capacitor phase}}$$

$$K = \frac{\text{effective conductors in capacitor phase}}{\text{effective conductors in main phase}}$$

$$\frac{J_s}{Jm} = \frac{\text{current density in capacitor phase}}{\text{current density in main phase}}$$

$$\frac{Wt_s}{Wtm} = \frac{\text{total weight of wire in capacitor phase}}{\text{total weight of wire in main phase}}$$

Having determined K from equation (**11**), equation (**7**) gives,

$$X_c = \frac{KE_m - BR_c - KK_a Br_{1m} + K^2 Br_{1m}}{A} \qquad (12)$$

$$C = \frac{10^6}{2\pi f X_c} \text{ or } \frac{2650}{X_c} \text{ for 60 cycles} \qquad (13)$$

where C = effective microfarads

$$E_c = \frac{I_m Z_c}{K} = \text{voltage across capacitor} \qquad (14)$$

PERFORMANCE CALCULATION OF BALANCED MOTOR

Since the motor has been designed to have rotor conditions identical to a two-phase motor with each phase like the main phase of the capacitor motor, the motor may be calculated like a polyphase motor. There will be, however, differences in some of the performance values.

Line Current

$$i_L = i_{1m} + i_{1s} = i_{1m} + \frac{j}{K} i_{1m}$$

$$i_L = A - jB + \frac{B}{K} + j\frac{A}{K}$$

$$i_L = \frac{AK + B}{K} + j\frac{A - BK}{K}$$

where i_L is the vector expression of line current

$$I_L = \sqrt{\left(\frac{AK + B}{K}\right)^2 + \left(\frac{A - BK}{K}\right)^2}$$

$$= \frac{\sqrt{A^2 + B^2}\sqrt{K^2 + 1}}{K}$$

$$I_L = \frac{I_m\sqrt{K^2 + 1}}{K} \qquad (15)$$

where

I_L = magnitude of line current

I_m = magnitude of main or polyphase current

Power Factor

$$\text{P. F.} = \frac{\dfrac{AK + B}{K}}{\sqrt{\left(\dfrac{AK+B}{K}\right)^2 + \left(\dfrac{A-BK}{K}\right)^2}}$$

$$= \frac{AK + B}{\sqrt{A^2 + B^2}\sqrt{K^2 + 1}}$$

$$\text{P. F.} = \frac{AK + B}{I_m\sqrt{K^2 + 1}} \qquad (16)$$

or

$$\frac{A + \dfrac{B}{K}}{I_L} \qquad (17)$$

The losses of the capacitor motor will differ from the losses in the polyphase motor only in primary copper loss, and the additional loss in the capacitor. The main

phase loss will be the loss per phase in the polyphase motor.

$$\text{Main phase loss} = \frac{\text{polyphase loss}}{2}$$

$$\text{Capacitor phase loss} = i_{1s}^2 r_{1s}$$

Only the magnitude of current is of interest

$$\text{Capacitor phase loss} = \left(\frac{I_m}{K} \right)^2 r_{1s}$$

$$= \left(\frac{I_m^2}{K^2} \right) K K_a \, r_{1m} = (I_m^2 \, r_{1m}) \frac{K_a}{K}$$

$$\text{Total primary copper loss} = I_m^2 \, r_{1m} \left(1 + \frac{K_a}{K} \right)$$

$$= \frac{K + K_a}{2K} \text{(polyphase loss)} \qquad \textbf{(18)}$$

$$\text{Capacitor loss} = \frac{I_m^2}{K^2} R_c \qquad \textbf{(19)}$$

Conclusion

The method of designing capacitor motors given in this paper is briefly as follows:

1. Design the main winding to give the necessary maximum torque.
2. Calculate the performance as a two-phase motor with each phase like the main winding determined above.
3. Solve equation (**11**) for K.
4. Choose K_a to be as close to K as possible and solve again for K.
5. Solve equation (**12**) for X_c.
6. Design capacitor from X_c and correct above solutions if R_c is too far in error.
7. Calculate performance by equations (**15**), (**17**), (**18**) and (**19**).

As mentioned above, this method is useful only when the starting torque required is not too great.

In many cases, starting torque requirements determine the windings, and the motor must operate slightly unbalanced at full-load with the resultant circulating current and lowered efficiency. A sample calculation on a typical motor is given in the appendix.

List of Symbols

E_m = main winding and line voltage
E_s = capacitor winding voltage
E_c = capacitor voltage
e_{2m} = induced voltage in main winding
e_{2s} = induced voltage in capacitor winding
i_{1m} = primary current in main winding
i_{1s} = primary current in capacitor winding
I_M = current per phase in identical two-phase motor
I_L = line current of capacitor motor
A = power component of main winding primary current
B = reactive component of main winding primary current
J_m = current density in main winding
J_s = current density in capacitor winding
Wt_m = weight of wire in main winding
Wt_s = weight of wire in capacitor winding

$$K = \frac{\text{effective conductors in capacitor winding}}{\text{effective conductors in main winding}}$$

$$K_a = \frac{\text{area of conductor in main winding}}{\text{area of conductor in capacitor winding}}$$

r_{1m} = primary resistance of main winding
X_{1m} = primary reactance of main winding
Z_{1m} = primary impedance of main winding
r_{1s} = primary resistance of capacitor winding
X_{1s} = primary reactance of capacitor winding
Z_{1s} = primary impedance of capacitor winding
R_c = effective resistance of capacitor
X_c = effective reactance of capacitor (positive)
Z_c = effective impedance of capacitor
C = effective capacity of capacitor (in microfarads)

Appendix

Following is given an example of a capacitor motor design illustrating the method described in this paper. It is assumed that the main winding has been determined from the maximum torque requirements, and that the performance as a two-phase motor with each phase like the main winding has been calculated.

E_m = 110 volts (applied voltage)
r_{1m} = 1.89 ohms primary resistance of main phase
I_m = 2.39 amperes current per phase
Primary copper loss = 21.6 watts
Input = 273.6 watts
Output = 186.7 watts (1/4-hp., 110-volt, 60-cycle, two-phase, 4-pole)
Losses 86.9 watts
Efficiency 68.2 per cent
Power factor 52.0 per cent
R_c = 8.00 ohms
I_m^2 = 5.7
$A = I_m PF = 2.39 \cdot 52 = 1.24$
$B = \sqrt{I_m^2 - A^2} = 2.045$
K_a and K are assumed = 1.26

$$K = \frac{2.045 \times 110 - 1.26 \times 1.89 \times 5.7 - \dfrac{8.0}{1.26} \times 5.7}{1.24 \times 110 - 1.89 \times 5.7}$$

$$= \frac{225 - 13.57 - 36.2}{136.1 - 10.79} = \frac{175.2}{125.31} = 1.40$$

Second trial

$$K = \frac{225 - 13.57 - \dfrac{8.0\ 5.7}{1.42}}{125.31} = 1.430$$

$$X_c = \frac{1.43 \times 110 - 2.045 \times 8 - 1.43 \times 1.26 \times 2.045 \times 1.89 + 1.43 \times 2.045 \times 1.89}{1.240}$$

$$= 114.3 \text{ ohms}$$

$$C = \frac{2650}{114.3} = 23.2 \text{ effective microfarads}$$

$$I_L = \frac{2.39}{1.43} \sqrt{1.43^2 + 1} = 2.915 \text{ amperes}$$

$$\text{P. F.} = \frac{1.24 + \left(\dfrac{2.045}{1.43}\right)}{2.915} = 91.5 \text{ per cent}$$

$$\text{Primary loss} = \left(\frac{1.43 + 1.26}{2\ \ 1.43}\right) 21.6 = 20.3$$

(decrease of 1.30 watts)

$$\text{Capacitor loss} = \left[\frac{2.39}{1.43}\right]^2 8 = 22.4$$

Total losses 86.9 − 1.3 + 22.4 = 108.0

Output	= 86.7
Input	= 294.7

$$\text{Efficiency} = \frac{186.7}{294.7} = 63.3 \text{ per cent}$$

Fig. 2 is plotted from the values obtained in this example.

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Discussion

L. A. Doggett: In this article as well as all previously published articles, the theory of the capacitor motor has been developed on the assumption of sine waves. The correctness of this assumption is challenged by the oscillogram of Fig. 1 which shows the current waves when a *RKS*-126, 2-hp., 1,800-r.p.m., 220-volt, 8.6-ampere, single-phase, 60-cycle, G. E. capacitor motor is operated from a practically sinusoidal source. (Analysis: $Y = \sin x + 0.01 \sin 2x + 0.00 \sin 3x + 0.01 \sin 4x + 0.01 \sin 5x$). These current waves are for the motor running light, curve $I_l = 4.4$ shows the line current; curve $I_c = 6.1$ shows the capacitor phase current; and curve $I_m = 2.4$ shows the main phase current

That these currents depart widely from sinusoidal shape throughout the whole range of operation is brought out by an I^2R, I^2X analysis of the load run test data. Since wattmeters measure the average power, not only of the fundamental but also of the harmonics, a reasonably close check was to be expected of the I^2R analysis. The I^2X values were calculated from the expression,

$$I^2X = (\overline{EI}^2 - [I^2R]^2)^{1/2}$$

which is true for sinusoidal cases only. The degree by which the I^2X analysis fails to check is a measure of the departure of the actual waves from sine waves. The wide discrepancies in the I^2X values, shown by Table I, indicate the presence of large harmonics.

So long as there is a transfer of power from one phase to the other there are bound to be large harmonics present. If the designs of Mr. Trickey avoid this transfer of power, oscillograms from motors of his design should show much better sine waves than those of Fig. 1. It is to be hoped that Mr. Trickey will submit oscillograms for some of his motors.

If there is a transfer of power, then one phase becomes a generator and the other phase becomes a receiver. It will hardly be claimed that such a generator is a sine-wave generator. To make matters worse, a sizeable condenser is included in the series circuit between the generator and the source. Thus the stage is set for the generation of harmonics, wide in variety and large in amplitude.

TABLE I*

Line			Cond. phase			Main phase		
I^2R	I^2X	I^2Z	I^2R	I^2X	I^2Z	I^2R	I^2X	I^2Z
1,020	−387	1,090	1,460	−716	1,598	−440	203	485
1,410	−387	1,462	1,420	−591	1,535	− 40	176	180
1,620	−412	1,673	1,400	−490	1,482	240	000	240
1,880	−490	1,944	1,360	−544	1,463	520	245	576
2,110	−374	2,140	1,320	−505	1,415	800	122	809
2,280	−400	2,313	1,280	−485	1,370	1,000	173	1,015
2,500	−374	2,525	1,240	−424	1,310	1,260	255	1,285
2,640	−316	2,660	1,200	−453	1,282	1,440	141	1,445
1,020	−387	1,090	1,480	−678	1,598	−480	232	533

SUMMARY OF ABOVE

Line I^2R	Branches I^2R	Line I^2X	Branches I^2H
1,020	1,020	−387	−513
1,410	1,380	−387	−415
1,620	1,640	−412	−490
1,880	1,880	−490	−299
2,110	2,120	−374	−383
2,280	2,280	−400	−312
2,500	2,500	−374	−169
2,640	2,640	−316	−312
1,020	1,000	−387	−446

I^2R = wattmeter reading.
I^2Z = product of voltmeter and ammeter readings.
$-I^2X$ – condensive reactance volt amperes.
*The data of this table were prepared by A. R. Mengel and D. W. McGill, Enrolled Students A.I.E.E. at Pennsylvania State College.

C. G. Veinott: Anyone who has designed, or has attempted to design capacitor motors recognizes that the problem is tremendously involved and that there is a considerable number of factors which must be taken into account. Mr. Trickey's paper throws much useful light on this complicated problem in a simple manner. He shows the conditions which must be met in order

to obtain operation equivalent to that of a two-phase motor and incidentally brings to light some interesting and important conclusions. For example, in Fig. 1, the winding ratio (ratio of capacitor phase turns to main phase turns) is of the order of the tangent of the power-factor angle of the motor when considered as a two-phase motor with both phases like the main winding. It is, therefore, necessary to have control of the winding ratio as well as the external microfarads in order to obtain balanced

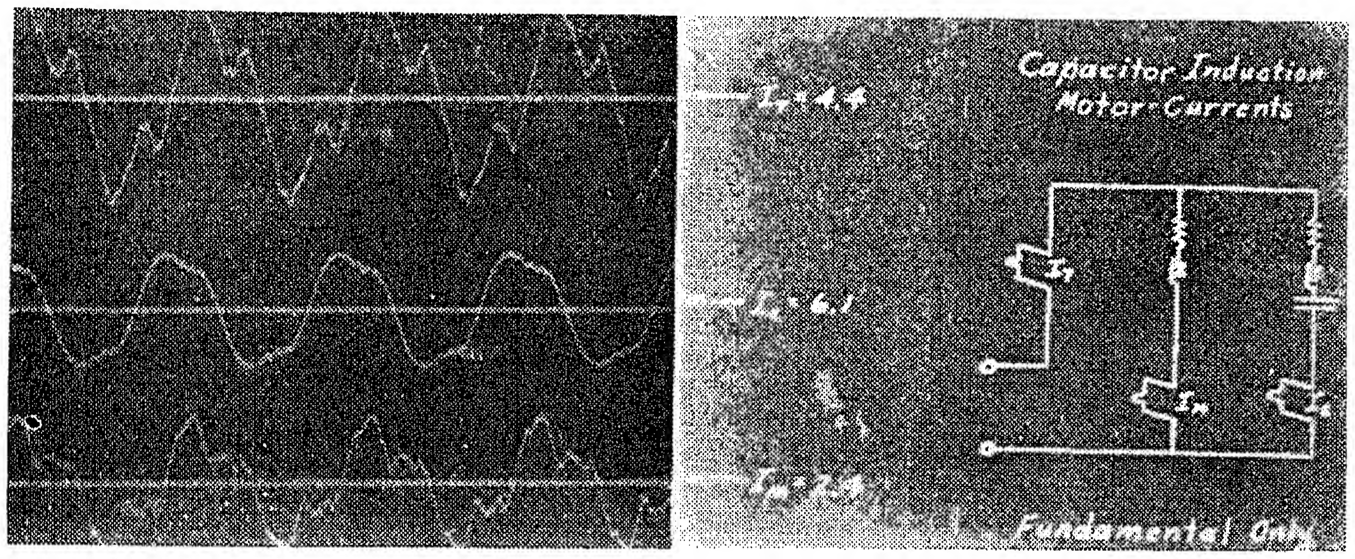

Fig. 1

operation. This means that any polyphase motor cannot necessarily be operated as a polyphase motor from a single-phase source merely by the addition of a capacitor of suitable value in one phase. This fact seems to be often misunderstood by those not familiar with capacitor motors.

P. H. Trickey: Regarding the advisability of operating a capacitor motor at its balanced operating point, Fig. 2 shows the effect of changing the voltamperes of the capacitor on efficiency, power factor, and apparent efficiency of a typical ¼-hp., 110-volt, 60-cycle motor. The balanced point is at *A*.

Having designed a motor for balanced operation, and calculated its performance by the method of this paper, the designer can choose actually to operate the motor at any value of capacitor voltamperes he wishes. Point *B* is the maximum efficiency point which will coincide with point *A* if $R_c = O$ and if $K = K_a$, point *C* gives the maximum power factor and point *D* gives the maximum apparent efficiency. The operator may operate at any one of these points, or if cost is of greatest importance the point *E* might be chosen, which gives a motor within 1½ per cent of the maximum efficiency and has only 59 per cent as many voltamperes. However, the power factor has dropped from 80 to 69 per cent.

If the designer chooses to operate at any point between *E* and *D*, this method is still of use to him in predetermining performance especially on new and radically different designs. Knowing the efficiency and power factor quite accurately at the balanced point, he can estimate very closely the performance at the actual operating point, from the general shape of the curves of Fig. 2.

In reply to Mr. Doggett's discussion the most prominent cause of harmonics in capacitor motor currents is saturation of iron, either in the motor itself or in the capacitor transformer. The transformer is the most likely cause because it has no air gap and on any voltage the magnetizing current may have a very bad third harmonic component. Unless a capacitor motor is returning power to the line through its main phase, the main phase current will have nearly the wave shape of the applied voltage except a slight distortion due to saturation in motor iron. The capacitor phase current will be distorted depending on the transformer saturation and since at no load the capacitor phase current is such a large proportion of the line current, the line current will also be distorted. The capacitor phase current will be symmetrical about a vertical axis, and since it adds approximately in quadrature to the main phase current, the line current will be distorted toward one side. Under no-load, overvoltage conditions this distortion may be very bad. In fact if the voltage is raised high enough, the capacitor phase fundamental magnetizing current will just balance the leading condenser current, and leave the third harmonic practically alone.

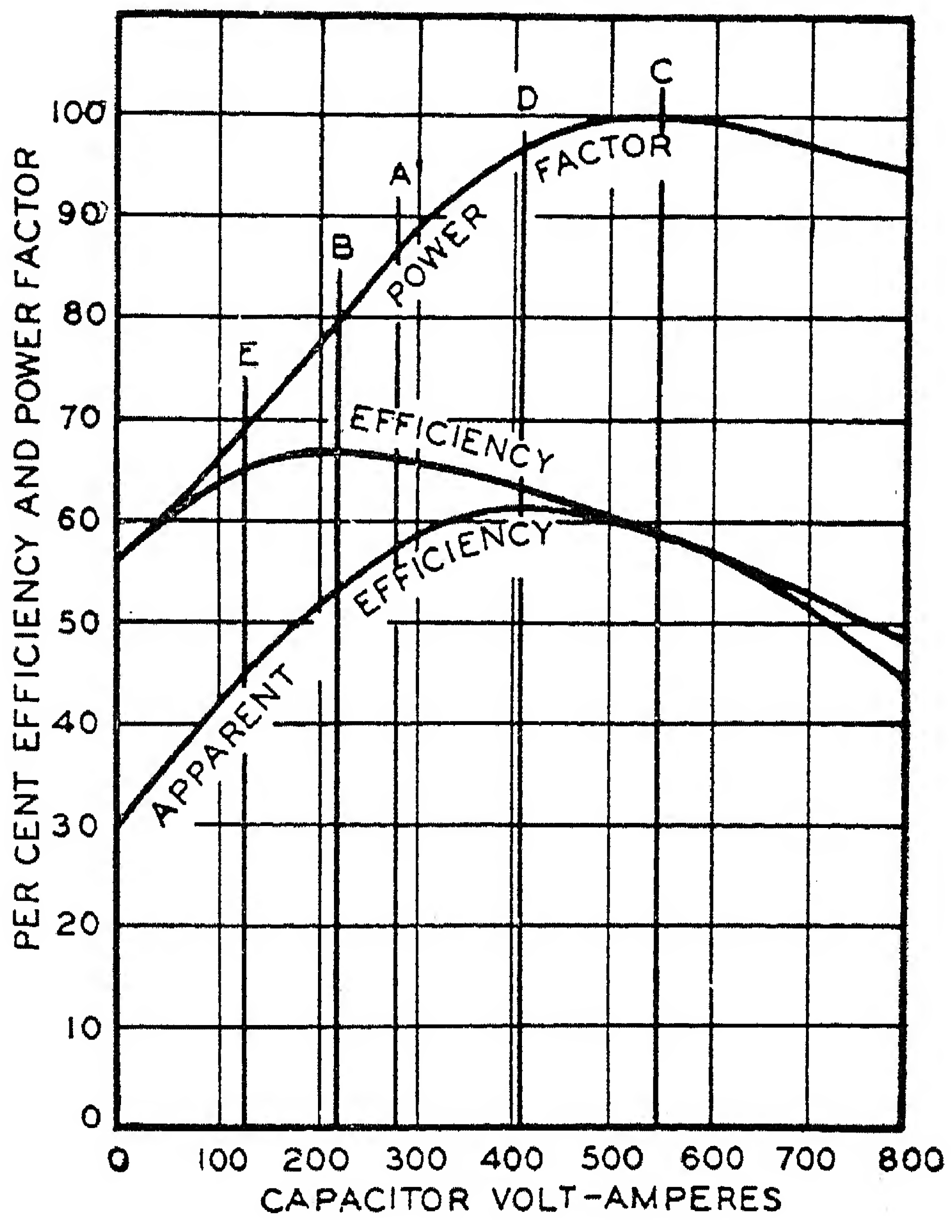

Fig. 2—Effect of Changing Capacitor Voltamperes on Capacitor Motor Performance

A. Balanced point
B. Maximum efficiency
C. Maximum power factor
D. Maximum apparent efficiency

However, I believe the wave shapes in a capacitor motor designed for balanced operation at full load, and with inductions sufficiently low in the transformer for operation at the usual overvoltages, will be close enough to a sine-wave for all practical purposes except at very light loads. At very light loads the motor is inherently unbalanced, and inefficient and a few harmonics more or less will not make it much worse.

The Flexible Progressive Traffic Signal System

BY H. I. TURNER*
Associate, A.I.E.E.

A FLEXIBLE progressive traffic signal system has been defined as one which provides a means of timing signals at different street intersections for operation in any desired relation with respect to each other, subject to the one limitation that the time of each cycle of operation must be the same at all intersections of the system. Such a range of adjustments usually makes it possible for an engineer to time the signals so that traffic may move east and west as well as north and south at certain predetermined uniform speeds without being obstructed by a "stop" signal after once having passed a "go" signal.

Most manufacturers have represented a flexible progressive system as one which may be timed to permit uninterrupted uniform movement of traffic after the traffic once has gotten into step with the signals. This is so often true that the statement is fairly well justified, but it does lead some persons to believe that a flexible progressive system possesses some mysterious timing characteristics which always make it possible,irrespective of the spacing of streets and certain other factors, for traffic to flow at uniform speed. This is not true as occasionally it is impossible or very impractical to time signals so traffic may maintain a uniform speed in both directions along a street and yet always reach an intersection while the "go" signal is directed toward it.

It is evident that, if the action of signals be properly coordinated, the fact that all signals must operate on the same time cycle (or in multiples of the same cycle) is not a weakness of the system. There otherwise could be no continued coordinated action of all signals. There are students of the subject who believe that it is theoretically impossible to provide any timing of signals within a given area which is an improvement over the timing that is possible in a genuine flexible progressive system, providing, of course, that facilities are furnished for readily varying timing to meet changes in conditions which develop during different times of the day. Some engineers believe that a single intersection, considered by itself, may often be dealt with more satisfactorily with signaling devices which are actuated by traffic than by any pretimed signals. A careful study of what can be accomplished by a properly-proportioned set timing of signals will, nevertheless, frequently indicate that even for many types of isolated intersections the controller with set timing deals with conditions as well or very nearly as well as the most elaborate system of traffic-actuated devices. Coordinated action of signals located at intersections of streets leading to a square having several entering streets is nearly always the most satisfactory way of dealing with these situations and the advantage of such a system becomes more evident as the volume of traffic entering the square increases.

In any event, traffic-actuated devices can rarely be introduced into the average flexible progressive system without upsetting the uniform flow of traffic to a more or less serious degree. In a great majority of congested districts the coordination of the movement of traffic through the entire district is of incomparably greater importance than the most efficient movement of traffic at any few intersections taken by themselves.

Signals may be timed for flexible progressive service by two general types of systems: *i. e.*, that which employs a central office type controller with no timing devices located at individual intersections, and that which employs a master controller at a central point and an individual controller at each intersection. A most outstanding example of the former is the controller which times the signals in the Chicago Loop District. Examples of the latter are found on Massachusetts Avenue, Cambridge; Olive Street, Saint Louis; Wilshire Boulevard, Los Angeles; Broad Street, Philadelphia, and in many other cities. All of these controllers have facilities for the following adjustments which entitle them to be classed as truly flexible progressive systems:

1. The time of each cycle of operation of the signals at any intersection may be adjusted from a central point. A cycle time anywhere between 30 and 120 seconds may be found useful in the average system.

2. The percentage of the cycle taken by the "go" signal and the "stop" signal may be adjusted over a wide range and adjustments made at one intersection are made independently of adjustments at any other intersections. In practise it has been found that ability to adjust both the "go" and "stop" periods from 15 to 85 per cent of the total cycle is all that is necessary.

3. The start of the cycle of operation of the signals at any intersection is 100 per cent adjustable with respect to the start of the cycle of operation of the signals at any other intersections of the system. The start of a cycle may be considered the instant when the "go" signal appears on any signal face and the total time of one cycle is the time which elapses between the appearance of one "go" signal and the appearance of the next "go" signal on the same face.

The chart A, Fig. 1, shows how it became necessary to time signals on one street to provide a progressive flow of traffic. The wide black marks indicate that traffic on the main street is facing a "stop" signal. At First Avenue a 25-second "stop" period for traffic on the main street was necessary. A 35-second "go" period was likewise necessary. In other words, during 58 per cent

*Eagle Signal Corporation, Moline, Ill.

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of the cycle the "go" signal shows to the main street. In order for traffic to flow in both directions without a stop, it was necessary for the signals at Second Avenue to show "go" to the main street during 75 per cent of the cycle. The adjustments necessary at the other avenues can be seen from the chart. These signals were so timed that traffic could start at either end of the street and proceed at 15 miles per hour past the other three intersections without being obstructed by a "stop" signal. A little study of the chart will make clear to the reader that traffic must not move much faster nor much more slowly than this speed or it will encounter a "stop" signal. This condition is one of the important features of a flexible progressive system—it discourages the movement of traffic at speeds either above or below that for which the system is timed.

Chart B shows what happens when the time of each cycle of the signals is changed from 60 seconds to 30 seconds. Traffic must now move 30 miles per hour to avoid "stop" signals instead of 15 miles per hour. The operator at the central station can, therefore, time the signals so as to discourage the movement of traffic at any speed other than the one he desires. No police officers on the street can so effectively control the speed of traffic as a well-engineered flexible progressive system. Traffic officers may restrict speed, but a flexible progressive system will actually make a driver try to hold his speed at that for which the signals are set. It is obvious that daily changes in traffic volume, slippery streets, and many other conditions which are constantly developing bring about excellent applications of the operator's ability to influence speed from a central point. Usually it has been found that a much larger volume of traffic can be handled by a comparatively low speed and systematic timing than by higher speeds without well-coordinated operation of signals.

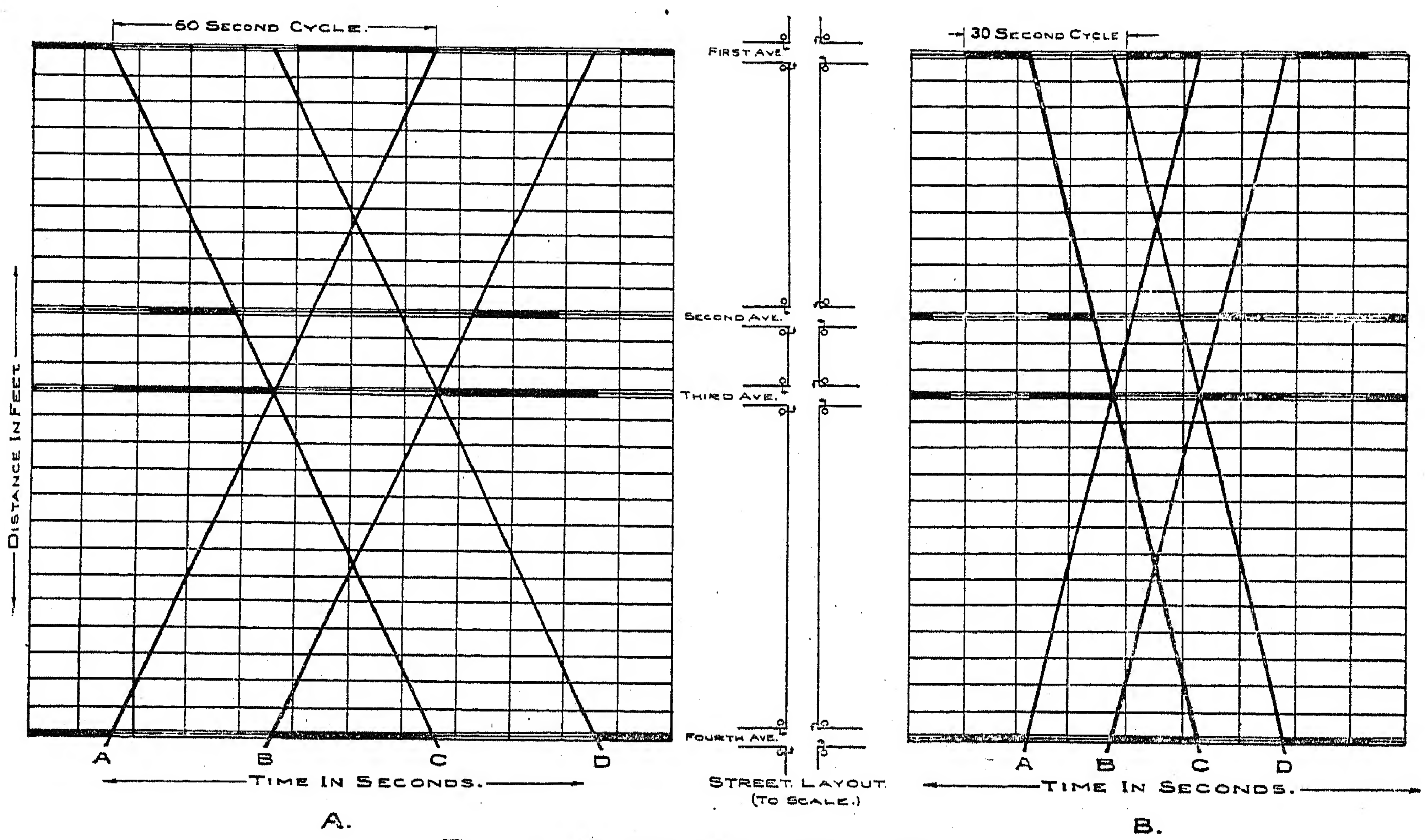

Fig. 1—Traffic Movement Chart

The following features or characteristics are important in any flexible progressive control devices:

1. Those which keep maintenance expenses and service interruptions at a minimum, such as: simplicity, sturdiness, smooth quiet operation, freedom from impact between parts.

2. Facilities for manual control. Usually the manual control of signals at one intersection of a system is objectionable because no officer can time the movements anywhere near as efficiently as can the controller itself. There are times when manual control is essential, however, such as immediately after an accident has occurred, when school children are leaving a nearby school and during times when very abnormal traffic conditions exist.

3. Facilities for shutting down the signals at each local controller and from a central point by merely throwing a switch. It is advisable to cause signals to flash amber lights when not operating as "stop" and "go" signals. Therefore, facilities should be provided at the central control point for causing signals to flash amber when not in normal operation.

4. Adjustments should be easily understood and of wide range. It is preferable that the controller be designed so adjustments may be made on a sliding scale and not in steps. A vehicle moving 30 miles per hour travels 44 feet per second and it is obvious that unless a controller can be adjusted to change its signals at the correct second it is not possible to handle traffic as smoothly as is possible when adjustments are set accurately as dictated by a traffic flow chart.

5. The duration of the amber light or warning that the signal is about to change should not increase as the total time cycle increases and should not decrease when the total time cycle is decreased. Fig. 1 shows that traffic has to travel faster when the time cycle is decreased in order to keep in step with the signals. If

decreasing the time cycle also decreases the amber or other warning signal, traffic, when it is traveling fast, receives less warning that it will be required to stop than when it is traveling more slowly. Obviously this is wrong. The amber period should either remain constant or else increase as traffic is speeded up and should either remain constant or decrease when traffic is slowed down. Any controller which causes its amber signal to increase or decrease as the total cycle increases or decreases will either provide too short an amber period at short cycles or too long an amber period at long cycles.

6. Means for exhibiting emergency signals by control from a central point. This usually consists of causing all signal faces to show red lights only and is for the benefit fo the fire department.

A description and brief history of two typical flexible progressive systems follow:

Chicago Loop System

This system was one of the earliest genuine flexible progressive systems to be placed into successful operation. It was so very substantially constructed that for six years it has functioned every day without an interruption worthy of mention. The flexible progressive system was installed in preference to other systems partly to minimize the peak load on the power station supplying power for the street railway system, as limited progressive and synchronized systems tend to make motormen start and stop many cars at the same instant at many street intersections.

From Fig. 3 the general construction of the controller is evident. It consists of individual units coupled together by a train of gears and driven by a motor. All of these units rotate at the same speed so that a complete cycle of signal changes at each intersection is accomplished in the same length of time irrespective of the relation of "go" periods to one another. Each dial shown in the illustration has associated with it four contacts, No. 3 closes circuit to north and south red lights and east and west green lights, No. 2 closes circuit to north and south green lights and east and west red lights. No. 1 and No. 4 close the circuit to amber lights in all four directions. The No. 3 contact and No. 2 contact are adjustable with respect to each other by arms in back of the dial, and by moving them to the proper positions, the red and green signals may be caused to show for any

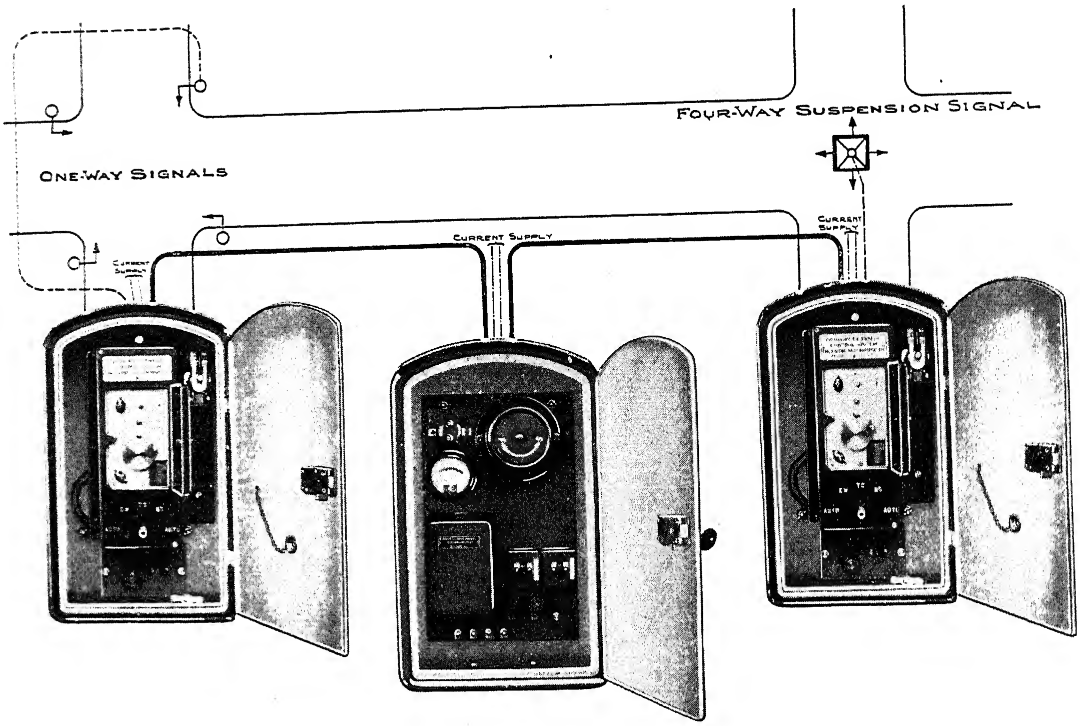

Fig. 2—Typical Master Controller Connected to Two Local Controllers

7. Means for aiding traffic movement during times of the day when traffic is very heavy on a particular street. Seldom does it happen that the ideal timing for conditions existing during most of the day is also ideal for peak hours. At such times facilities for increasing the "go" signals showing on the main thoroughfare or facilities for changing with respect to each other the times at which "go" signals at different intersections appear may meet the situation. If the "go" period is increased for the benefit of the main street, it gives cross street traffic less chance to move. If "go" signals at different intersections are shifted in time relation with respect to each other in order to favor traffic in one direction, whatever benefit is derived by traffic in that direction is at the expense of traffic in the other direction.

desired percentage of the cycle and the signals controlled by one unit may be caused to operate at any time with respect to the signals controlled by any other unit.

The total cycle may be increased or decreased by means of either one of two power units, each power unit consisting of a motor, an adjustable speed transmission and clutch which drive the entire train of gears. One advantage of the central office type controller is that as many and as wide variations in timing of signals as may be required to meet variations in traffic conditions

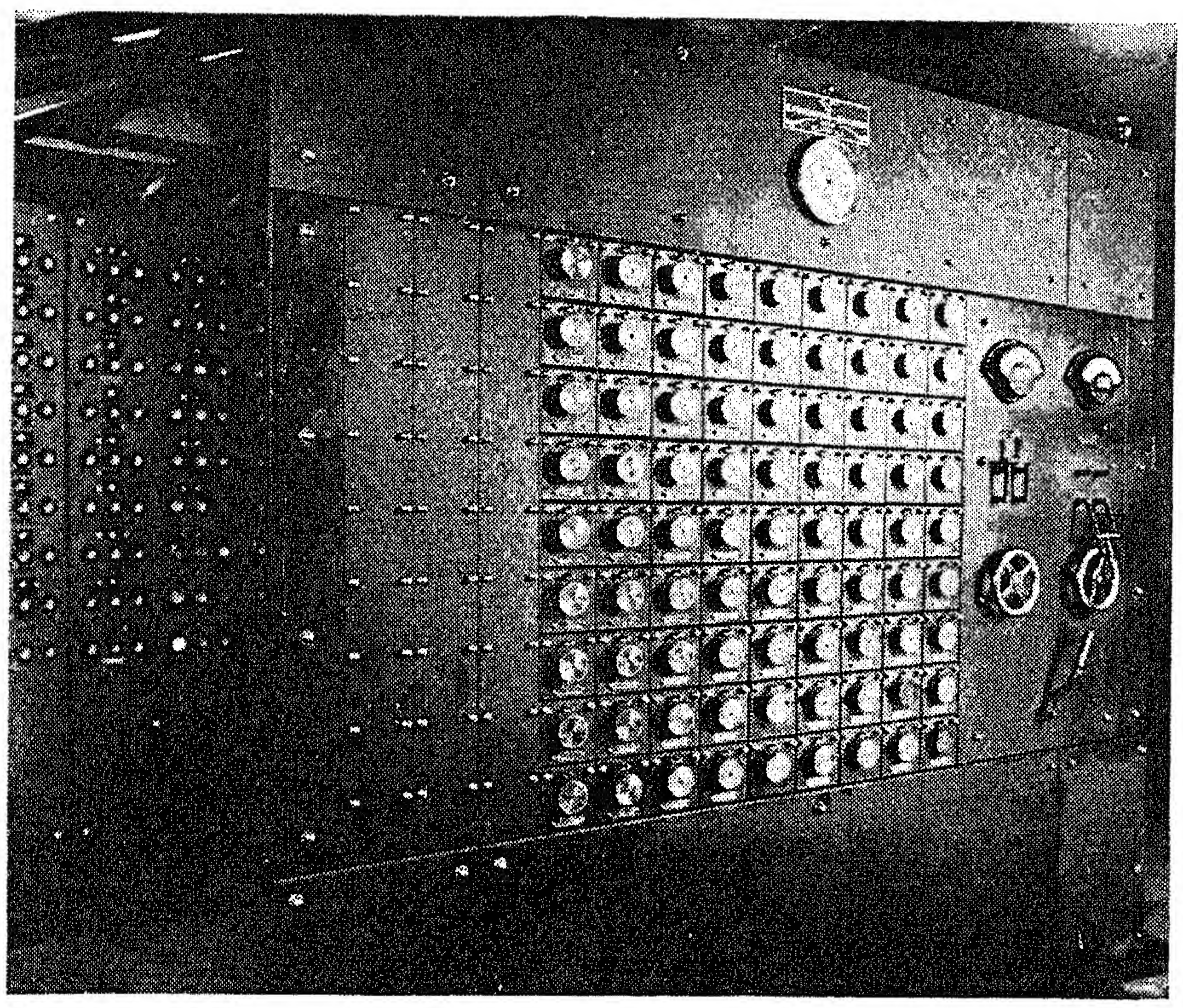

Fig. 3—Central Office Type Controller of Chicago Loop System

during the day may quickly be effected by the operator in charge. The controller is located near the central point of the Chicago Loop District and all power for the lights is conducted over wires issuing from this central office. This system is probably the most sturdy and simplest flexible progressive system in existence.

Wilshire Boulevard, Los Angeles, California

The flexible progressive system was specified for Wilshire Boulevard only after a long and intensive study. It has been stated that both the density and speed of traffic on this thoroughfare are higher than on any other avenue in the country. In any event, Los Angeles claims more automobiles in proportion to its population than any other large American city and this is reflected in an unusual traffic problem on Wilshire Boulevard.

Wilshire Boulevard uses the Coordiplex system. This system employs at each intersection controllers having two induction disk motors. One of these motors drives a cam shaft intermittently and its rotation causes the lights to change from "stop" signal to "go" signal or vice versa. The other motor called the "cycle motor" closes the contacts which start the first motor when it is time for signal lights to change. The cycle time is controlled by a master timer located at a central point. Once during every cycle of operation of the lights, all cycle motors are brought to a stop for about one second and by means of an impulse sent out from the master timer are caused to start into motion at the same instant. Errors in the speed of these motors are thereby cancelled by the beginning of each cycle of operation. The speed of these motors as well as the speed of the master timer which sends out the re-synchronizing impulse is varied by the voltage impressed on the system by an adjustable voltage auto-transformer located at the master switchboard. This system provides flexible progressive operation in its strictest sense. The time of the "go" and "stop" periods, also the timing of one controller with respect to others is set by self-locking arms on the cycle motor dial of each intersection controller. The system enables an operator instantly to cause all signals to show red lights for the

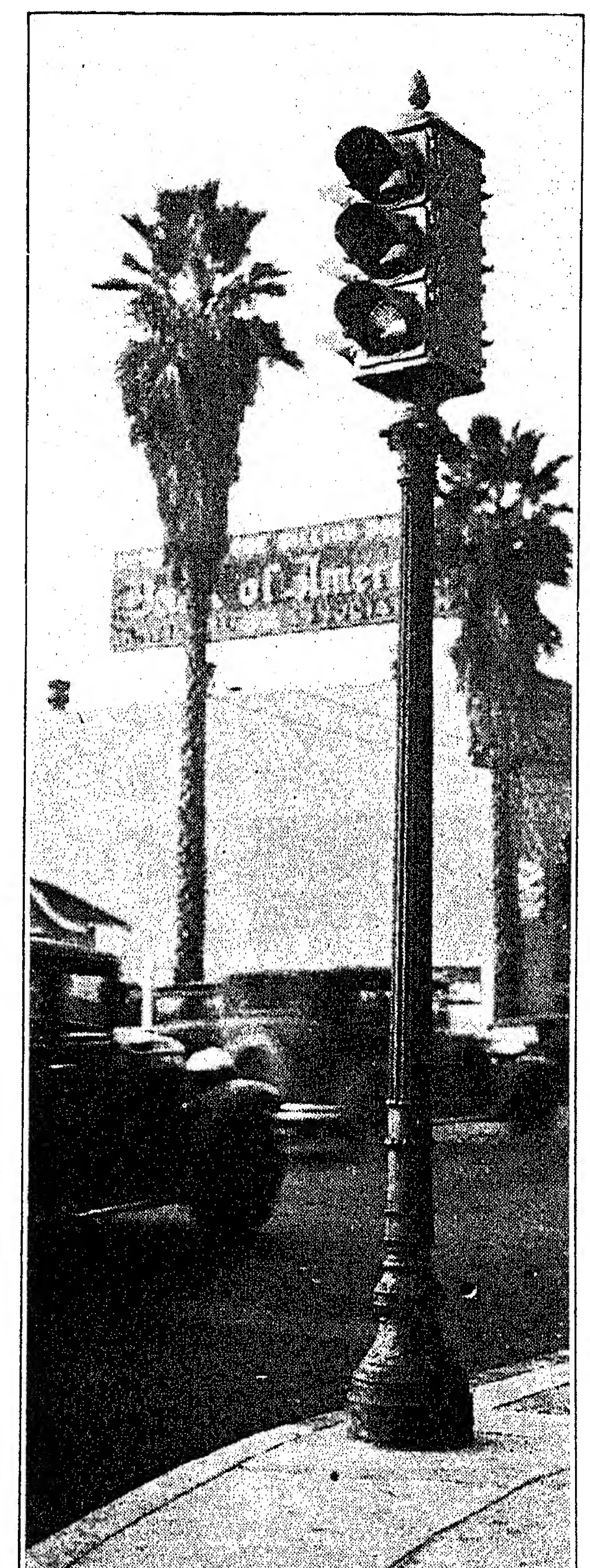

Fig. 4—Wilshire Boulevard Signal

benefit of the fire department. It may be operated at late hours of the night with flashing amber showing on Wilshire Boulevard and flashing red showing on the cross street. Amber lights preceding the color change may be set at any desired interval, but when once set the interval does not vary when the cycle time is changed at the master controller. All controllers may be operated manually and one position on the manual control provides for red lights in all directions at the intersection under manual control.

The Electro-Matic Traffic Dispatching System

BY H. A. HAUGH, JR.*

Associate, A.I.E.E.

THE electro-matic vehicle-actuated traffic-dispatching system, as its name implies, is a system of traffic control by which the traffic in a given area controls itself. In its operation it approaches the ideal system in which the right-of-way or "green light" period is assigned to the approaches to an intersection in proportion to the volume of traffic on each approach. For example, if at the junction of A and B streets there is twice as much traffic on A street as on B street, the timing of the traffic-light cycle will be so adjusted that, within certain limits, the A street green light period will be twice as long as the B street green light period. A few minutes later the distribution of traffic may be reversed. B street may carry twice as much traffic as A street. When this condition occurs the traffic light cycle will be readjusted in its timing to give B street the longer green light period.

Originally designed for use at the intersection of only two streets the system has since been expanded and improved until it is now in use at intersections where five or six streets meet. Provision is made for handling trolley cars and for protecting pedestrians. Under traffic conditions varying from instant to instant it operates to produce a smooth flow of vehicles, using the intersection at its greatest efficiency.

The electro-matic system consists of three parts:

1. The detectors, which are switches operated by vehicles approaching the intersection.
2. The signal lights, which may be any one of the standard types of red, amber and green traffic lights. The amber light may be eliminated if so desired.
3. The dispatcher, which receives electrical impulses from the vehicle detectors and controls the signal lights in accordance with the traffic requirements as indicated by the detector impulses from all streets.

THE DETECTOR

The detector may be any one of several types such as a mechanical switch, an electromagnetic unit, or a beam of light which will be interrupted by approaching vehicles. Experiments have been made with all of these types of vehicle detectors and all of them are now in use in electro-matic systems.

The mechanical type of detector is the simplest, cheapest and the most practical. The mechanical detector used in this system consists of a pair of thin flexible steel plates separated by a strip of rubber around the edge which holds them normally one-quarter of an inch apart. The passage of a vehicle over the detector flexes the upper plate so that it makes contact with the lower plate thereby causing a current to flow to the dispatcher. These steel plates are completely enclosed in a rubber package which is vulcanized to them, thus making the whole a water-tight unit. A lead wire is attached to each plate and brought out from the package through a rubber pig tail. Fig. 1 is an illustration of a detector package. The detector package is laid in an iron casting, a strip of rubber one-half inch thick is bolted over the top of this casting to protect the package from mechanical damage by the cars which cross it, and the casting is ready to be placed in the street for service. At one end of the iron casting is a junction box in which the wires running to the dispatcher are connected by a water-tight joint to the leads from the detector package. This construction gives an extremely simple detector unit absolutely water-tight and thoroughly reliable (there are no levers, pinions, or bearings to wear or get out of order). Operating with an electro-matic dispatcher this type of detector will "pick up" a vehicle traveling at sixty miles an hour. Fig. 2 shows a mechanical detector imbedded in the pavement.

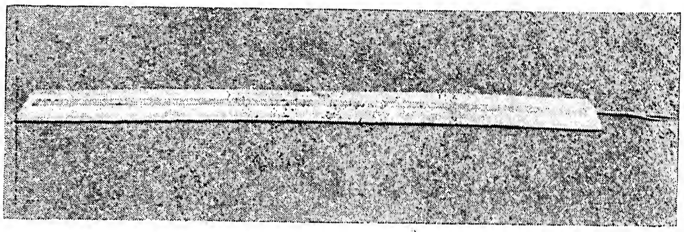

FIG. 1—A DETECTOR PACKAGE

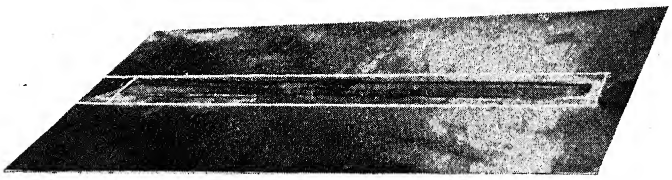

FIG. 2—A DETECTOR IMBEDDED IN THE PAVEMENT

The casting which holds the detector package is twelve inches wide, two inches deep and is made in three lengths, four, six, and eight feet. The shortest is used between the rails of a trolley track and the other two sizes are used to "cover" streets of different widths. In "covering" the side of a street on which vehicles approach an intersection it is not necessary to place the detectors immediately adjacent to each other. If the street is wide enough to require two or more detectors they are located along a line at right angles to the curb and are spaced four feet apart end to end. Since the

*Vice President, Automatic Signal Corp., New Haven, Conn.

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tread of a standard vehicle is more than four feet, one wheel is bound to cross one of the detectors thus sending an impulse to the dispatcher. If a vehicle crosses the detector in such a way as to send in several impulses some caused by the front wheels and some by the rear wheels, the operation of the system is not impaired.

Occasionally a street will be so narrow that vehicles going in either direction must perforce travel the middle of the highway. To prevent the sending of impulses by vehicles leaving such an intersection a special "directional" detector is used. The directional detector sends an impulse to the dispatcher from those vehicles approaching the intersection but does not send any impulse when crossed by a vehicle leaving the intersection. This effect is accomplished by splitting the top steel plate lengthwise and bringing out three leads from the detector package instead of the usual two. One of these leads is attached to each half of the top plate and the third is attached to the single bottom plate. By longitudinally splitting and electrically separating the top plate into the two parts, one on each side of the center line of the detector package, two impulses are sent to the dispatcher by each vehicle crossing the detector. Calling these impulses A and B it is evident that a vehicle crossing the detector will send in the two impulses, one shortly after the other. If a car moving toward the intersection sends the impulses in the order A followed by B, a car moving away from the intersection will send them in the reverse order, B followed by A. A relay system is arranged to pass on to the dispatcher the impulse A followed by B but it will stop that of B followed by A. Thus the directional effect is achieved.

The Dispatcher

The outstanding characteristic of the electro-matic system is its extreme flexibility. The timing of the signal light cycle is constantly changing to meet the irregular demands of traffic. The secret of this flexibility is to be found in the dispatcher, which is essentially a group of time-delayed relays combined with a switching mechanism. The switching mechanism operates the traffic signal lights. It receives and stores until used all impulses sent from the vehicle detectors. It also switches the group of time-delayed relays and vehicle detectors into different parts of the circuit as the green light is shifted from street to street. The time-delayed relays act as watchmen who keep an eye on each street while it has the green light, to see that it does not hold the right-of-way longer than is necessary. These "watchmen" will go so far as to stop a line of moving cars, when occasion requires, in order to give traffic on a cross street a chance to move. The functioning of these relays is worthy of some consideration.

There are three time-delayed relays in this group. One of them measures the amber light period and the "initial period" which will be discussed later in this paper. The operation of the second of these relays (called the vehicle interval relay) signifies that a line of cars has completely passed the intersection and puts the system into such a condition that a "call" from the cross street will cause immediate transfer of the right-of-way thereto. This transfer, however, takes place only when cross traffic "requests" it.

The operation of the third time-delayed relay (called the maximum-interval relay) signifies that a car has been kept waiting on the cross street for a period of time sufficiently long to warrant arbitrarily stopping the moving traffic to let it move into the intersection. When the maximum-interval relay operates it produces an effect beyond that of the vehicle-interval relay. The vehicle-interval relay merely makes the right-of-way subject to call by cross-street traffic. The maximum-interval relay, on the other hand, causes immediate transfer of the right-of-way to the cross street.

The vehicle-interval relay serves to protect a moving line of cars against unwarranted interruption by cross traffic. The maximum-interval relay serves to protect waiting cross traffic against being unreasonably delayed by a "solid" line of moving cars.

As might be expected from the difference in their functions these two relays are energized through separate circuits. The vehicle interval relay is energized through a rather peculiar connection in order to permit a "solid" line of moving cars to protect its right-of-way by preventing this relay from operating. As each street is given the green light the vehicle detectors located in that street are connected by the dispatcher switching mechanism to the vehicle-interval relay in such a way that impulses from these detectors will reset this relay. Hence the vehicle-interval relay will not operate to yield the right-of-way to cross traffic so long as cars coming into the green light continue to reset it by operating the detectors at short intervals. A break in the line of moving cars will, however, give the vehicle-interval relay time to operate and put the system into a condition to respond immediately to "calls" from cross-street traffic. Whenever the intersection is free of moving traffic this relay causes the right-of-way to be subject to call by cars on the cross street. But, on the other hand, it will not operate and therefore will not release the right-of-way to cross traffic so long as cars continue to move through the intersection.

The maximum-interval relay is energized as soon as a car has indicated its presence on the cross street by operating the cross-street detector. This relay is not reset by moving cars. Therefore, if at the end of a predetermined period subsequent to the arrival of cross traffic, there has been no break in the moving traffic, the maximum period relay will operate to transfer the right-of-way to the waiting cross traffic.

It can be seen that the vehicle-interval relay and the maximum-interval relay working together cause the right-of-way to be yielded to cross-street traffic when the intersection is clear, or even when it is not clear if the cross traffic has waited for a predetermined period, but

they never interrupt moving traffic unless the right-of-way has been called to the cross street by waiting traffic or a "pedestrian period."

Time Intervals

The time intervals measured by this group of time-delayed relays are basic units in the electro-matic system and may be described as follows:

1. The *amber interval*, the period during which the amber light is shown.
2. The *vehicle interval*, the time required for a single vehicle in motion to pass through the intersection.
3. The *initial interval*, an additional interval inserted at the beginning of each green light period in order to give those cars which are not in motion a little extra time to get under way.
4. The *maximum interval*, the time during which the traffic in one street can hold the right-of-way against waiting cross traffic.

The vehicle interval is such an important unit in the electro-matic system that it merits more detailed consideration.

It is measured by the vehicle-interval relay and is the means by which moving cars can extend the right-of-way period between the minimum limit (set by the initial interval plus one vehicle interval) and the maximum limit fixed by the maximum interval. Each car which crosses the detector in the street which has the right-of-way cancels, by resetting the vehicle interval relay, the remainder of the vehicle interval allotted to the preceding car and starts a new vehicle interval to hold the green light while it passes through the intersection. The cancellation of the remainder of each vehicle interval by the succeeding vehicle is a very important factor in providing for the highest efficiency in the use of the intersection. In operation, this cancellation means that the cross traffic does not have to wait for a series of accumulative vehicle intervals to expire. Right-of-way will be transferred to the waiting cross traffic immediately after the last car of a moving line has passed through the intersection.

To demonstrate how these time intervals are used to produce a smooth flow of traffic assume four different traffic conditions at the intersection of A street and B street. These traffic conditions may be designated as cases 1, 2, 3, and 4, described as follows:

1. Cars moving on A street with no traffic on B street.
2. Cars moving on A street with one car waiting on B street.
3. Cars moving on A street with several cars waiting on B street.
4. Heavy traffic on both A street and B street.

In case 1, the cars on A street have the right-of-way and there is no waiting traffic on B street. The maximum interval is not being measured since no B street traffic has called for the right-of-way. The initial interval has expired. The vehicle interval is being intermittently renewed through the resetting of the vehicle interval relay by the cars crossing the detectors on A street. The vehicle interval relay is operating from time to time. However, since the right-of-way has not been called to B street, the operation of the vehicle interval relay does not act to transfer the right-of-way thereto, the green light continues to shine on A street and A street traffic flows without interruption.

In case 2 traffic is moving out of A street and a single car is waiting on B street. The A street initial interval has expired. Let us assume that the cars on A street are crossing the detectors at sufficiently short intervals to reset invariably the vehicle interval relay before it can operate. The vehicle interval relay therefore tends to hold the right of way on A street. However, the A street maximum interval is being measured because the car in B street, when it crossed the B street detector, energized the maximum interval relay. The transfer of the right-of-way to B street will be made in this case when the A street maximum interval has expired. If, on the other hand, we assume that the cars on A street are "straggling" or that the A street line of traffic completely passes the intersection before the maximum interval expires, the right-of-way will be yielded by the operation of the vehicle interval relay as soon as the A street traffic breaks. It can be seen that, with waiting cross traffic, the vehicle-interval and the maximum-interval relay are both energized, the right-of-way being yielded by the first to operate. Inasmuch as in this case there is only one car on B street, the green light will revert to A street by action of the vehicle-interval relay, after the initial interval and one vehicle interval have elapsed.

In case 3 the right-of-way will be transferred to B street exactly as in case 2. The instant that the B street green light is illuminated the timing of the initial interval begins. This will be succeeded by the first vehicle interval. However, in this case there are several cars waiting on B street. Before the end of the first B street vehicle interval this line of cars will be in motion and will be crossing the B street detector thereby always resetting the vehicle interval relay each time before it has time to operate. These cars will therefore hold the right-of-way on B street until they have had sufficient time to pass through the intersection, the time being assumed to be less than the maximum interval allowed for B street traffic against waiting A street traffic. As the last B street car passes the intersection the right-of-way will be transferred to A street by the operation of the vehicle interval relay.

In case 4 where traffic was assumed to be heavy on each street, the vehicle interval relay will be repeatedly reset by successive cars thereby holding the right-of-way on each street until the operation of the maximum relay yields it. It is surprising how seldom this condition occurs even when traffic seems to be very heavy. Interruptions constantly occur in the flow of traffic. A trolley or a bus must, in providing the public with adequate service, stop at any street corner when

passengers require it. The electro-matic system permits cross traffic to take advantage of all such breaks in the moving traffic.

The electro-matic dispatcher has been described in some detail from a functional standpoint. A short description of its actual makeup may prove interesting. Fig. 3 is a front view of a dispatcher. The dispatcher was described earlier in this paper as a combination of a switching mechanism with a group of time-delayed relays. The switching mechanism consists of a shaft on which are fixed several cams and a separate group of contacts which are either opened or closed by the action of these cams. A cam shaft with some associated contacts is shown in Fig. 4. In completing one signal cycle this cam shaft makes one revolution. It is operated by a solenoid-actuated ratchet and pawl, Fig. 5, which turns it through an angle of 60 deg. at a time. The solenoid which drives the cam shaft is in turn controlled by the time-delayed relays. Hence, when the time-delayed

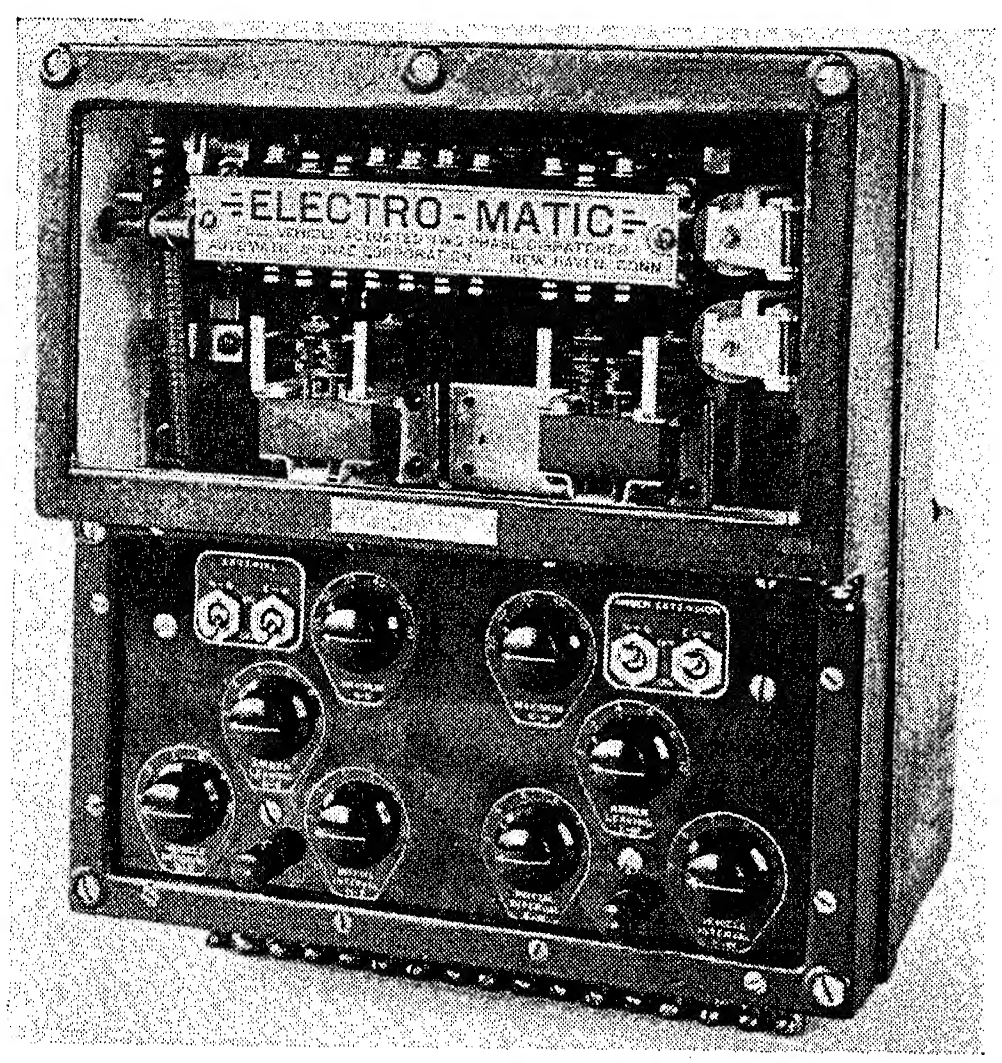

Fig. 3—The Front View of a Dispatcher

relays say in effect that traffic conditions warrant it, the solenoid advances the cam shaft into its next position thereby also advancing the signal lights one step in their cycle. In addition, as stated before, the vehicle detectors and the time-delayed relays are by this act switched into the particular place in the circuit which they should occupy during this step of the traffic signal cycle. Presently the time-delayed relays will again act to advance the cam shaft into its next position thereby advancing the system one more step in its cycle. Six such steps constitute a complete cycle which is repeated as often as traffic conditions may require it. In following the operation of the cam shaft through one cycle assume a standard red, amber and green color cycle with amber displayed on both streets between the red and the green periods.

In each cam-shaft position one of the time intervals upon which the electro-matic system is based is being measured to determine when to move the shaft into its next position. Again referring to two intersecting streets as A and B streets, let us designate as position 1 the position which the cam shaft holds when A street has just been given the right-of-way. The shaft remains in position 1 during the A street initial interval. The time-delayed relay measuring this interval is energized continuously while the cam shaft is in position 1. To end this period the initial-interval relay operates the

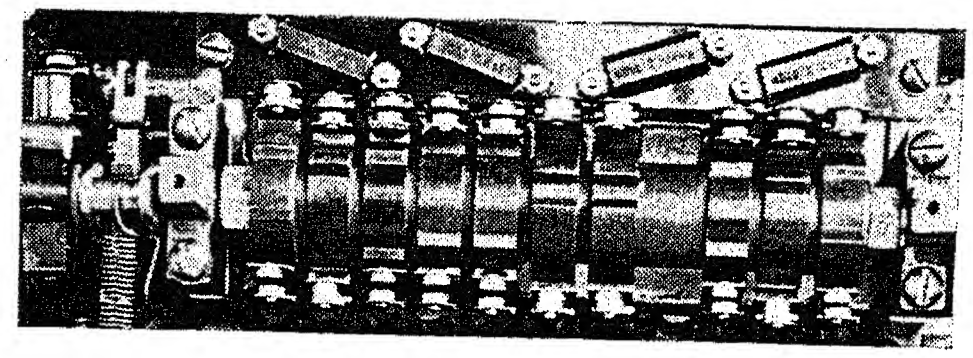

Fig. 4—A Cam Shaft with Some Associated Contacts

solenoid to move the cam shaft into position 2. No change in the signal lights accompanies the change from position 1 to 2. Position 2 might aptly be called a waiting position. The cam shaft does not move again until traffic requires the right-of-way to be shifted to B street. While the shaft is in position 2 the vehicle interval relay is energized and if there is waiting cross traffic the maximum interval relay is also energized. When the right-of-way is to be transferred to B street either the vehicle-interval relay or the maximum-interval relay will cause the solenoid to move the cam shaft into position 3. In this position the cam contacts close the circuit to the amber lights, energize the amber interval relay and open the circuits feeding all red or green lights.

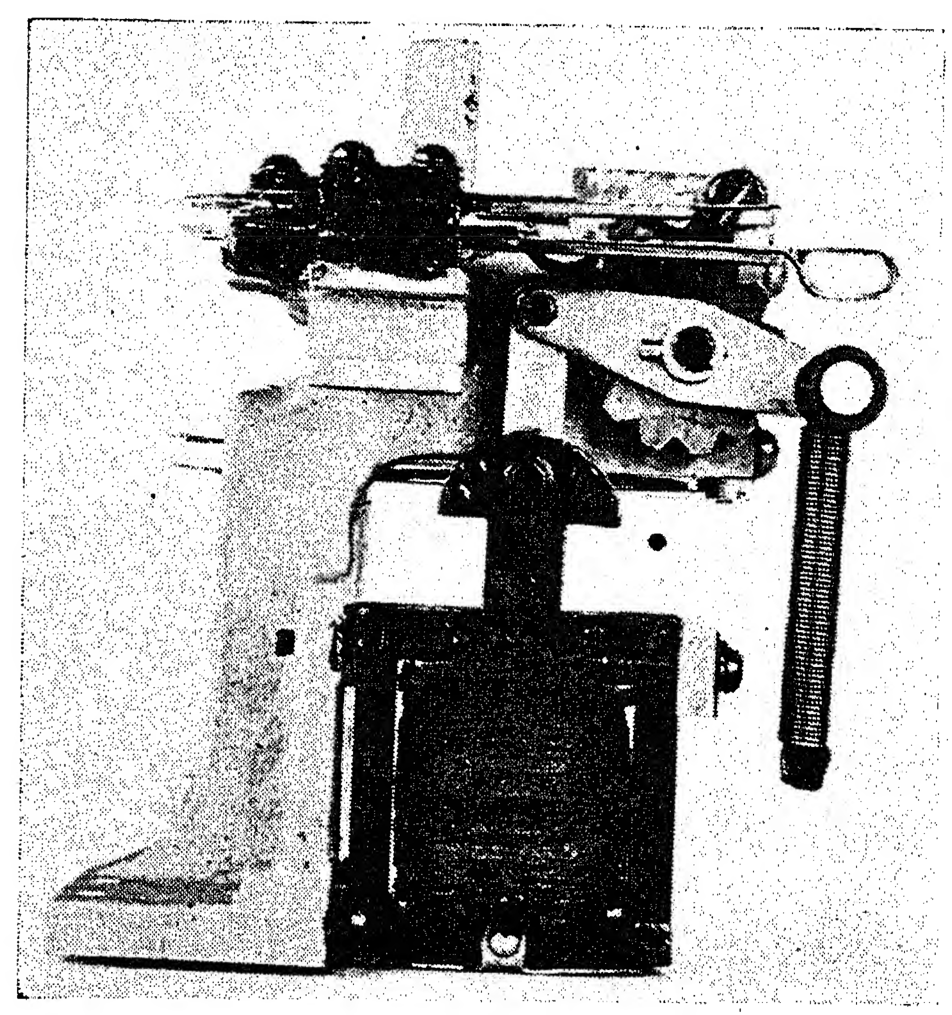

Fig. 5—The Cam Shaft Solenoid Ratchet and Pawl

At the end of this period the amber interval relay operates to cause the solenoid to advance the cam shaft to position 4. Positions 4, 5, and 6, correspond respectively to positions 1, 2, and 3. Inasmuch as the latter three positions differ from the former only in that the right-of-way is being transferred to A street instead of to B street, there is no need to follow this operation through positions 4, 5 and 6.

The Timing Units

It is desirable that each of the four basic time intervals of the electro-matic system should be variable in magnitude as they are applied to traffic in different streets. A slight grade on one approach may cause cars entering the intersection from that street to be somewhat slower in starting than those on the cross street. To compensate properly the initial interval and perhaps the vehicle interval on the "slow" street should be somewhat longer than the corresponding intervals on the other. Or it may be desirable to allow a longer maximum period for one street than for the other. "Static timing" as used in the electro-matic system greatly facilitates such variations.

The static timing unit (the type of time-delayed relay now used in the electro-matic system) consists of a

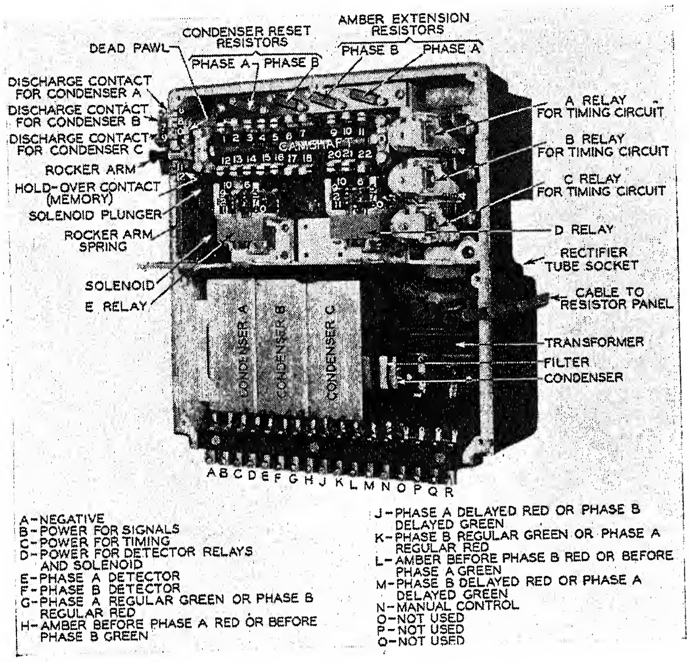

Fig. 6

Strowger relay, a condenser, a resistance unit and a grid-glow flashover tube. The timing is actually accomplished by applying the well-known fact that the period required to charge a condenser to a given potential, under a uniform impressed e.m.f., depends upon the magnitude of the resistance in series with the condenser. The grid-glow tube is shunted across the condenser through the coil of the Strowger relay. When the condenser voltage reaches the critical voltage of the tube the tube "breaks down" and becomes a conductor. This permits the energy stored in the condenser to discharge through the relay coil which causes the relay to operate, thus signifying that the interval being measured has expired. As this timing unit is switched from place to place in the circuit, the charging resistance in series with the condenser is changed to meet the different timing requirements. Thus, for instance, the vehicle-interval timing unit, which serves each street in turn, may be set for a five second period on one street, whereas on another street it may operate after eight seconds. In a like manner the initial- and the maximum-interval relays can be used to time their particular intervals for one street after another, giving a different period for corresponding intervals on each street. The parts of these timing units are so located in the dis-

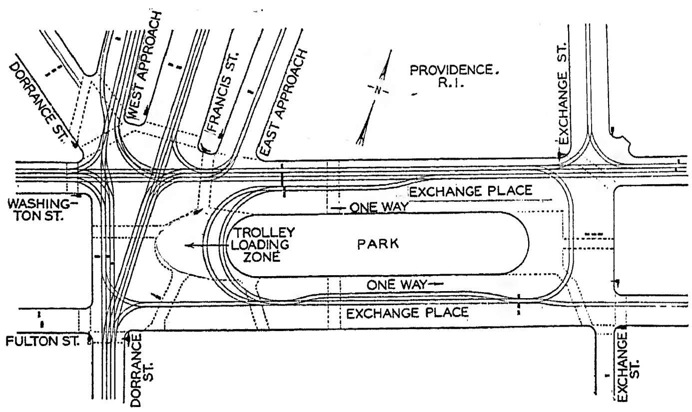

Fig. 7

patcher that no photograph can include them all. Three condensers, one for each timing unit, with their associated Strowger relays may be seen in Fig. 6.

The important function of resetting the vehicle-interval relay is accomplished by discharging the condenser in this timing unit by action of a vehicle-detector relay.

Each timing interval, the initial, the vehicle, the maximum and the amber interval is independently adjustable for each street. This adjustment is made by

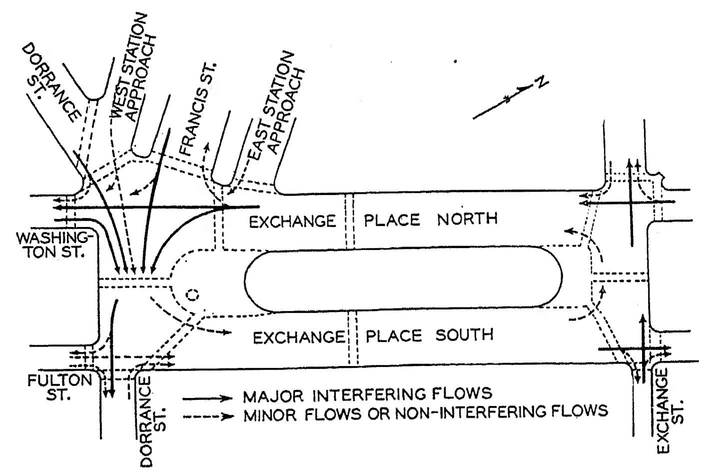

Fig. 8—Present Traffic Flows Exchange Place

turning a knob on a panel until the arrow thereon points to the number of seconds desired for the particular interval controlled by that knob. As a knob is turned it introduces resistances of different magnitudes into the condenser charging circuit which it controls thereby changing the time required to charge the condenser to the critical voltage of the grid-glow tube.

This paper describes a standard two-phase system which provides complete vehicle actuation for signals governing two interfering lines of traffic. By using a somewhat more complicated switching mechanism in conjunction with the three basic timing units the system has been adapted to control traffic at intersections where as many as six or seven interfering flows cross. Furthermore, adjacent intersections may be interconnected so as to operate with a certain degree of coordination without unduly interfering with the free vehicle actuation of each. Exchange Place in the center of Providence, Rhode Island, provides a good example of this type of installation. The location of vehicle detectors and signal lights is shown in Fig. 7. The various traffic flows are shown in Fig. 8, a study of which shows that there are several one way streets.

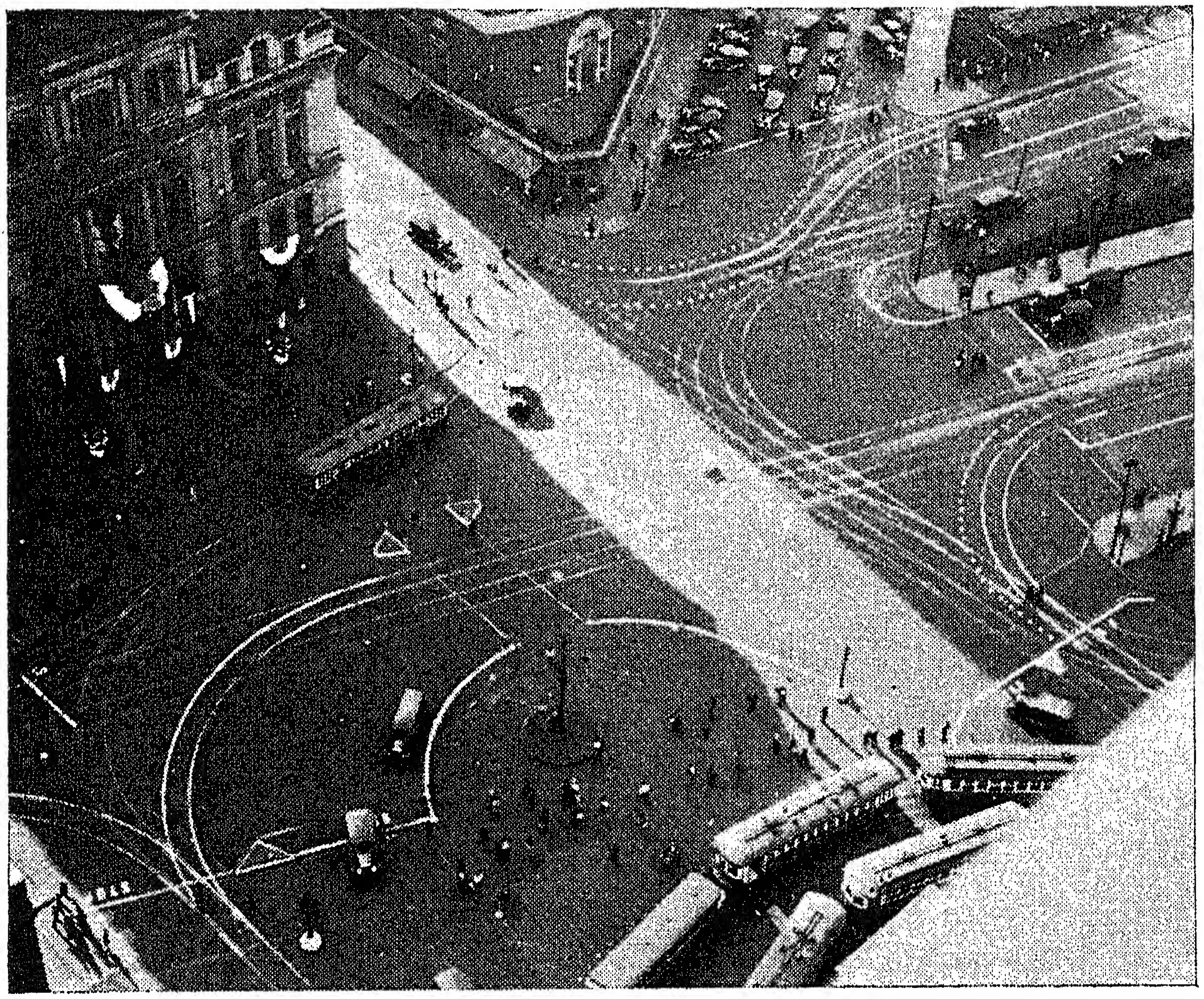

FIG. 9—EXCHANGE PLACE SHOWING TROLLEY LOADING STATION

The traffic in each corner of Exchange Place is controlled by a separate independent vehicle-actuated dispatcher which gives the right-of-way in general to north-south traffic and to east-west traffic independently in order to eliminate crossing flows. The right turn traffic out of Washington Street, that from the southern end of Exchange Place into Fulton Street, and that out of Dorrance Street South, as well as the left turn traffic around the central section, moves at all times except during pedestrian protection periods. Pedestrians are a major factor in Exchange Place traffic. A trolley loading station located in the southern end of the Place, shown in Fig. 9, makes it necessary for thousands of people to cross the streets clustered about this point. To give these people a maximum of protection with a minimum of inconvenience to the motorists a "progressive" pedestrian period is provided. This period is inaugurated by showing the red light on all streets at the southwest corner, thereby stopping all traffic inbound to the southern end of Exchange Place. At the same instant the dispatcher controlling the traffic at the southeast corner begins to limit signal changes there in order to be ready for its pedestrian period which will commence 15 seconds later. This 15 seconds between the inception of the pedestrian periods at the two corners permits cars which were already in the southern end of Exchange Place when the pedestrian period was started at the southwest corner to pass out of this district without being stopped at the southeast corner. Hence the whole southern end of Exchange Place is cleared of traffic to give the pedestrians unhampered use of this district. At the termination of the pedestrian period traffic is first allowed to move at the southwest corner. A few seconds later, just as the first cars moving through the southern end reach the southeast corner, the pedestrian period is ended there and traffic again flows smoothly.

A Recent Development in Traffic Control

BY H. W. VICKERY*
Non-member

and

V. W. LEONARD*
Non-member

Synopsis.—The large number of automobiles in use today has introduced new problems in the control of vehicular traffic. Timing devices with increased flexibility are required so that signals can be operated more nearly in accordance with varying conditions of traffic flow. A recently developed timing device, containing new features and having increased flexibility, offers a solution of the problems involved. The timer is described and its advantageous features are discussed.

Introduction

CONGESTED traffic conditions of the present day indicate the necessity for greater flexibility in timing apparatus so that signals can be set in step with traffic and adjusted to accommodate variations in traffic flow. Rapidly changing traffic conditions emphasize the need for timing devices capable of adjustment to meet future requirements. As a solution of these problems, a timing device, with new characteristics and extremely wide ranges of adjustment, has recently been developed.

Types of Traffic Control Systems in Use

Traffic control systems now in use are of the following general types: first, the isolated system for controlling traffic at a single intersection; second, the non-interconnected progressive system in which timing devices at various intersections, though not interconnected in any way, can be adjusted to operate the signals in a progressive relation; and third, the interconnected progressive system in which not only a progressive relation of the signals is maintained but, in addition, certain features are obtained by remote control, such as change of total time cycle, selection of various progressive relationships, shut down, flashing amber and fire control.

A fourth type of interconnected system is that in which timing apparatus at the central point alone controls signals at a number of intersections. One form, in which a single timer controls the system, is extremely limited as far as flexibility is concerned. In its other form, where a timer for each intersection is located at the central point, the installation cost is so high as almost always to be prohibitive.

The discussion, therefore, is confined to the first three systems mentioned, in which individual timing devices are located at each intersection.

Advantages of Synchronous Motor Drive

Timing devices in non-interconnected progressive systems must, of necessity, be driven by synchronous motors in order that the progressive relation between intersections be maintained.

The advantages of synchronous motor drive in isolated and in interconnected progressive systems are brought out by consideration of a practical form of timing device. Such a device is shown in Fig. 1.

The synchronous motor (Fig. 2) operates on the hysteresis principle. Thus, unusual torque is developed with the expenditure of a small amount of power. The rotor is unconventional in the fact that it simply consists of hardened steel laminations spun into a drawn brass shell. All windings are stationary, surrounded and protected by the rotor. This construction produces a sturdy motor unit, insusceptible to damage in operation or handling as opposed to the inherently frail disk type of motor commonly used.

Any setting which may be made on a synchronous timer, whether it be total time cycle, proportion of periods in the color sequence, or progression setting is exact. All dials are marked in exact values due to the fact that the speed of a synchronous motor is independent of voltage, temperature and even fairly wide friction changes.

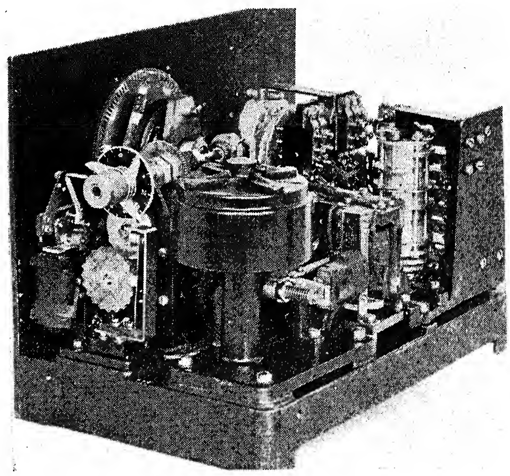

Fig. 1—Rear View of Synchronous Timer with Cover Removed

Any setting which may be made on an induction timer, on the other hand, is inexact. The total cycle dial must be calibrated, that is, various total cycles must be measured by means of a stop watch or some other standard and the dial marked to correspond. Changes in temperature, voltage or friction render such dial markings inaccurate.

Induction timers in interconnected systems require a compensating period, which is introduced once each

*Meter and Instrument Engg. Dept., General Electric Co., West Lynn, Mass.

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cycle, to correct for the inaccuracies in speed of the various timer motors. The master controller is set to operate at a time cycle slightly longer than that of the intersection timers. The intersection timers complete one total cycle at which time they stop and wait for the master controller to overtake and release them.

Synchronous timers in interconnected systems, on the other hand, due to their time keeping accuracy, require no waiting or compensating period. A short resynchronizing interval is provided once each cycle, simply

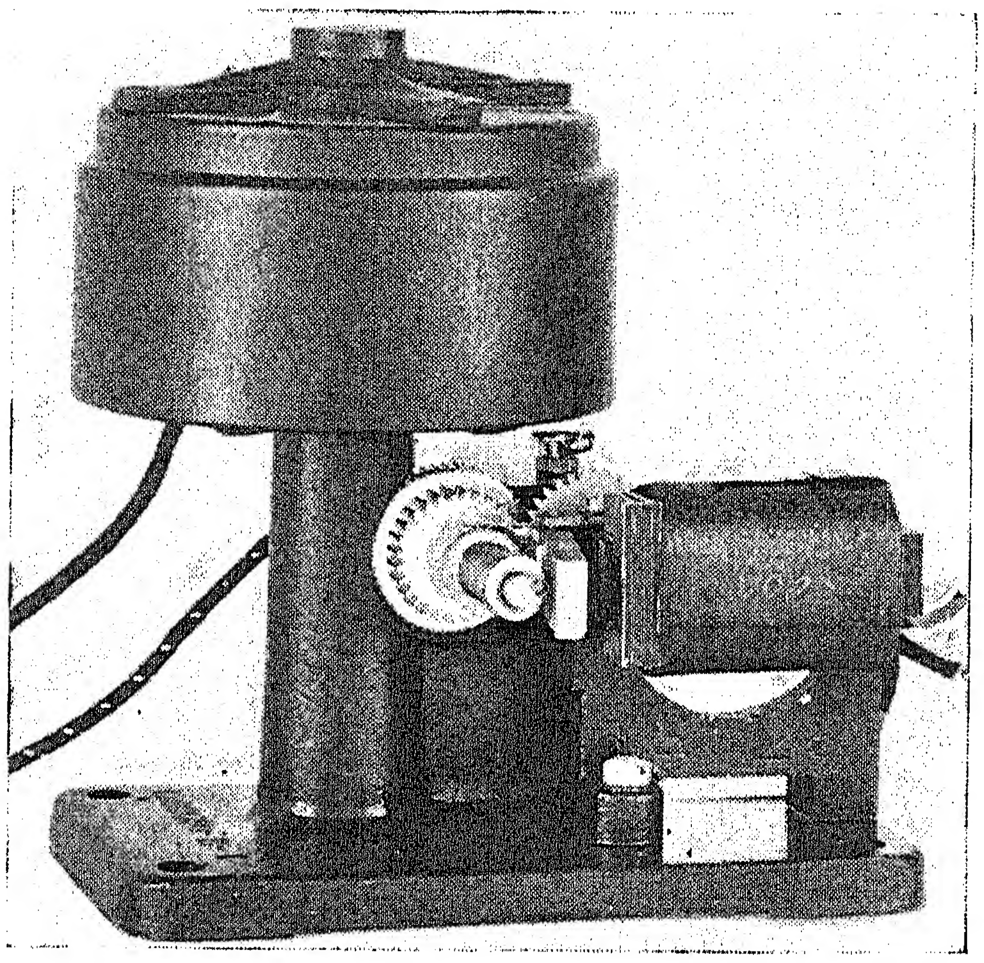

Fig. 2—Synchronous Motor Unit

as a supervisory check, so that any timer which may be out of step, due to servicing, tampering or other reasons, will immediately be placed back in step. The master controller is merely a supervisory device, whose function is not to keep the timers in step, but simply to place them in step.

While it is true that interconnected induction timer systems, in some cases, continue to operate even though central power fails, they usually speed up in case of such failure and, due to their non-synchronous principle, are no longer able to maintain progressive relationship. The interconnected synchronous timer system, on the other hand, not only continues to operate when central power fails but, due to its synchronous principle, maintains the same total cycle, the same progressive relationship and the same percentage values as before.

Another advantage of synchronous motor drive is the fact that flashing signal lights can be made to operate at a given, constant and exact speed, regardless of any change in total cycle which may be made. Such operation is not possible with an induction timer because in such case, the rate of flashing changes in proportion to the total cycle.

Change of Total Time Cycle

The gear shift unit used for changing the total time cycle is shown in Fig. 3. It consists of a cone of gears of varying diameters and an idler gear which can be shifted to engage any one of the cone gears. The gears can be shifted manually at each timer or, in the case of interconnected systems, from a remote point. The total cycle range is flexible, usually consisting of ten steps with five seconds between steps.

While it is true that an induction timer, in which the total time cycle is usually changed by varying the motor speed, can apparently be set in steps much smaller than five seconds, it is probable that, due to voltage, temperature and friction changes, the actual total time cycle varies as much if not more than five seconds from the time cycle indicated on the dial.

Selection of Progressive Relationships

In order that traffic may move along a street at a given uniform speed and with a minimum number of stops, it is necessary that the signals be set in step so that traffic arrives at each intersection during the green or "go" indication. Setting the signals in step gives a progressive movement to traffic, the traffic moving in groups or platoons along the street.

To give ideal progressive movement to traffic in a given direction, it is necessary that the beginnings of the green indications of the signals occur successively one after the other in the given direction, the time elapsing between the beginning of the green indication of any one signal and the beginning of the green indication of the next adjacent signal in the given direction being equal to the time required to travel the distance between the two signals at the given speed.

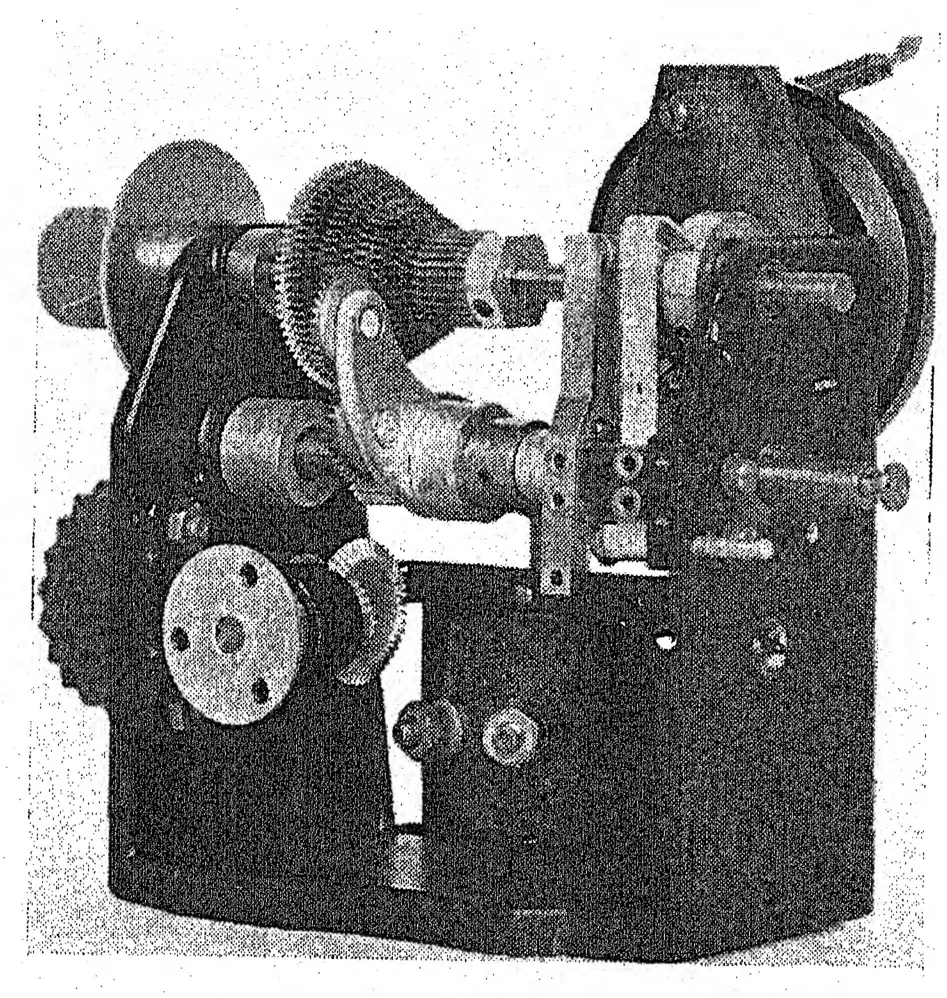

Fig. 3—Gear Shift Unit

Unfortunately, due to unequal distances between signals and due to the particular speed at which it is desired to have traffic move, setting the signals in step to give ideal progression in one direction usually results in very poor progression in the opposite direction (Figs. 4 and 5). The beginnings of the green indications of the signal, almost invariably, do not occur in a sequence in the opposite direction to allow as large a number of automobiles to travel without stopping, as is possible in the direction of ideal progression.

The usual procedure, where only one progressive relation of the signals is available, is to set the signals in step so as to give average progression in both directions (Fig. 6). With average progression, however, the number of automobiles which can travel in either direction without stopping is smaller than would be possible in the direction of ideal progression as shown in Figs. 4 and 5. Furthermore, continual stopping and starting

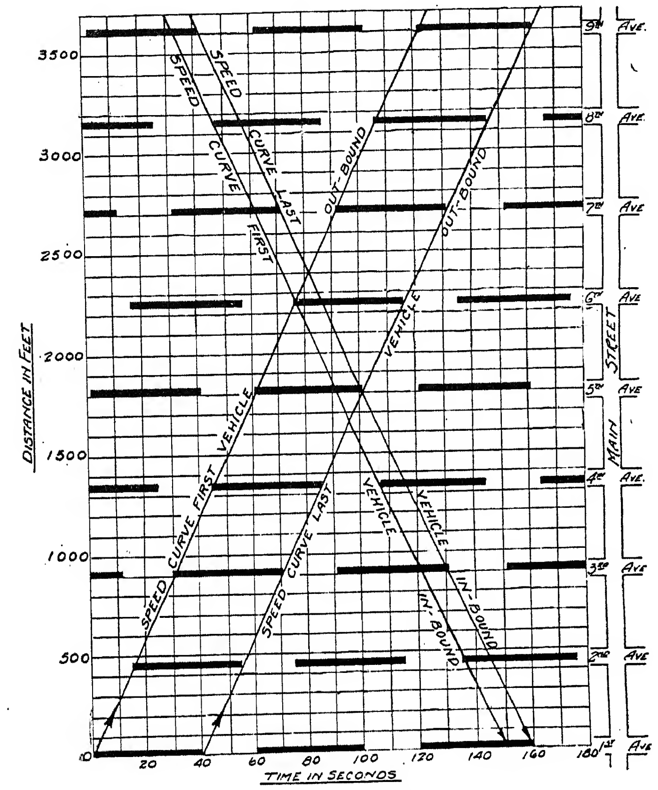

FIG. 4—SCHEMATIC TIMING DIAGRAM SHOWING IDEAL PROGRESSIVE RELATION FAVORING OUT-BOUND TRAFFIC

Traffic speed—20 miles per hour. Total time cycle—60 seconds
Main street "green"—40 seconds indicated by heavy lines

of some of the traffic, due to the beginnings of the green indications of the signals being out of step, creates confusion and delay, thereby diminishing the number of automobiles which otherwise would be able to proceed without stopping.

Since, in large urban districts, traffic on a given street may be mostly inbound in the morning, mostly outbound in the later afternoon, and about equally divided in both directions at other hours of the day and since the volume of traffic in such districts is usually much greater during the morning and late afternoon than at other hours of the day, it is obvious that a single progressive relation of the signals does not meet these three fundamental traffic conditions in the best manner possible. A means, therefore, has been developed in the timer to give three separate and distinct progressive relations of the signals. Thereby, the progressive relation can be changed remotely to give ideal progression to inbound traffic in the morning, ideal progression to outbound traffic in the late afternoon and average progression in both directions at other hours of the day.

Adjustment Flexibilities

Since traffic conditions change from month to month and from year to year, sufficient flexibility should be built into traffic signal timing apparatus to insure that such apparatus shall not shortly become obsolete. In other words, a traffic signal timer should have sufficient flexibility to allow of adjustment to meet changing and, as far as possible, new requirements which may develop. For example, a traffic signal timer should be capable of being adjusted to meet changes in color sequence which might be authorized by federal or state legislation or to meet certain special local requirements. The various periods of the color sequence should be capable of percentage adjustment over very wide ranges to provide for unforseen conditions which might occur. Pedestrian periods, bell or green turning arrow indica-

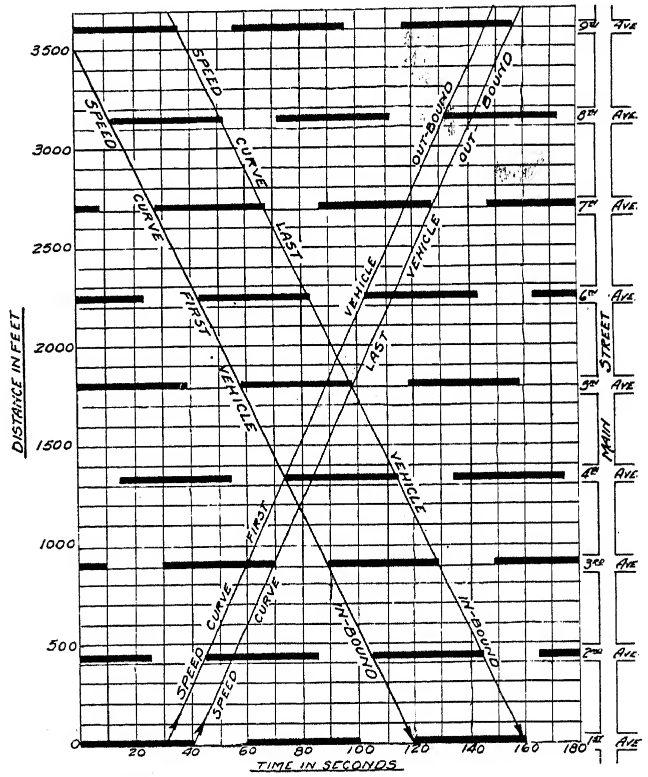

FIG. 5—SCHEMATIC TIMING DIAGRAM SHOWING IDEAL PROGRESSIVE RELATION FAVORING IN-BOUND TRAFFIC

Traffic speed—20 miles per hour. Total time cycle—60 seconds
Main street "green"—40 seconds indicated by heavy lines

tions which might later be required should be easily obtained by minor changes in the timer as opposed to intricate relay combinations used at present to secure such features.

A solution of these problems is represented by a drum unit (Fig. 7), a rotating switch attached to the gear shift unit in Fig. 3 and a cycle percentage dial having 100 slots and containing a predetermined arrangement of

easily removable keys as shown on the timer panel in Fig. 8. The drum contains a predetermined arrangement of cams for opening and closing the signal light contacts and is rotated in steps by means of a solenoid-operated ratchet. As the finger of the rotating switch strikes a key in the cycle percentage dial, contact is momentarily made and an impulse applied to the operating solenoid of the drum, thus causing the drum to advance to a new position or period in the color sequence.

Extreme flexibility is obtained by such construction inasmuch as any of the modern color sequences can be obtained merely by setting up the proper arrangement

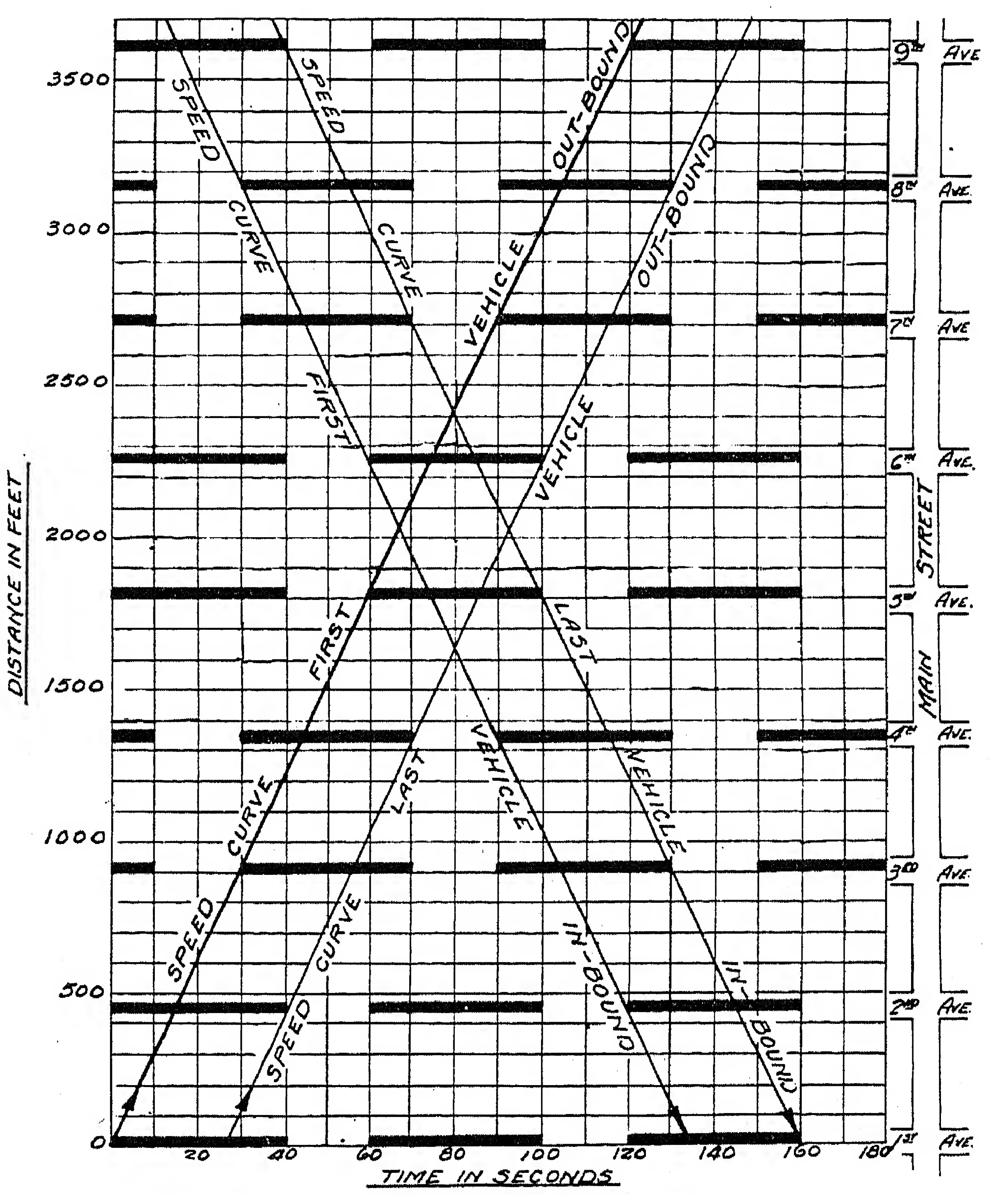

FIG. 6—SCHEMATIC TIMING DIAGRAM SHOWING PROGRESSIVE RELATION FOR AVERAGE TRAFFIC FLOW

Traffic speed—20 miles per hour. Total time cycle—60 seconds
Main street "green"—40 seconds indicated by heavy lines

of cams on the drum. Also, any desired percentage adjustments of the periods in the color sequence can be obtained simply by proper arrangement of the keys in the cycle percentage dial.

The drum unit is mechanically independent of the cycle percentage dial as opposed to conventional construction in which fanned cams are used, not only to determine the length of each color period, but, by means of drops in these cams, to change the lights from one condition to another. Such fanned cam construction is usually limited as far as changes in color sequence or percentage adjustment of color periods is concerned.

A unique feature of the cycle percentage dial is the fact that there are no moving parts on the front of the timer and yet direct visual indication is given at all times, not only of the relative length of each color period, but also of the position of each period in the color sequence. This is not true of fanned cam construction since, to obtain the length of the cross-street green period, it is necessary to subtract the sum of all of the other periods from 100 per cent. Furthermore, with such construction, no indication whatever is given of the positions of the periods in the color sequence. Also, in some cases, the existence of certain auxiliary periods,

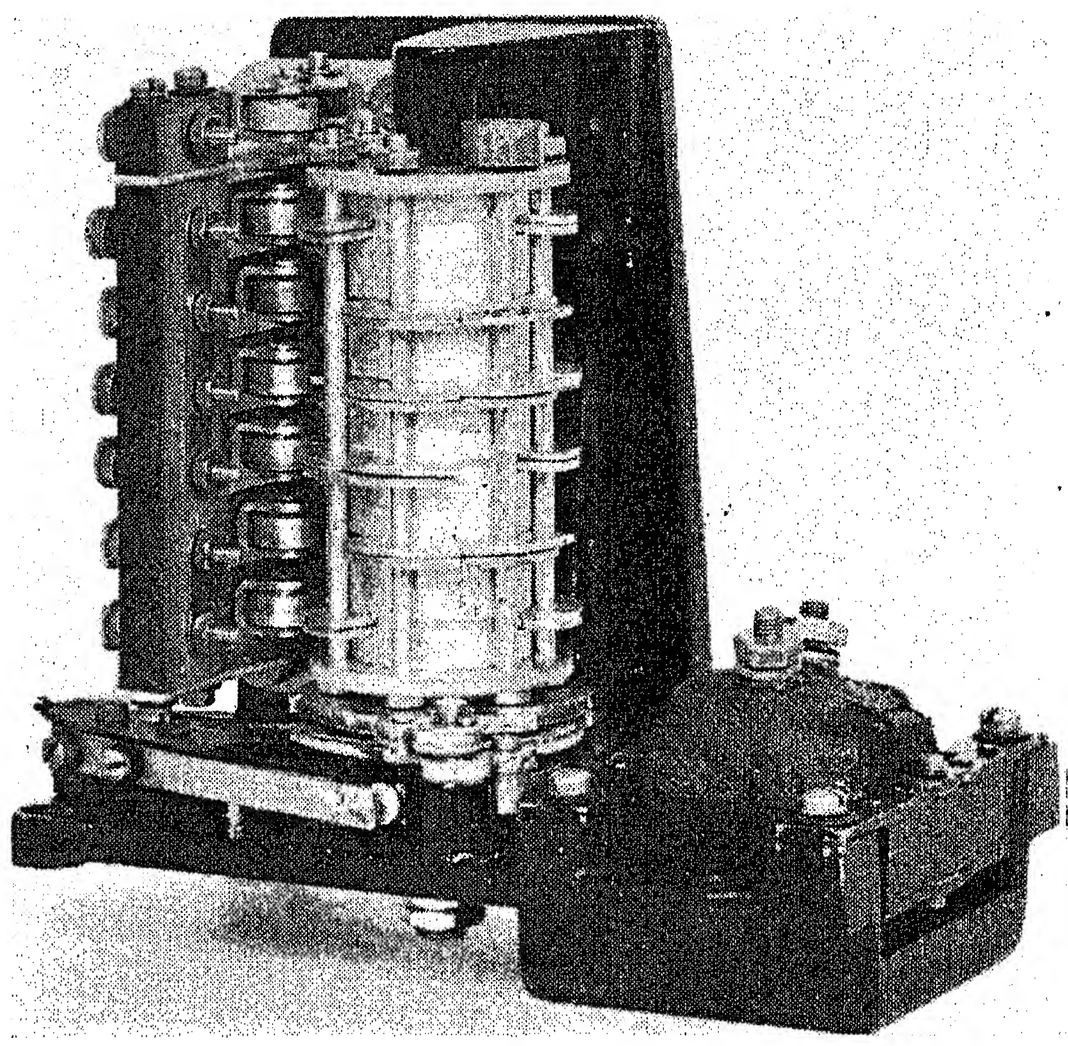

FIG. 7—DRUM UNIT

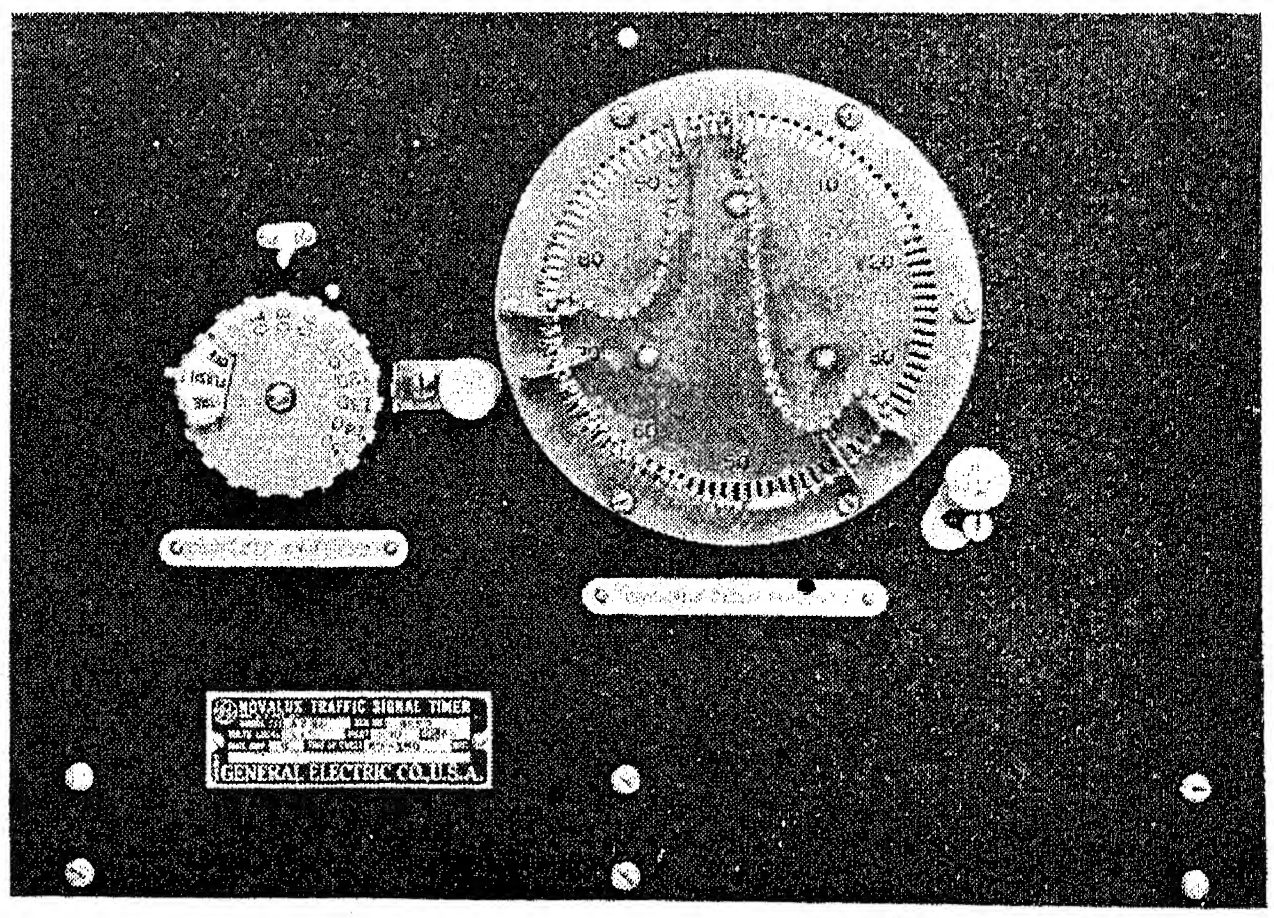

FIG. 8—FRONT VIEW OF TIMER PANEL

such as warning periods, pedestrian periods, or bell periods, is not indicated at all on the front of the timer, inasmuch as such periods are usually obtained by the internal adjustment of slotted cam sectors inside the timer.

The operation of the drum and rotating switch and the visual picture of the color sequence as presented by the cycle percentage dial are exceedingly simple and easy to understand as opposed to the somewhat complex and inflexible fanned cam construction.

Manual Control

Manual control of the signal lights is made possible by means of a push-button switch attached to a flexible cord. Successive, momentary depressions of the push-button switch impart electrical impulses to the drum solenoid and cause the drum to advance through the various periods in the color sequence. An important point is the fact that the traffic officer is thus forced to operate the lights in exactly the same sequence as is obtained with automatic operation. He cannot skip certain change periods as officers are sometimes prone to do with manual handles of the conventional type.

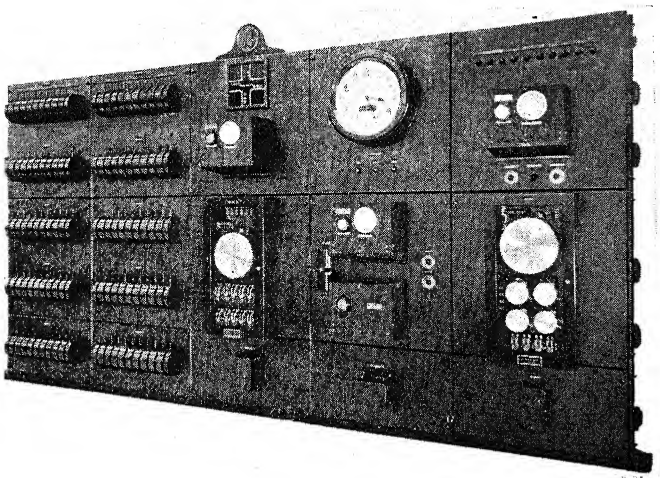

Fig. 9—Traffic Control Supervisory Board in Large Eastern City

Inherent Design Features

Auxiliary features, such as remote shut down, flashing amber and fire control, are usually obtained in interconnected systems by means of relays in each control box, one additional interconnecting cable or wire being required for each of these three features. Elimination of such relays and additional interconnecting cables is made possible by an "off, flash, fire" device in each synchronous timer. This device, while being essentially a multi-contact relay, is operated over the same wires used for remote shifting of the gears, thus resulting in a very important saving in the number of interconnecting cables required.

The timer is of universal application, that is, timers of the same identical construction can be used at isolated intersections, in non-interconnected progressive systems and interconnected progressive systems. This is of extreme importance where it is desired to operate timers in a non-interconnected progressive system initially, interconnection to be made at a later date as warranted by traffic conditions, since the necessity of replacing present timing apparatus with new and radically different construction is avoided.

The master controller, due to the fact that its supervisory feature is obtained by means of a simple pair of contacts, can, with the exception of these contacts, be of the same identical construction as an intersection timer, if so desired. Thus it can be made to serve as an intersection timer to control a set of signal lights in addition to performing its supervisory function.

Installation in Large Eastern City

A complete traffic signal installation, having all the flexibility described, has recently been installed in a large eastern city on one of the most congested traffic arteries in the world. The system is interconnected and automatically controlled from a supervisory board (Fig. 9). Ideal progression is given to the inbound traffic peak in the morning. Outbound traffic is similarly accommodated in the late afternoon, while average progression is maintained at other hours of the day. Program clocks automatically change the timing in accordance with the desired daily schedule for each day of the week.

Conclusion

It is felt that the synchronous timing device described is a real contribution in the art of efficient, flexible traffic control. Being built upon principles which are fundamentally sound and correct, it should take care of traffic requirements for years to come.

The Traffic Flow Regulator

BY C. H. BISSELL*
Member, A.I.E.E.

and

J. G. HUMMEL*
Non-member

THE annual cost of automobile accidents in the United States is more than the annual cost of public school education, five times the country's average yearly fire loss, and more than half the amount required to maintain all the agencies of the federal government. In an attempt to promote safety, many cities have installed a large number of traffic signals at isolated corners. While the installation of these signals has perhaps promoted safety it has at the same time severely penalized traffic by needless delays and congestion during the major portion of the day, thus contributing to the two billion dollar loss previously mentioned. This discussion is confined primarily to the reduction of this loss through the installation of an efficient and properly timed control.

During certain hours of light traffic a "go" period of 15 seconds on each street may be found very satisfactory, while during rush hours, especially on Saturdays and Sundays, a "go" period of 60 seconds on each street may be demanded. With the equipment available to date, the only solution is a compromise, with the result that generally a timing of 30 seconds on each street is used. This compromise is maintained all day long, and every day of the week. During the hours when a 15-second "go" is best, the 30-second compromise "go" is 100 per cent too long, and during the times when a 60-second "go" is best, the 30-second compromise "go" is 100 per cent too short. This practise quite naturally leads to the severe criticism of causing needless delays and congestion, and the continued operation of signals in this manner may break down the obedience of motorists to traffic signals in general.

There has always been a great deal of discussion about the extreme variation in traffic. One group of adherents has contended that there was absolutely no regularity in traffic flow and has cited several freak instances of momentary variation to bear out its contention in an attempt to justify the expenditure of large amounts of money. Another group took traffic surveys and counts and found that these freak conditions occurred at not more than one intersection in twenty, or not over 5 per cent. These surveys also showed the existence of very definite peaks and valleys in the volume of traffic.

For the past six years an independent and impartial traffic survey has been carried on by the University of Wisconsin in the research of the Barney Link Fellowship and the result has just been published in its 1931 report. This report contains the result of six years of traffic research and study including the counting of millions of cars in a large number of different cities of various sizes, studied during all kinds of weather. The report shows that the occurrence of traffic peaks and valleys at 95 per cent of all intersections is so regular that definite formulas for determining the vehicular traffic have been derived for various size cities. In fact, characteristic curves for the traffic flow can be obtained by taking only five traffic counts of 12 or 18 hours each. The characteristic curve is normally typical of the traffic on all boulevards and primary streets for the city in which the data are taken.

It is a well-known fact that traffic counts and surveys have demonstrated that there are very definite traffic peaks and valleys for every week-day as well as Saturdays and Sundays. Experience has shown that these variations in traffic occur with definite regularity each day of the week. By this we mean that the light and heavy movements of traffic at each intersection occur with definite regularity at that intersection. While the traffic may differ very materially on a week-day from that on Saturday and Sunday, the average flow on every normal Monday, for instance, will be the same. Likewise the peaks and valleys of the traffic flow will occur with definite regularity on each and every day of the week, perhaps different for week-days and week-ends but with a very definite repetition. It has not been found practical or economical to send out officers to change the timing on every isolated traffic timer in the city or county to take care of each changing condition.

The only attempt to take care of this variation to date has been an attempt to control the timing simultaneously in accordance with the number of vehicles on each street. This requires some sort of electrical eye or ear or a mechanical finger in the street or highway, generally requiring the breaking up of the highway and the installation of considerable underground ducts, cables, etc., with the attendant expense of installation and equipment varying from approximately one to five thousand dollars per intersection depending upon the complexity of the installation. The result has been that few cities are able, or have felt justified, in going to this expense to take care of momentary or simultaneous variations. The most recent reaction has been that certain cities which arranged to install vehicle-actuated equipment on a rental basis feel that the expense is not justified and are looking for a more economical manner of controlling traffic efficiently.

During the past few years a group of traffic engineers has been studying this problem in an attempt to solve it in an economical manner. This study lead to the traffic flow regulator shown in Fig. 1. This device makes it possible to vary automatically the timing to

*Crouse-Hinds Co., Syracuse, N. Y.

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take care of these regular traffic peaks and valleys that observation or a traffic survey and count show so distinctly. Essentially this device consists of the following principal units each of which is labeled in Fig. 1:

1. An automatic timing switch.
2. One or more time allowance units.
3. An electrically-wound time switch.
4. A seven-day program unit.

The function of the automatic timing switch is the same as with any automatic traffic signal, that is, to change the signal indications at predetermined intervals and in a desired sequence.

The function of the time allowance unit is to prolong the duration of the desired or preset signal indications

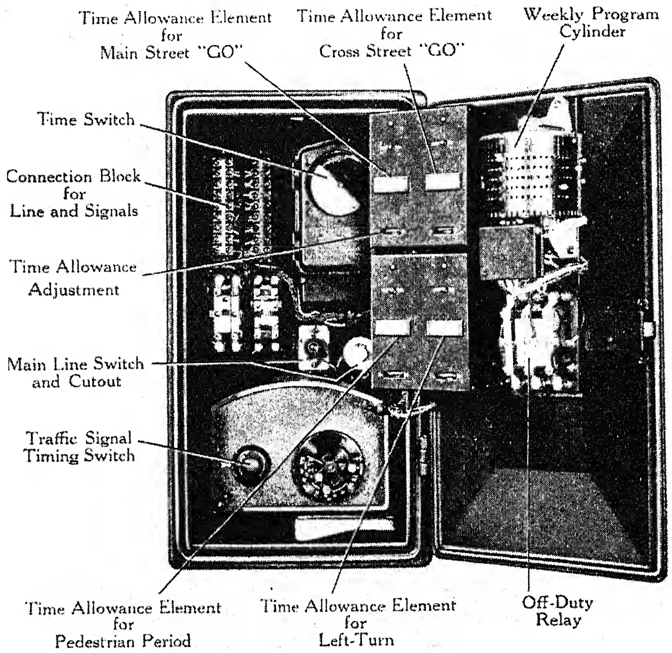

Fig. 1

or to introduce indications different than those normally used.

The function of the time switch is to cause the desired alterations in signal indications to occur at a desired time and to continue as long as predetermined.

The function of the seven-day program unit is to permit the setting up of a program for an entire week. This program, therefore, may be different for different days of the week.

When traffic is light on both streets, the Traffic Flow Regulator operates on a short total period thus passing vehicles and pedestrians safely and rapidly. As the traffic increases, the timing is automatically extended to take care of this increase. When the rush is over, the timing is automatically shortened.

Briefly, there are fifteen different cases, or symptoms, a traffic engineer (or doctor) may be called upon to prescribe for. The nine average conditions which are encountered at practically every intersection are:

1. *The Period of Operation* by which we mean the time at which the signal is turned "on" and "off" or on a steady or flashing indication for off-duty.
2. *The Early Morning Light Traffic* which may extend up to 7:30 a. m.
3. *The Morning Rush Period* generally occurring from 7:30 to 9:00 a. m.
4. *The Morning Off-Peak Period* extending from 9:00 to 12:00 noon.
5. *The Noon-Hour Rush* generally occurring from 12:00 to 1:30 p. m.
6. *The Afternoon Off-Peak Period* occurring from 1:30 to 4:30 p. m.
7. *The Afternoon Rush* lasting from 4:30 to 6:00 p. m.
8. *The Dinner Off-Peak Period* usually evident from 6:00 to 8:00 p. m.
9. *The Evening Pleasure Driving Period* extending from 8:00 to midnight.

In the above we have simply assumed certain hours, however, it should be pointed out that there is no limitation whatsoever. Any time can be selected for the various periods; likewise, the number of periods can be varied.

In addition to the above regular conditions that are met at practically every intersection there are certain special conditions that depend more upon the location of the intersection. For instance, a timer located adjacent to a school should be capable of allotting protection to the school children while passing to and from school and to regulate the traffic expeditiously and safely at all other times. A timer located adjacent to a church should be capable of giving protection to the church-goers on Sunday and at all other times control traffic in a manner dictated by the traffic conditions.

At certain intersections a great deal of out-of-town traffic must be cared for on Saturdays, Sundays and holidays.

At an intersection where a particularly bothersome left-turn is encountered it should be possible to insert a period to take care of this left-turn whenever, and as often as desired. Likewise, the control should be capable of inserting a period for pedestrians at a corner where the pedestrian travel is severe.

At intersections where there are more than four streets or entrances, it should be possible to control the traffic in two movements during hours of light traffic and during rush hours the timer should automatically change to three or four movements or phases of control.

All of the above conditions are taken care of by the Traffic Flow Regulator. Its flexibility is almost limitless and it is possible to furnish a Traffic Flow Regulator to take care of any traffic condition.

Double Conductors for Transmission Lines

BY H. B. DWIGHT* and E. B. FARMER*
Fellow, A.I.E.E. Non-member

Synopsis.—*When two overhead conductors a few inches apart are used for each phase of a transmission line circuit, instead of one larger conductor, the reactance is reduced 20 per cent or more. Where voltage drop determines the maximum power load, as it does in many cases, reduced reactance is advantageous and increases the allowable power rating. Current-carrying capacity and corona voltage are increased. The advantages are to be balanced against the extra cost due to mechanical features, including hangers, increased cost of stringing and additional wind and ice load. Since increase in ice load is possibly the greatest disadvantage, the use of double conductors is of most interest for southern districts, where ice load is not encountered.*

BY the term double conductors employed in this paper, is meant the use of two overhead conductors instead of one, for each phase. The construction is the same as in usual transmission lines except that a second conductor is hung a few inches below the first by metallic hangers, similar to those used in electric railway catenaries. The two conductors are therefore electrically in parallel and form one effective conductor of large cross-section, without increasing the number of insulators or cross-arms.

The advantages and disadvantages of this type of construction and the classes of lines on which it might be advantageous, will be described. Formulas will be given for the calculation of the electrical characteristics, which will enable comparisons to be more easily made.

Reactance

An overhead double-conductor line has approximately 20 per cent less reactance than a single-conductor line of the same weight of conducting metal. In many usual cases, especially where there is not complete control of the voltage by synchronous condensers, the reactance is the most important item in determining the power rating of the line, for both the voltage drop and the stability limit of the load depend principally on the reactance. Therefore, in many instances, without increasing the weight of conductor metal, a line can be built for about one-fifth greater power rating at very little increase in cost where ice load is absent, by using double-conductor construction. This is particularly advantageous where the cost of the right-of-way is large, and it is desirable to transmit as much power per circuit as possible.

The reduction in reactance, amounting to approximately 20 per cent on practical lines, obtained by using double-conductor construction, is graphically shown in Fig. 1. The formula for the reactance of a double-conductor line, suitable for usual overhead spacings, is quite simple and is given in Appendix I, equation (**1**). Fig. 1 is drawn for copper conductors. It may be noted that hollow conductors and aluminum conductors of equivalent resistance have from 5 to 7 per cent less reactance than the values shown for single conductors in Fig. 1. The two small conductors of the double-conductor line may be of aluminum if desired. A formula and curve for calculating reactance of hollow conductors are given in Appendix II.

The effect of changing the distance between the two wires of the same phase, in a double-conductor line, is shown in Fig. 2. It is seen that for larger spacings than 6 inches there is little further improvement. Accordingly the discussions in this paper for the most part are based on a value of 6 inches. Curves for other con-

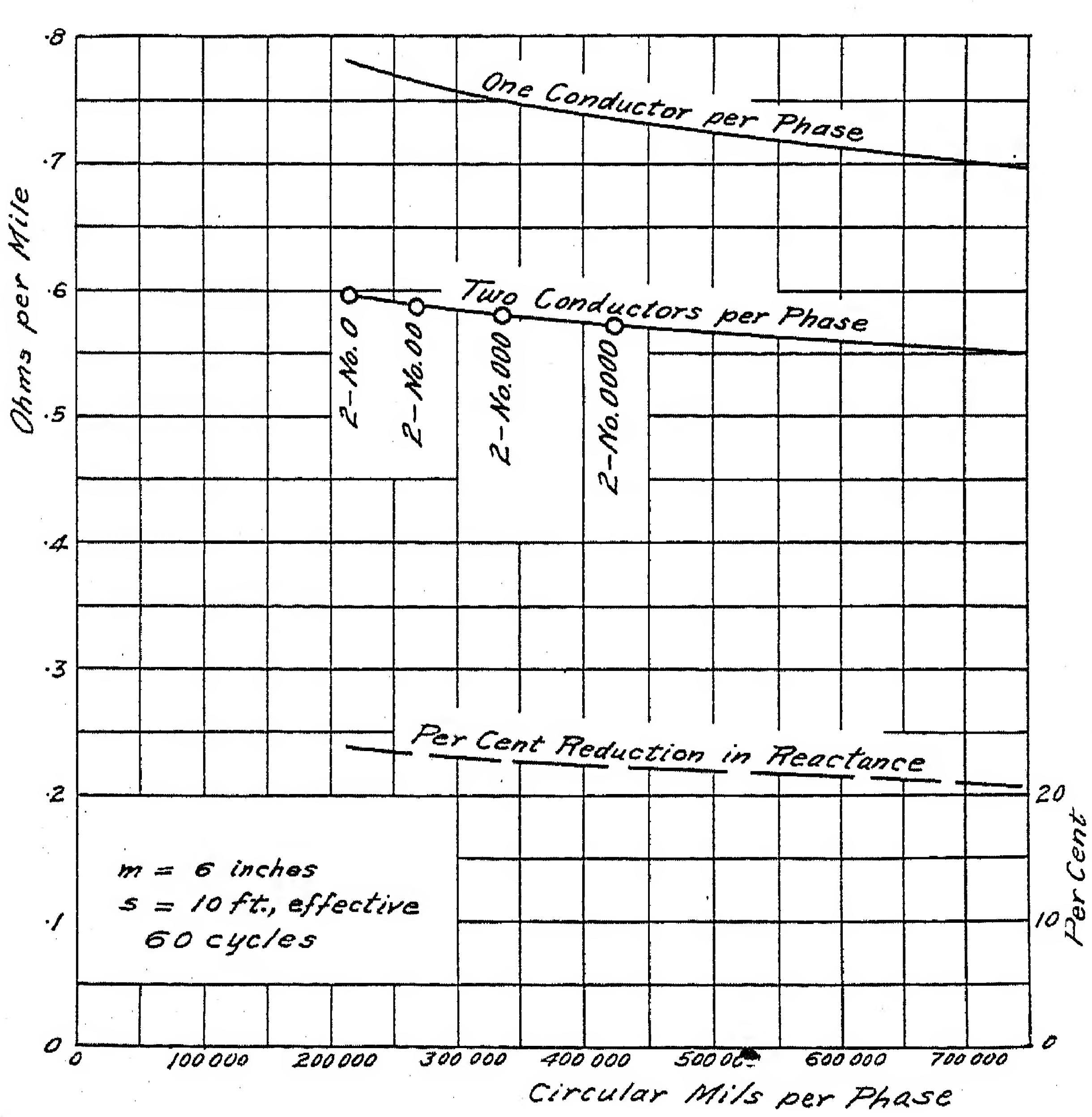

Fig. 1—Comparison of Reactance of Single- and Double-Conductor Lines

ductor sizes and for other spacings between phases are very much the same as the curves of Fig. 2.

Resistance

Another limit to the power rating of a transmission line is the cost of resistance losses. The greatest advantage of double-conductor construction in this connection is in the case of a line already built, which has higher resistance than desired and which has some mechanical margin of safety. It is possible to add a second conductor and cut the resistance in half without the necessity of taking down or scrapping the old con-

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*Massachusetts Institute of Technology.

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ductor. In other words it is possible to make the reduction in resistance with only one-half as much new conductor metal as would be needed if single large-size conductors were installed. It is of course necessary that the line be strong enough to stand the extra weight including wind and ice on the second conductor. The proposition to add to the conductor metal of existing lines may be attractive at a time when the cost of conductors per pound is low.

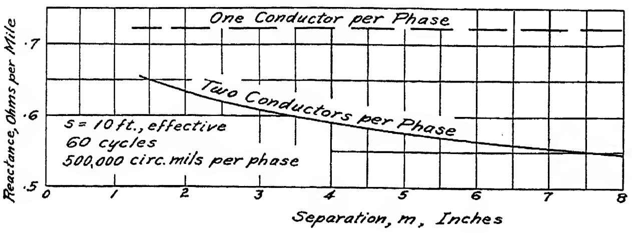

Fig. 2—Change of Reactance with Separation of Paralleled Conductors

For new transmission lines the difference in resistance between single- and double-conductor lines of a given weight of conductor metal is due to skin effect. The amount of the difference is shown in Fig. 3. It is seen to be a matter of minor importance with overhead conductors of usual sizes. For obtaining values of skin effect of hollow conductors, see reference 1 of Bibliography.

Heating of Conductors

In the preceding two paragraphs the cost of resistance losses was considered, as a limitation to the allowable power load of a line. With some short lines, and notably with many tie-lines, voltage drop and the cost of resistance losses are not serious limitations to the allowable load, and heating of the overhead conductors

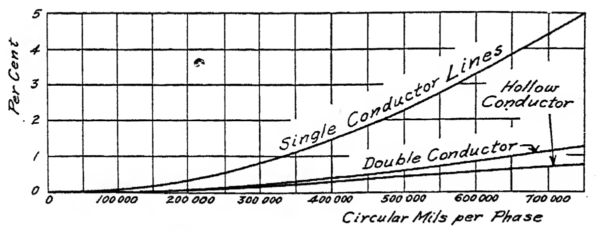

Fig. 3—Increase in Resistance Due to Skin Effect at 60 Cycles

is the practical limit encountered. In many cases, twice the normal load must be carried by one circuit as an emergency condition. It is stated in reference 4 that hard-drawn copper starts to anneal at 100 deg. cent. and that there is a reduction of 12 per cent in tensile strength due to a temperature of 130 deg. cent. for 72 hours, while a temperature of 150 deg. cent. for the same time results in a reduction of tensile strength of 30 per cent. The possibility of oxidation of joints in connectors is another reason for keeping the conductor temperature down. The current-carrying capacity of overhead conductors listed in reference 5 is based on 40 deg. cent. rise above 40 deg. cent. ambient temperature.

Tie lines have been built with very heavy overhead conductors for the purpose of avoiding overheating of the conductors. A 132-kv., 30-mile tie line from Chicago to Waukegan has been built with 750,000-cir. mil hollow copper conductors. It is stated in reference 6 that the main condition in designing the line was the heavy current to be carried.

The curves of Fig. 4 have been drawn from the table of current ratings for bare conductors, outdoor service, published in reference 3, page 466, using the ratings for 40 deg. cent. rise. It is seen that double-conductor construction increases the current-carrying capacity nearly 30 per cent, for a given weight of conductor metal. This is because the two small conductors have more

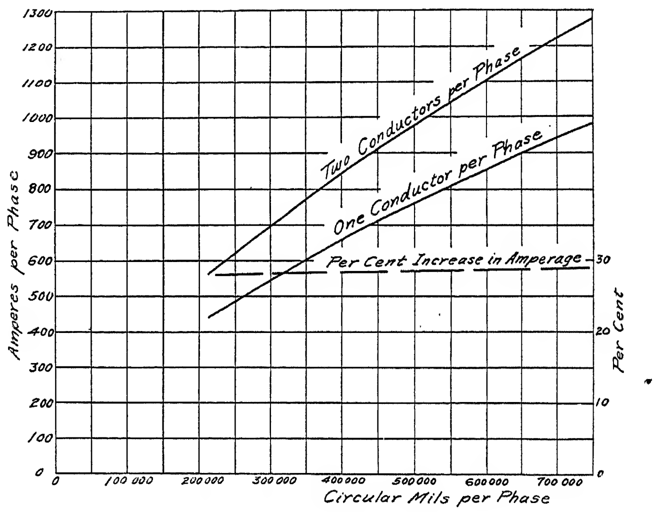

Fig. 4—Current-Carrying Capacity

surface than the single large conductor. If current ratings from the table for indoor service were used, the percentage difference between single- and double-conductor construction would be nearly the same. Hollow copper conductors, because of their large diameter, show an increased current-carrying capacity, over corresponding standard cables, though the increase is usually a small percentage. Aluminum conductors also have larger diameter and therefore greater current-carrying capacity than standard copper cables of the same resistance.

Capacitance

As is shown by Fig. 5, the capacitance of an overhead double-conductor line is 20 per cent or more greater than that of a single-conductor line of the same weight of conductor metal. This is an advantage in the case of power networks in well-settled parts of the country, where the loads and generating stations are scattered to

some extent throughout the network. In such networks it is a matter of experience that synchronous condensers are used almost entirely with strong field currents, and very little with lagging power factor load and weak field currents. Consequently, an increase in line capacitance of 20 per cent means a saving of a definite amount of synchronous condensers which would otherwise need to be installed.

In other cases, where transmission lines are very long and the loads are widely separated from the generators, special attention is required to handle the line capacitance current. A sufficient number of generators must

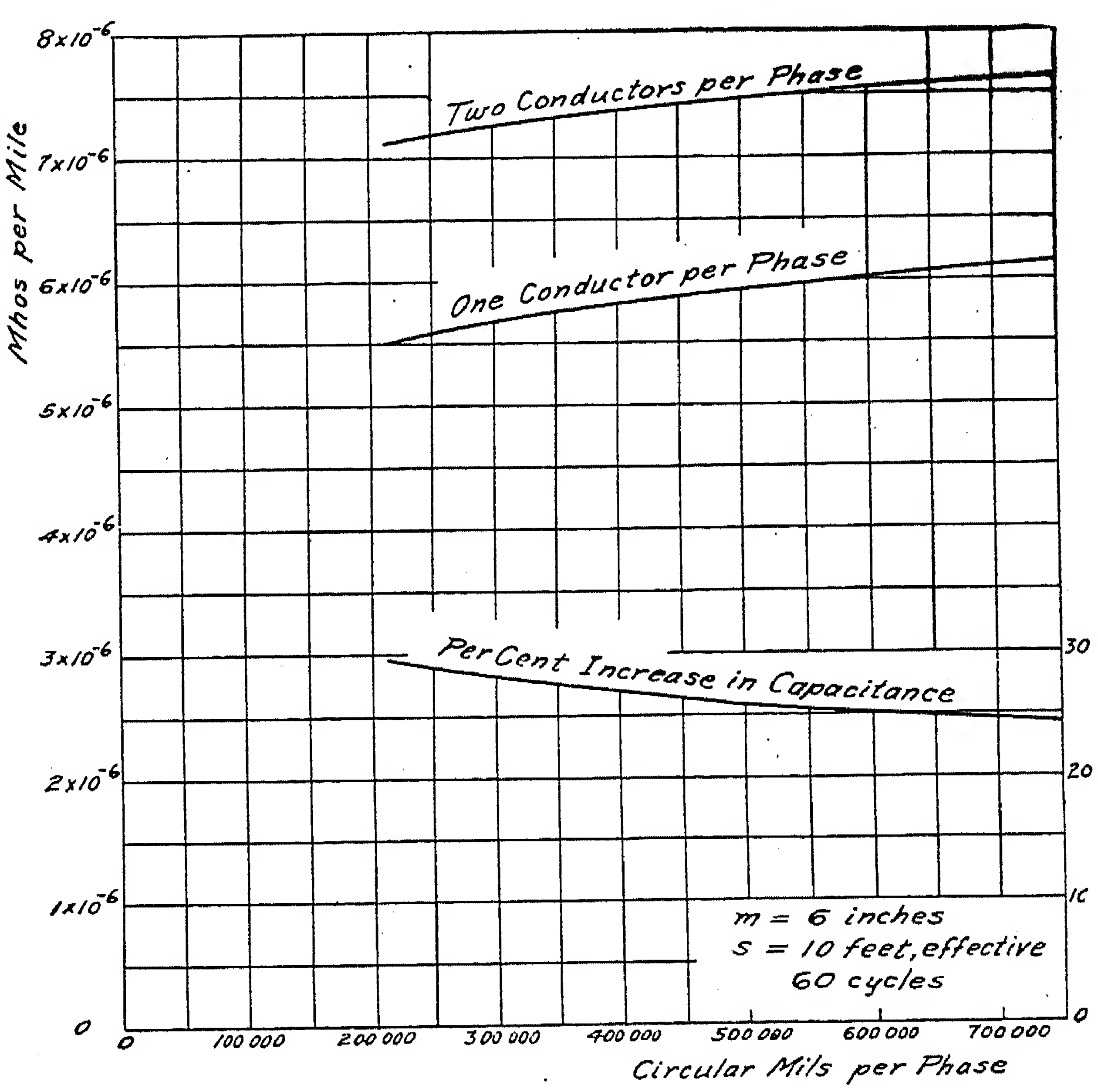

Fig. 5—Comparison of Capacitive Susceptance of Single- and Double-Conductor Lines

be kept always connected to the system, so that the no-load current is not an overload for them and so that the field current at rated voltage and leading power factor load does not become so small as to endanger the stability of the system. While these matters require attention, yet they seldom involve expense, particularly in the case of water-power systems where they are most likely to be noticeable, for there are practically always plenty of generators available. Accordingly even in such cases, an increase in line capacitance by using double-conductor construction, is not an economic disadvantage.

Corona

Values of corona voltage, that is, the voltage at which corona starts, are shown in Fig. 6 for one and two conductors per phase, where m, the separation of the two conductors of the same phase, is 6 inches. Approximate formulas for calculating these results are given in Appendix IV. Very little increase in corona voltage is obtained by using two small conductors of the same type as the single large one. This is in general true at other separations than 6 inches. More increase in corona voltage is obtained by using 3 or more conductors per phase. The use of a hollow or cored conductor gives a considerable increase in corona voltage.

Measurements of the corona voltage with two and three transmission line conductors per phase are given by F. W. Peek, Jr., in "Dielectric Phenomena in High Voltage Engineering" (reference 8), page 82, and also in the first edition of the same book, pages 71 and 72, published in 1915.

If a double-conductor line and a single-conductor line have the same calculated corona voltage, the operating voltage of the double-conductor line should be kept lower than that of the other, because when corona does start, as in stormy weather, the double-conductor line has twice as many conductors on which loss takes place.

Reference should be made to the paper by P. H. Thomas, Trans. A.I.E.E., 1909, (reference 7), in which a number of features relating to the use of two and three conductors per phase was discussed. A rather large spacing between the paralleled conductors, namely 18 inches, was assumed.

A transmission line with two conductors per phase could be constructed using a type of hanger already developed and tried out for electric railway catenaries. It would be more expensive than a single-conductor line of equal mechanical strength. The duty on the towers,

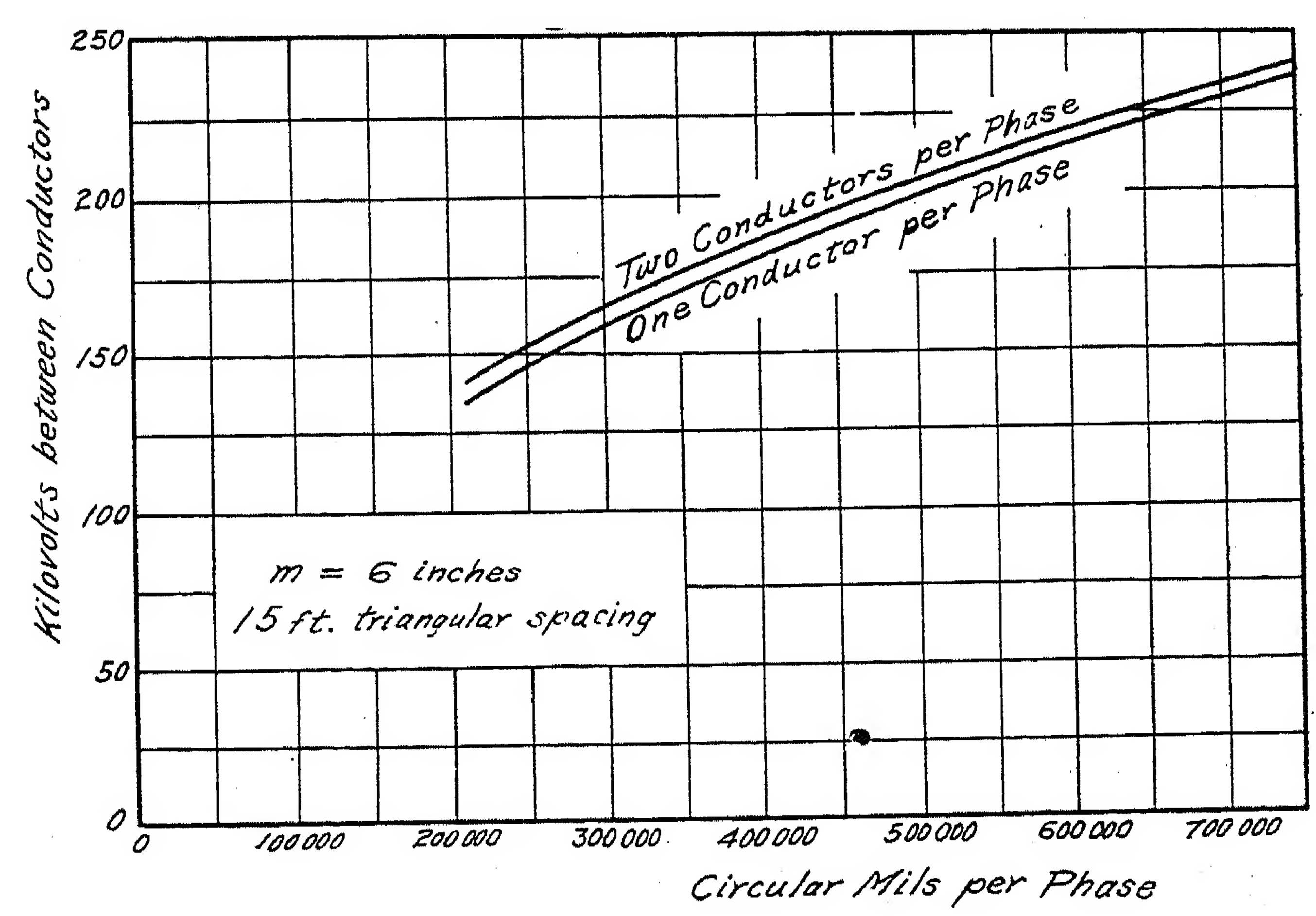

Fig. 6—Corona Voltage at Sea Level

and their height, would be greater due to approximately twice as much wind and ice load. More clearance would be required and shorter spans might be needed. More margin of safety would be required in the tensile strength of the conductors, for although each might be pulled up to the proper tension, it would not be certain that one conductor would not take more than its share of the tensile load at some later time. The cost of stringing the conductors would be greater. Since the increase in ice load is perhaps the greatest disadvantage, the use of double conductors is of most interest for southern districts, where ice load is not encountered.

The additional cost due to the mechanical features of the double-conductor construction is to be balanced against the value of the advantages to be obtained. If circumstances are such that the voltage drop in the line must not be more than a certain percentage, then 20 per cent reduction in reactance permits a corresponding increase in the allowable power load. The additional cost, is, in such a case, to be compared with 20 or 25 per cent of the entire cost of the transmission line. Such a case assumes that methods of controlling the voltage as by synchronous condensers are not considered applicable, and that the only way in which the power can be increased is by decreasing the reactance or providing additional circuits.

An increase in current-carrying capacity or in corona voltage might be an additional advantage, but such an increase would probably not by itself justify the special construction, for ratings now in use. The current-carrying capacity or corona voltage of a single-conductor line could usually be increased 30 per cent without increasing the cost of the entire transmission line more than a few per cent, by using a heavier conductor or one of about 30 per cent larger diameter. The cost of the conductors is only a fraction of the cost of the entire line. Such a conductor, however, will not give 20 per cent less reactance. The large reduction in reactance is therefore the most important feature of the double-conductor construction.

In conclusion, the advantages shown are as follows:

Approximately 20 per cent less reactance; the opportunity in some cases to add to the conductivity and power rating without scrapping the old conductors; the elimination of skin effect almost entirely; and the increase in current-carrying capacity of nearly 30 per cent. The disadvantages are the greater clearance and mechanical strength required, particularly where ice load is important, the cost of hangers, and the increased cost of installation.

Appendix I

Reactance for Two Conductors per Phase

Let there be two equal wires per phase, at an axial distance m inches apart. Let the geometric mean of the spacings between the phases be s inches. Then the reactance will be approximately the same as for equilateral spacing of s inches. If s is considerably larger than m, the current will divide practically equally between the two wires of each phase.

Reactive drop in one wire is approximately, for a three-phase system,

$$= j 2 \pi f\, 10^{-9} \left[\frac{I}{2} \left(\frac{1}{2} + 2 \operatorname{logn} \frac{p}{\rho} \right) + \frac{I}{2}\, 2 \operatorname{logn} \frac{p}{m} \right.$$

$$\left. + I(-0.5 + j0.866)\, 2 \operatorname{logn} \frac{p}{s} + I(-0.5 - j0.866)\, 2 \operatorname{logn} \frac{p}{s} \right]$$

volts per cm.

where ρ is the radius of the wires, and p is a certain large distance up to which flux is counted. As p cancels out, the result is the same no matter how large p may be. The term logn denotes natural logarithm. Reactive drop

$$= j \frac{2 \pi f\, 10^{-9}}{2}\, I \left(\frac{1}{2} + 2 \operatorname{logn} \frac{s}{\rho} + 2 \operatorname{logn} \frac{s}{m} \right)$$

$$= j \pi f I\, 10^{-9} \left(\frac{1}{2} + 2 \operatorname{logn} \frac{s^2}{m\,\rho} \right) \text{ volts per cm.}$$

Changing units,

$$\text{Reactance} = \pi f\, 10^{-6} \left(80.5 + 741 \log_{10} \frac{s^2}{m \rho} \right)$$

ohms per mile (1)

If the conductor is a 7-strand cable of diameter 2ρ, the figure 80.5 is changed to 103.3 and if there are 19 strands, the figure becomes 89.3. (See reference 9, page 127, for other numbers of strands.)

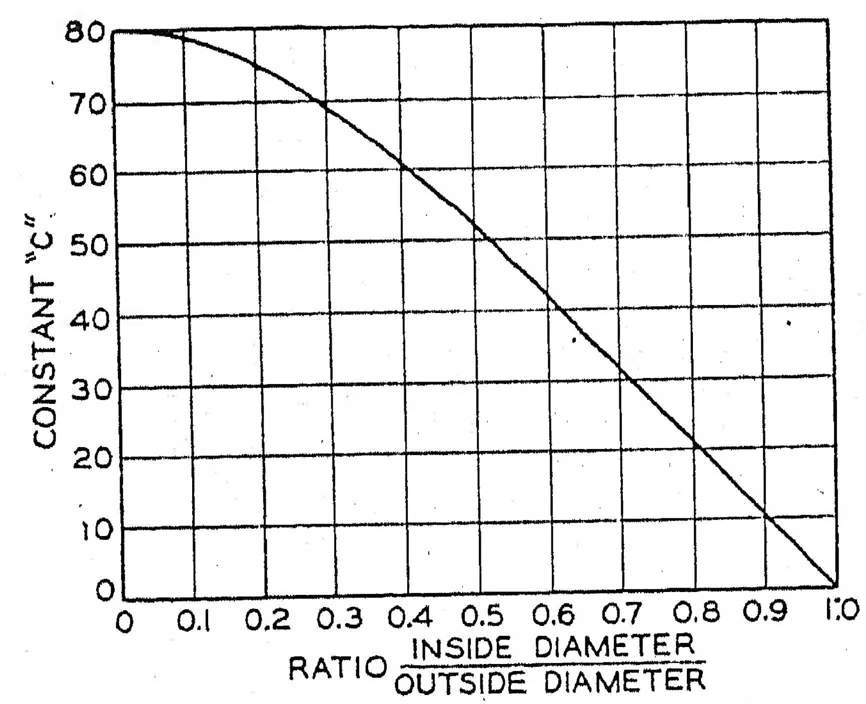

Fig. 7—Constant c for Use in Equation (2), Reactance of Tubular Conductor

Appendix II

Reactance of Tubular or Hollow Conductor

The reactance of a tubular conductor is

$$2 \pi f\, 10^{-6} \left(c + 741 \log_{10} \frac{s}{\rho} \right) \text{ ohms per mile} \quad (2)$$

where s is the geometric mean of the axial spacings between phases, 2ρ is the diameter of the conductor and c is read from Fig. 7. The formula used in drawing the curve for c is given in reference 9, page 137, equation (5), or in other books.

Formula (2) is in most practical cases a close approximation for the reactance of a hollow stranded conductor. It may be noted that the reactance of a standard conductor having 37 strands, and 18 strands in the outer layer, is given very closely by the formula for a solid wire.

Appendix III

Capacitance for Two Conductors per Phase

Let there be a three-phase system with two equal conductors per phase of radius ρ and at an axial distance m. Let the geometric mean of the spacings between the phases be s. Then the capacitance will be approximately the same as for equilateral spacing s. If s is considerably larger than m, the charge will divide almost equally between the two conductors of each phase. Let the charge on each of the conductors of the first phase be $q/2$ per cm. For the fundamentals of this type of calculation see reference 9, pages 142, 143 and 152, or other books.

Carry a unit charge from a conductor of one phase to one of another, assuming equilateral spacing. The potential between phases is approximately

$$q \operatorname{logn} \frac{s}{\rho} + q \operatorname{logn} \frac{s}{m}$$

$$- (-0.5 + j.866)\left(q \operatorname{logn} \frac{s}{\rho} + q \operatorname{logn} \frac{s}{m} \right)$$

The absolute value of the potential between phases is

$$1.732\, q \operatorname{logn} \frac{s^2}{m\rho} \qquad (3)$$

The capacitive susceptance required in connection with the three-phase circuit is equal to the charge per phase divided by the voltage to neutral and multiplied by $2\pi f$, and is

$$\frac{2\pi f}{\operatorname{logn} \dfrac{s^2}{m\rho}} \text{ in absolute units.}$$

$$\text{Changing units, } b = \frac{2\pi f \times 38.8 \times 10^{-9}}{\log_{10} \dfrac{s}{\sqrt{(m\rho)}}} \qquad (4)$$

$$= \frac{4\pi f\, 38.8 \times 10^{-9}}{\log_{10} \dfrac{s^2}{m\rho}} \text{ mhos per mile} \qquad (5)$$

The capacitance of a double-conductor line, to the first approximation, is therefore the same as that of a single-conductor line the radius of whose conductors is $\sqrt{(m\rho)}$.

Appendix IV

Corona Voltage

By corona voltage is meant the voltage at which corona starts to form on transmission line conductors in fair weather. This is an advisable limit of operating voltage. It depends on the potential gradient in volts per cm. at the surface of the conductors.

If in a double-conductor transmission line s/m is comparatively large, the charge may be taken to be equally divided between the two conductors per phase. If $q/2$ is the charge per cm. of conductor, the gradient in absolute units at the surface of one of the wires is $\dfrac{q}{\rho}$ due to the charge on that wire. This component of the gradient is, for equilateral triangular spacing between phases,

$$\frac{q}{\rho} = \frac{e_o}{2\rho \operatorname{logn} \dfrac{s}{\sqrt{(m\rho)}}} \text{ in absolute units,}$$

$$\text{since} \qquad e_o = 2q \operatorname{logn} \frac{s}{\sqrt{(m\rho)}} \text{ to neutral,}$$

from equation (**3**), Appendix III.

In order to include the component of gradient due to the other conductor of the same phase, the above value of gradient should be multiplied by $1 + \dfrac{2\rho}{m}$ according to equation (**15**) and Table II of the paper *Three-Phase Multiple-Conductor Circuits*, by Edith Clarke, see page 809 of this issue. This factor was given for use when the value of $\dfrac{m}{2\rho}$ is as low as 5.

Then if g_o is the gradient at which corona forms,

$$e_o = \frac{2\, g_o\, \rho}{1 + \dfrac{2\rho}{m}} \operatorname{logn} \frac{s}{\sqrt{(m\rho)}}$$

If g_o is in kilovolts per inch and ρ in inches, e_o is in kilovolts to neutral. The expression for e_o should be multiplied by an irregularity factor m_o, an average value of which for stranded conductors is 0.85, and by an air density factor δ, the value of which for standard temperature and pressure is unity. The value of g_o for smooth wires and standard temperature and pressure is 53.6 kv. per inch. Then

$$e_o = \frac{53.6 \times 2.30\, m_o \delta \rho}{1 + \dfrac{2\rho}{m}} \log_{10} \frac{s^2}{m\rho} \qquad (6)$$

For stranded conductors at normal temperature and pressure,

$$e_o = \frac{105\, \rho}{1 + \dfrac{2\rho}{m}} \log_{10} \frac{s^2}{m\rho} \text{ kilovolts to neutral,} \qquad (7)$$

for double-conductor lines, dimensions being in inches.

By equation (**35′**), page 302, reference 8,

$$e_o = 105\, \rho \log_{10} \frac{s}{\rho} \qquad (8)$$

for single-conductor lines, with $m_o = 0.85$. For a de-

scription of calculations of corona, see reference 8, Chapter X.

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6. "Application of Anaconda Hollow Conductors," by C. S. Anderson, presented at St. Louis, Feb. 1931, Engineering National Section, N.E.L.A.

7. *Output and Regulation in Long Distance Lines*, by P. H. Thomas, Trans. A.I.E.E., 1909, p. 615.

8. "Dielectric Phenomena in High-Voltage Engineering," by F. W. Peek, Jr., ed. of 1929.

9. "Transmission Line Formulas," by H. B. Dwight, ed. of 1925.

10. *Three-Phase Multiple-Conductor Circuits*, by Edith Clarke, Trans. A.I.E.E., September, 1932., p. 809.

Discussion

For discussion of this paper see page 821.

Three-Phase Multiple-Conductor Circuits

BY EDITH CLARKE*

Associate, A.I.E.E.

Synopsis.—During the past year the subject of power transmission by multiple-conductor circuits has received considerable attention. The formulas and estimating curves given in the paper may therefore be of interest at this time.

Formulas are developed for the inductance and capacitance to neutral per phase and the approximate corona starting voltage of perfectly transposed, multiple-conductor, three-phase transmission lines having any number of conductors per phase.

For certain special arrangements of the conductors, curves are given for the 60-cycle reactance, capacity susceptance, and corona starting voltage. These curves show the effect of variations, throughout their practical range, in conductor diameter, spacing between phases and between conductors of the same phase. Two, three, four and five conductors per phase are considered.

A comparison is made of multiple- and single-conductor circuits with respect to charging current, voltage at no load, power which can be carried with the same voltage drop, and stability.

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INTRODUCTION

MULTIPLE or split conductors were proposed in 1909 by P. H. Thomas in a paper[1] before the Institute. In this paper Mr. Thomas pointed out the advantage of split conductors in increasing the power which may be transmitted at a given voltage drop. Since then, at irregular intervals, the use of two or more conductors per phase has been suggested by transmission engineers for various contemplated projects. At such times it was found advantageous to have available for estimating purposes, curves giving the 60-cycle reactance, the capacity susceptance and the disruptive critical voltage for various proposed arrangements of the multiple-conductor circuits. These curves were made up from time to time as new arrangements of the conductors were considered.

The recent revival of interest in the subject has suggested that these curves be assembled and brought to the attention of those who may find use for them in estimating the performance of multiple-conductor circuits. The curves include various arrangements of two, three, four, and five conductors per phase.

I—Electrical Characteristics of Multiple-Conductor Circuits

TRANSPOSED LINES

When the conductors of a three-phase transmission line are unsymmetrically arranged, unless the conductors are transposed, the inductance and capacitance to neutral of the three phases will be unequal and the currents under normal operating conditions unbalanced. When a multiple-conductor line is so transposed that each conductor occupies each of the tower positions or its equivalent for equal distances, the phases being rotated in cyclic order, the total inductance and capacitance to neutral of the three phases will be equal. Also, the total inductance and capacitance of each conductor will be the same. Therefore, with balanced voltages applied, the phase currents will be balanced and each will divide equally among the conductors of that phase. When the line is not transposed, or only partly transposed, an exact solution may be obtained by the use of symmetrical components. In the general case, with balanced voltages applied, there will be negative and zero sequence currents as well as positive sequence currents. Approximate results may be obtained by considering only the positive sequence currents and by using for inductance and capacitance to neutral the average of the three positive sequence inductances and capacitances, respectively. This is equivalent to using the values of inductance and capacitance which the line would have if it were perfectly transposed.

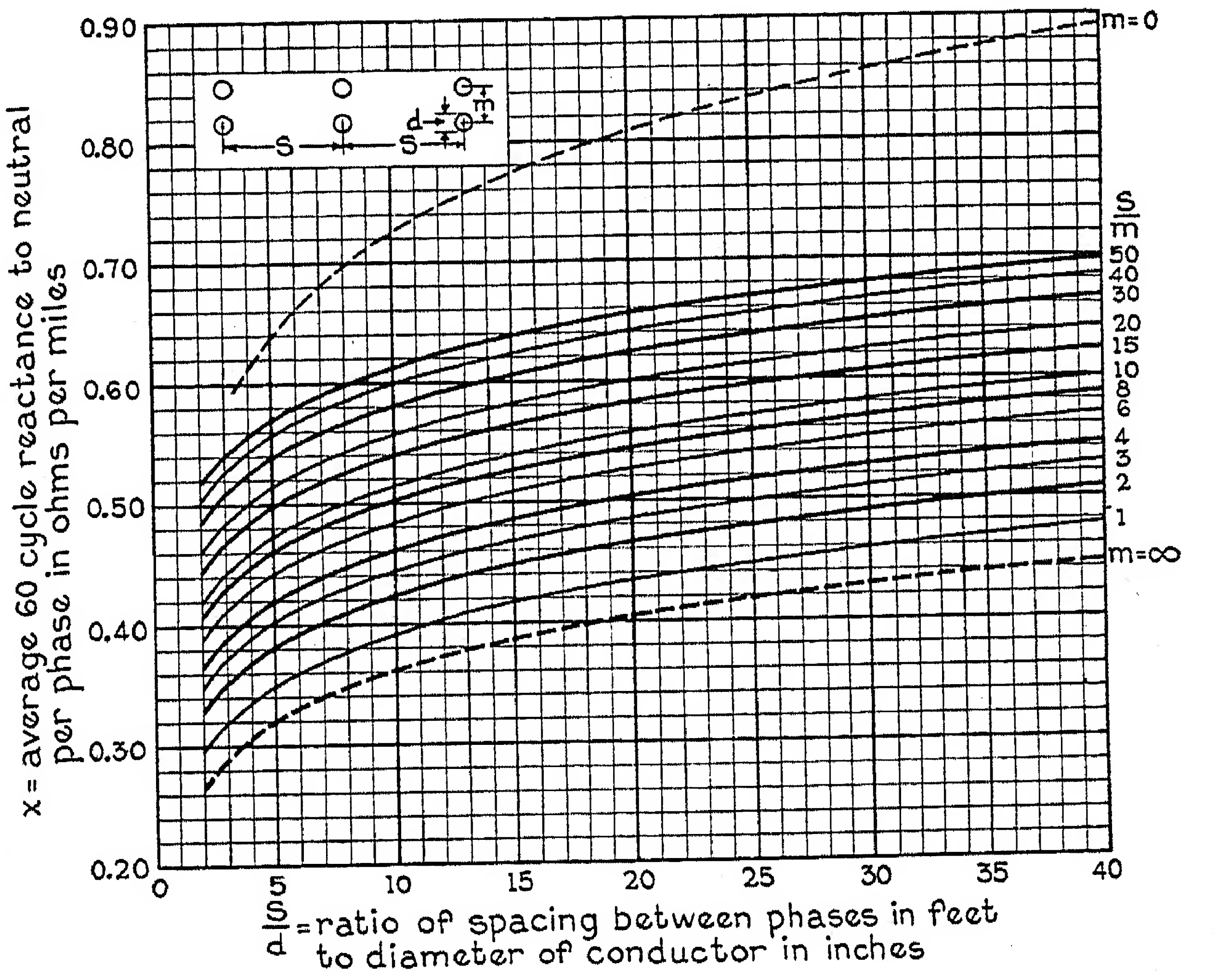

FIG. 1—TWO CONDUCTORS PER PHASE ARRANGED VERTICALLY

60-cycle reactance as affected by S/d and S/m

S = horizontal distance between adjacent phases in feet

m = distance between conductors of the same phase in feet

d = diameter of the conductor in inches

In the work which follows a completely transposed line will be assumed.

INDUCTANCE

A formula for the average inductance to neutral per mile of each conductor of a perfectly transposed three-phase transmission line having n conductors per phase

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*Central Station Engg. Dept., General Electric Co., Schenectady, N. Y.

1. For references see Bibliography.

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is derived in Appendix A. The inductance will vary along the line with the transpositions, but the average inductance per mile of all conductors to neutral will be the same. Since the n conductors of the phase are in parallel, the average inductance per phase will be the *nth* part of the inductance per conductor.

It has been found convenient in using curves for estimating purposes to express the diameter of the conductors in inches while the spacings between phases and between conductors of the same phase are in feet.

From (9a), Appendix A, the average inductance to neutral per mile of each conductor is

$$L_c = 0.08047 + 0.74113 \log_{10} \frac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}} \quad \text{millihenrys per mile} \qquad (1)$$

The average inductance per phase, L, will be the *nth* part of the inductance per conductor, therefore

$$L = \frac{1}{n}\left[0.08047 + 0.74113 \log_{10} \frac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}\right] \quad \text{millihenrys per mile} \qquad (2)$$

where

n = number of conductors per phase.

d = diameter of the conductor in inches.

S_{gm} = geometric mean of the distances between conductors of different phases in feet.

m_{gm} = geometric mean of the distances between conductors of the same phase in feet.

60-Cycle Reactance

The average 60-cycle reactance to neutral per mile of each conductor is

$$x_c = 2\pi f L_c = 377\,L_c = 0.03034 + 0.27941 \log_{10} \frac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}} \quad \text{ohms per mile} \qquad (3)$$

The average 60-cycle reactance to neutral per mile of each phase is

$$x = 377\,L = \frac{x_c}{n}$$

$$= \frac{1}{n}\left[0.03034 + 0.27941 \log_{10} \frac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}\right] \quad \text{ohms per mile} \qquad (4)$$

Solid, Stranded and Hollow Conductors

The internal inductance[2] of a stranded conductor is greater than that of a solid conductor of the same outside diameter, while that of a hollow conductor is less. The internal 60-cycle reactance of a solid conductor in ohms per mile is given by the first term on the right hand side of (3), while the external reactance is given by the second term.

Table I gives a plus or minus correction, ϵ, to be added to the 60-cycle phase reactance calculated by (4) to obtain the correct reactance per phase for stranded and hollow multiple conductors. These corrections were calculated by using for stranded conductors the formulas given in "Transmission Line Formulas," by H. B. Dwight, page 127, and for hollow conductor, equation (14) p. 827, Pender's "Handbook for Electrical Engineers."

For hollow or stranded multiple conductors

$$x = \epsilon + \frac{1}{n}\left[0.03034 + 0.27941 \log \frac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}\right] \quad \text{ohms per mile} \qquad (5)$$

Capacitance

Dr. Kennelly in his discussion[3] of Mr. Thomas' paper,[1] by applying the theorem of the propagation of electric disturbances in free space at the speed of light to multiple conductor circuits, points out that the capacitance to neutral in statfarads per cm. is the reciprocal of the external inductance to neutral in abhenrys per cm. The first term on the right hand side of (8a), Appendix A, is the external and the second the internal inductance. Therefore, for a perfectly transposed multiple-conductor circuit the average capacitance to neutral per cm. for each conductor will be

$$C_c = \frac{1}{2\ \log_e \dfrac{2\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}} \quad \text{statfarads per cm.} \qquad (6)$$

where d, S_{gm} and m_{gm} are expressed in the same units.

Expressing (6) in practical units with S_{gm} and m_{gm} in feet and d in inches, the average capacitance to neutral per mile for each conductor is

$$C_c = \frac{0.03883}{\log_{10} \dfrac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}} \quad \text{microfarads per mile} \qquad (7)$$

TABLE I—ϵ—CORRECTION FOR STRANDED AND HOLLOW CONDUCTORS

Conductors per phase	Stranded conductors, No. of strands: 7	19	37	61	Hollow conductors, Ratio of internal to external diameter: 0	0.20	0.4	0.6	0.8	1.0
1	0.0086	0.0033	0.0017	0.0010	0	−0.0022	−0.0075	−0.0145	−0.0222	−0.0303
2	0.0043	0.0017	0.0009	0.0005	0	−0.0011	−0.0038	−0.0073	−0.0111	−0.0152
3	0.0029	0.0011	0.0006	0.0004	0	−0.0007	−0.0025	−0.0048	−0.0074	−0.0101
4	0.0021	0.0008	0.0004	0.0003	0	−0.0005	−0.0019	−0.0036	−0.0056	−0.0076
5	0.0017	0.0007	0.0003	0.0002	0	−0.0004	−0.0015	−0.0029	−0.0044	−0.0061

The average capacitance to neutral per mile, C, for each phase will be n times the capacitance per conductor, therefore

$$C = \frac{0.03883\, n}{\log_{10} \dfrac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}} \text{ microfarads per mile} \tag{8}$$

60-Cycle Capacity Susceptance

The average capacity susceptance to neutral per mile of each conductor for a frequency of 60 cycles will be

$$b_c = 2\pi f C_c = 377 C_c = \frac{14.64}{\log_{10} \dfrac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}} \text{ micromhos per mile} \tag{9}$$

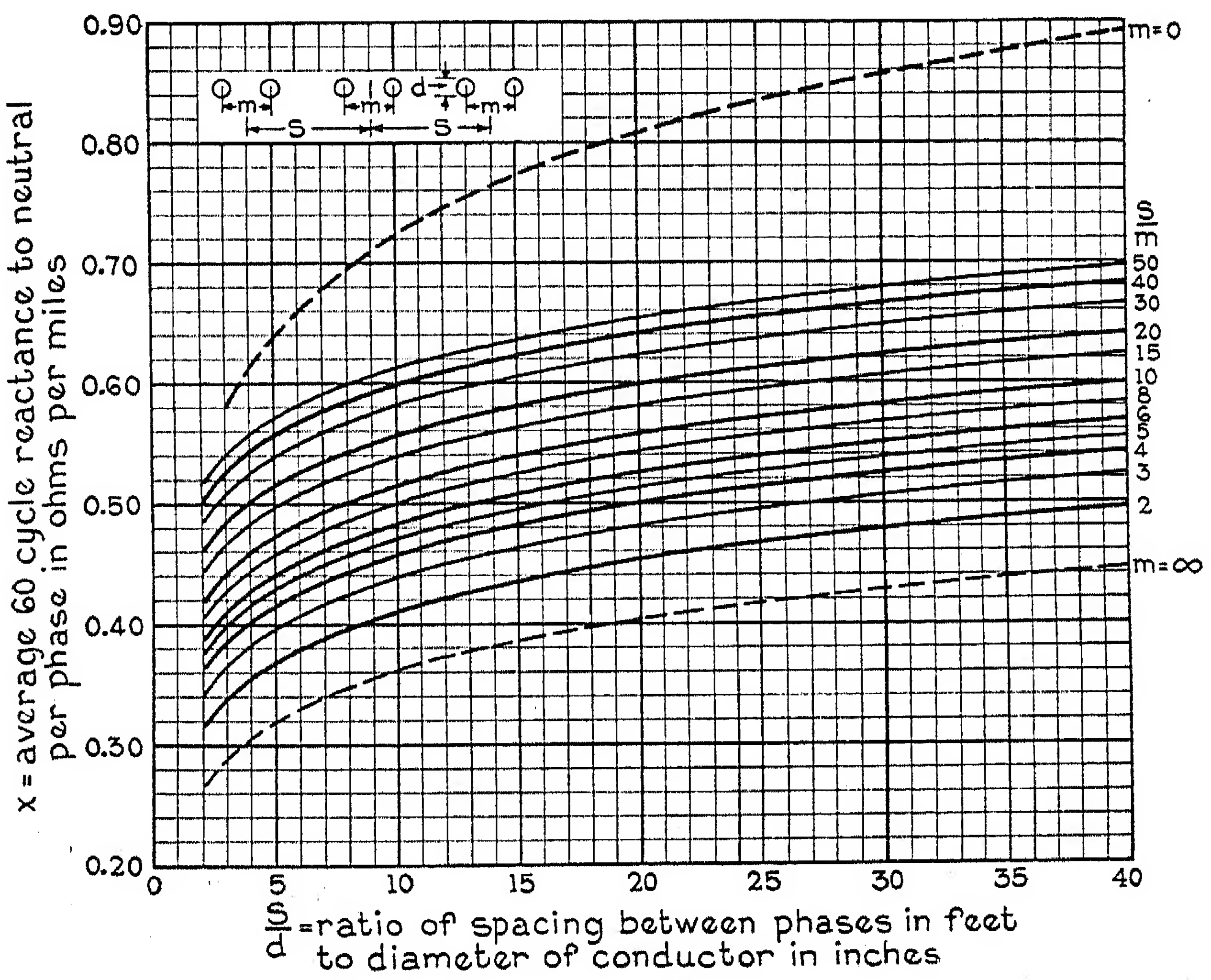

Fig. 2—Two Conductors per Phase Arranged Horizontally

60-cycle reactance as affected by S/d and S/m
S = distance between centers of adjacent phases in feet
m = distance between conductors of the same phase in feet
d = diameter of the conductor in inches

The average 60-cycle capacity susceptance to neutral per mile of each phase, will be

$$b = 377 C = n b_c = \frac{14.64\, n}{\log_{10} \dfrac{24\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}} \text{ micromhos per mile} \tag{10}$$

Relation Between 60-Cycle Reactance and Capacity Susceptance

If the product of capacitance and external inductance is unity in absolute units, it follows that there is a definite relation between 60-cycle reactance and capacity susceptance.

Transposing (5) and multiplying by (10) there results

$$b\left[x - \frac{0.0303}{n} - \epsilon\right] = 4.09 \tag{11}$$

When x, the 60-cycle reactance to neutral per phase in ohms is known, b, the capacity susceptance per phase in micromhos may be calculated from the following formula:

$$b = \frac{4.09}{x - \left(\dfrac{0.0303}{n} + \epsilon\right)} \text{ micromhos per mile} \tag{12}$$

Should b be known, x may be obtained, thus,

$$x = \frac{4.09}{b} + \frac{0.0303}{n} + \epsilon \text{ ohms per mile} \tag{13}$$

60-Cycle Reactance and Capacity Susceptance Curves for Multiple-Conductor Circuits

Equations (4) and (10) give the average 60-cycle reactance and capacity susceptance to neutral per mile for each phase of a perfectly transposed multiple conductor circuit, having any number of conductors per phase and any arrangement of the conductors.

For certain symmetrical arrangements of phases and conductors in the phases, the reactance and capacity susceptance may be expressed in terms of the ratios S/d and S/m,
where
S = the horizontal distance between the center lines of adjacent phases in feet.
m = the distance between adjacent conductors of the same phase in feet.
d = the diameter of the conductor in inches.

In Appendix B, formulas for 60-cycle reactance and capacity susceptance for special multiple-conductor circuits are given in terms of the ratios S/d and S/m.

The following cases have been considered and the average 60-cycle reactance to neutral per phase in ohms per mile plotted in Figs. 1 to 7:

1. Two conductors per phase
   a. Vertical arrangement of conductors in the same phase.
   b. Horizontal arrangement of conductors in the same phase.
2. Three conductors per phase
   a. Vertical arrangement of conductors in the same phase.
   b. Conductors of the same phase at the corners of an equilateral triangle.
3. Four conductors per phase
   a. Vertical arrangement of conductors in the same phase.
   b. Conductors of the same phase at the corners of a square.
4. Five conductors per phase
   a. Vertical arrangement of conductors in the same phase.

These curves are applicable for all practical conductor diameters, spacings between phases and between conductors of the same phase. Since they are for solid conductors, the correction, ϵ, Table I, should be applied when necessary.

Fig. 8 gives the average 60-cycle capacity susceptance to neutral per phase in micromhos per mile for two con-

ductors per phase, arranged vertically, case *Ia*. Fig. 1 gives the corresponding 60-cycle reactance.

Since 60-cycle capacity susceptance is readily calculated from equation (**12**) when the reactance is known, curves for the capacity susceptance of the other conductor arrangements are not given.

Corona

In a perfectly transposed multiple-conductor circuit, since each conductor occupies each of the tower positions for equal distances, the corona-starting voltage for

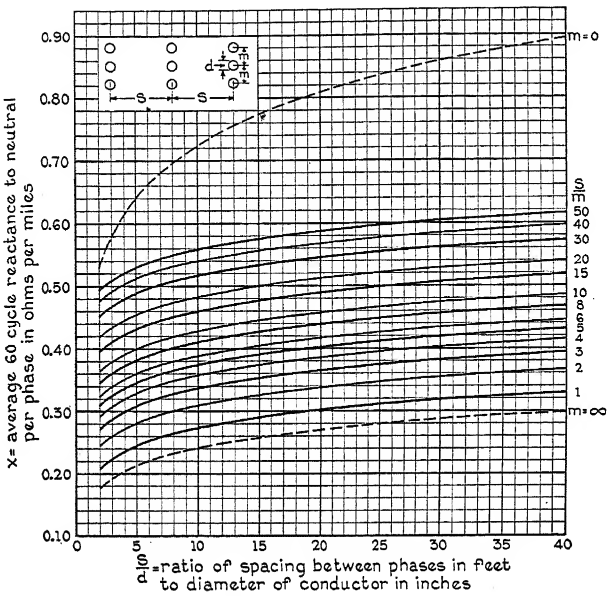

Fig. 3—Three Conductors per Phase Arranged Vertically

60-cycle reactance as affected by S/d and S/m
S = horizontal distance between adjacent phases in feet
m = distance between adjacent conductors of the same phase in feet
d = diameter of the conductor in inches

all conductors will be approximately the same. As a conductor is changed from one tower position to another, in general, it will have a different corona-starting voltage in each position.

In Appendix C, by Mr. S. B. Crary, a formula is developed for the disruptive critical voltage, or the corona-starting voltage, for any conductor of a multiple-conductor circuit. If the conductor which has the lowest disruptive critical voltage is designated *II-1*, as in Fig. 10, the lowest disruptive critical voltage of the system will be given by (**8c**), if 1 and *1* are interchanged.

Introducing the irregularity and air density factors,[4] M_0 and δ, respectively, and multiplying numerator and denominator of the fraction in (**8c**) by d, e_0, the lowest corona starting voltage of the system will be given by (**14**).

$$e_0 = \frac{61.5 M_0 \delta d \log_{10} \dfrac{2\sqrt{(S_{11}.S_{12}\ldots S_{1n})(S_{11}.S_{12}\ldots S_{1n})}}{d m_{12}\ldots m_{1n}}}{1+\sqrt{\left[\sum \dfrac{d}{m_{1n}}\cos\theta_{1n}\right]^2+\left[\sum \dfrac{d}{m_{1n}}\sin\theta_{1n}\right]^2}} \text{ kv. to neutral} \qquad (14)$$

where

e_0 = line-to-neutral corona starting voltage in kv.
M_0 = irregularity factor = 0.98 − 0.93 for roughened or weathered wires. = 0.87 − 0.83 for seven-strand cables.
δ = air density factor = $3.92b/(273 + t)$ = 1.00 at 76 cm. barometer and 25 deg. cent. where b = barometric pressure in cm. and t = temperature in deg. cent.

d, m, and S are in inches.

The formulas for disruptive critical voltage, given by (**8c**) and (**14**), are based on the assumptions that the charges on the three phases are balanced, and the charge per unit length is the same for all conductors of the same phase. The field distortion in the vicinity of a conductor due to charges on conductors of the other two phases will depend upon the ratio of diameter of conductors to spacing between phases, $\frac{d}{S}$. Since $\frac{d}{S}$ will be small for all practical arrangements of the con-

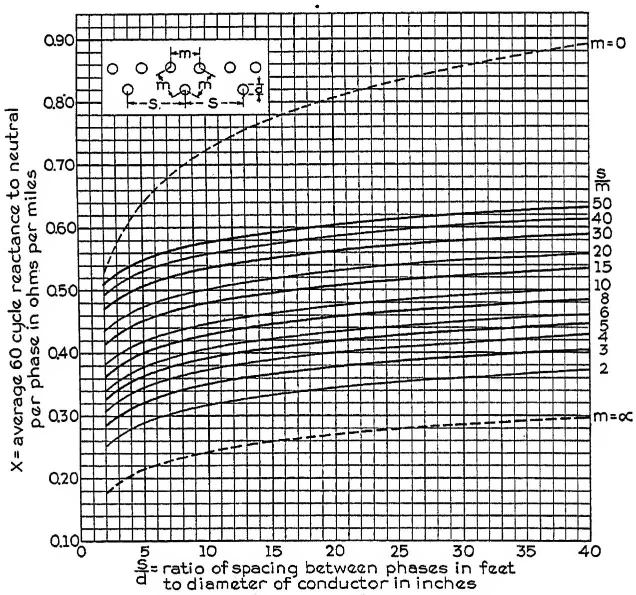

Fig. 4—Three Conductors per Phase Arranged at the Corners of an Equilateral Triangle

60-cycle reactance as affected by S/d and S/m
S = distance between centers of adjacent phases in feet
m = side of triangle in feet
d = diameter of conductors in inches

ductors in a multiple conductor circuit, the field distortion due to the charges on the other two phases will have a negligible effect on the corona starting voltage. The field distortion due to the charges on conductors of the same phase will depend upon $\frac{d}{m}$, the ratio of diameter of conductors to spacing between conductors of the same phase, and may have an appreciable effect upon the corona starting voltage. This is evident from

inspection of the denominator of the fraction in (**14**). As the ratio of diameter to spacing between conductors decreases, the field distortion will decrease and finally become negligible for very small values of $\frac{d}{m}$. When $\frac{d}{m}$ is small relative to unity, the denominator of the fraction in (**14**) approaches unity; the numerator of the fraction will then give the corona starting voltage.

As a first approximation, the corona starting voltage may be calculated from the numerator of the fraction in (**14**). When greater accuracy than this is required, the approximate corona-starting voltage may be corrected by dividing it by D, the denominator of the fraction in (**14**).

Let

$$D = 1 + \sqrt{\left[\sum \frac{d}{m_{1n}} \cos \theta_{1n}\right]^2 + \left[\sum \frac{d}{m_{1n}} \sin \theta_{1n}\right]^2}$$

$$= 1 + k\frac{d}{m} \qquad (15)$$

where d and m are in the same units and k depends upon the arrangement and the number of conductors in the phase. Values of k corresponding to seven special arrangements of conductors in multiple conductor circuits were calculated by (**15**) and tabulated in Table II.

TABLE II—VALUES OF k AND x_k TO BE USED IN CONNECTION WITH FIG. 9 TO OBTAIN CORONA STARTING VOLTAGE

Conductors per phase	Arrangement of conductors in the phases	k To be substituted in equation (15) to obtain D	x_k To be substituted in equation (18) to obtain x_m
2	vertical	1.0	0.056
2	horizontal	1.0	0.056
3	vertical	1.5	0.112
3	at corners of an equilateral triangle	1.732	0.084
4	vertical	1.833	0.179
4	at corners of a square	2.121	0.112
5	vertical	2.083	0.251

If the numerator of the fraction in (**14**) is compared with the formula for minimum 60-cycle reactance given in (**11a**), Appendix A, it will be noticed that the argument of the logarithm is the same in both equations. For convenience, corona-starting voltage will be expressed in terms of x_m, the minimum 60-cycle reactance of any conductor.

Replacing the logarithm in (**14**) by its value from (**11a**), and replacing the denominator by D,

$$e_0 = \frac{61.5\, M_0 \delta d \dfrac{x_m - 0.03034}{0.27941}}{D} \qquad (16)$$

Substituting 0.87 for M_0 and 1.0 for δ in (**16**) and multiplying both sides of the equation by $\sqrt{3}$, the line-to-line corona-starting voltage, E_0, is obtained.

$$E_0 = \frac{332\, d\, (x_m - 0.03034)}{D} \text{ kv. line-to-line} \qquad (17)$$

In Appendix B, formulas are given for x, the average 60-cycle reactance per phase for special arrangements of the multiple conductor circuits. These are plotted in Figs. 1 to 7. In Appendix A it is shown that x_m the minimum 60-cycle reactance for a conductor in ohms per mile may be obtained from x_c its average 60-cycle reactance by subtracting a value, x_k, which is approximately constant for any given number of conductors per phase and arrangement of conductors in the phase. Values of x_k for special conductor arrangements are

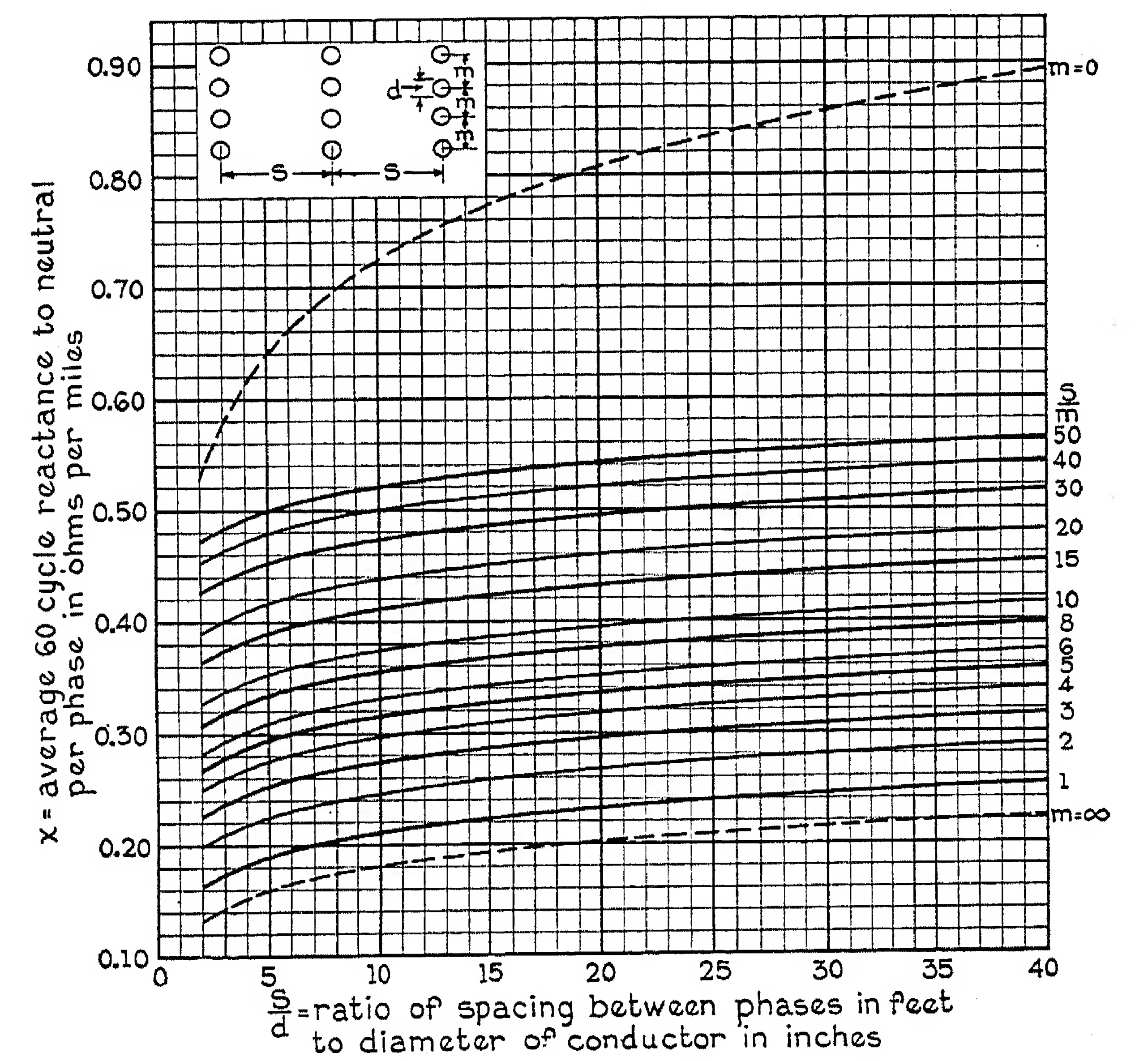

FIG. 5—FOUR CONDUCTORS PER PHASE ARRANGED VERTICALLY

60-cycle reactance as affected by S/d and S/m
S = horizontal distance between adjacent phases in feet
m = distance between adjacent conductors of the same phase in feet
d = diameter of the conductor in inches

derived from (**13a**), Appendix A and tabulated in Table II. x_m may be calculated from Figs. 1 to 7 and Table II, by means of (**18**).

$$x_m = x_c - x_k = nx - x_k \qquad (18)$$

Fig. 9, plotted from (**17**) when $D = 1$, gives a first approximation of the line-to-line corona-starting voltage in kv., expressed in terms of minimum 60-cycle reactance to neutral of any conductor in ohms per mile, and the diameter of the conductor in inches. It is applicable to both multiple- and single-conductor circuits. The reactance to be used is that for solid conductors of the given diameter and spacings.

From Fig. 9 and Figs. 1 to 7 the approximate diameter which is required to avoid corona at specified spacings and voltage for any of the special conductor

arrangements considered, may be determined by the following procedure:

a. Assume d, the diameter of the conductor.

b. From Figs. 1 to 7 read x, the average 60-cycle reactance per phase corresponding to the specified number of conductors per phase, arrangement of conductors, and calculated values of S/d and S/m.

c. Multiply x, the reactance per phase, by n, the number of conductors per phase, to obtain x_c, the average reactance per conductor.

d. From x_c, the average reactance per conductor, calculate x_m, the minimum reactance, by subtracting the appropriate value of x_k given in Table II. (See equation (18).)

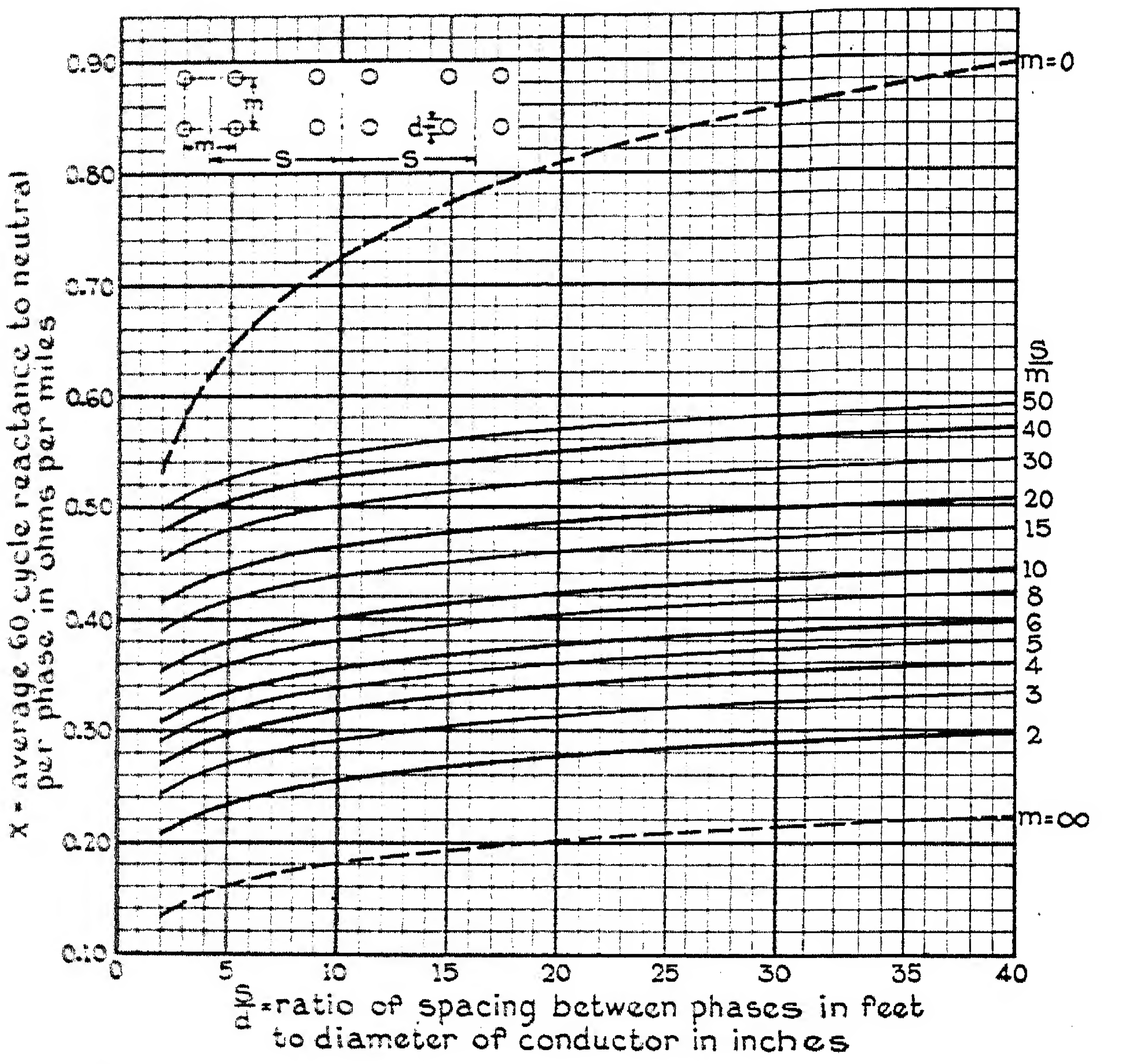

Fig. 6—Four Conductors per Phase at the Corners of a Square

60-cycle reactance as affected by S/d and S/m
S = distance between centers of adjacent phases in feet
m = side of square in feet
d = diameter of the conductor in inches

e. From Fig. 9 read E_0', a first approximation of the line-to-line corona-starting voltage corresponding to the assumed diameter, d, and the minimum reactance, x_m. This will be very nearly the correct value of the corona-starting voltage when the ratio of d/m is small relative to unity.

f. Calculate d/m and read k from Table II corresponding to the given number and arrangement of conductors. Substitute d, m and k in (15) to obtain D, d and m being in the same units.

g. Divide the value of E_0' read from Fig. 9 by D to obtain the corrected corona-starting voltage, E_0.

h. If E_0 obtained from (g) is greater than the operating voltage, corona will be avoided. If E_0 is less than the operating voltage, a larger diameter should be selected and the process repeated.

II—Comparison of Multiple- and Single-Conductor Circuits

Reactance and Capacity Susceptance

The upper dotted curve in Figs. 1 to 7 for $m = o$ gives the average 60-cycle reactance to neutral of the three phases of a single-conductor circuit with flat arrangement of conductors. It also gives the average reactance per mile of each phase of the perfectly transposed line. The reactance is plotted in terms of S/d, the ratio of the distance between adjacent conductors in feet to the diameter of the conductor in inches. The curve, which is the same on all seven figures, has been repeated for convenience in comparing the reactance of the single-conductor circuit with that of multiple-conductor circuits. Since the curve is for solid conductors, the correction from Table I corresponding to $n = 1$ should be applied for stranded or hollow conductors.

As m increases from 0 to ∞, the reactance per phase of the multiple conductor decreases. The lower dotted curves for $m = \infty$ apply when the conductors of the phase are so far apart that all mutual effects disappear and there are n separate parallel circuits.

From Figs. 1 to 7 it may be seen that the reactance per phase of a multiple-conductor circuit is always less than that of the single-conductor circuit, the diameter and spacing between adjacent phases being the same. The reactance of one conductor of the multiple-conductor circuit, however, is greater than that of a conductor of the single-conductor circuit.

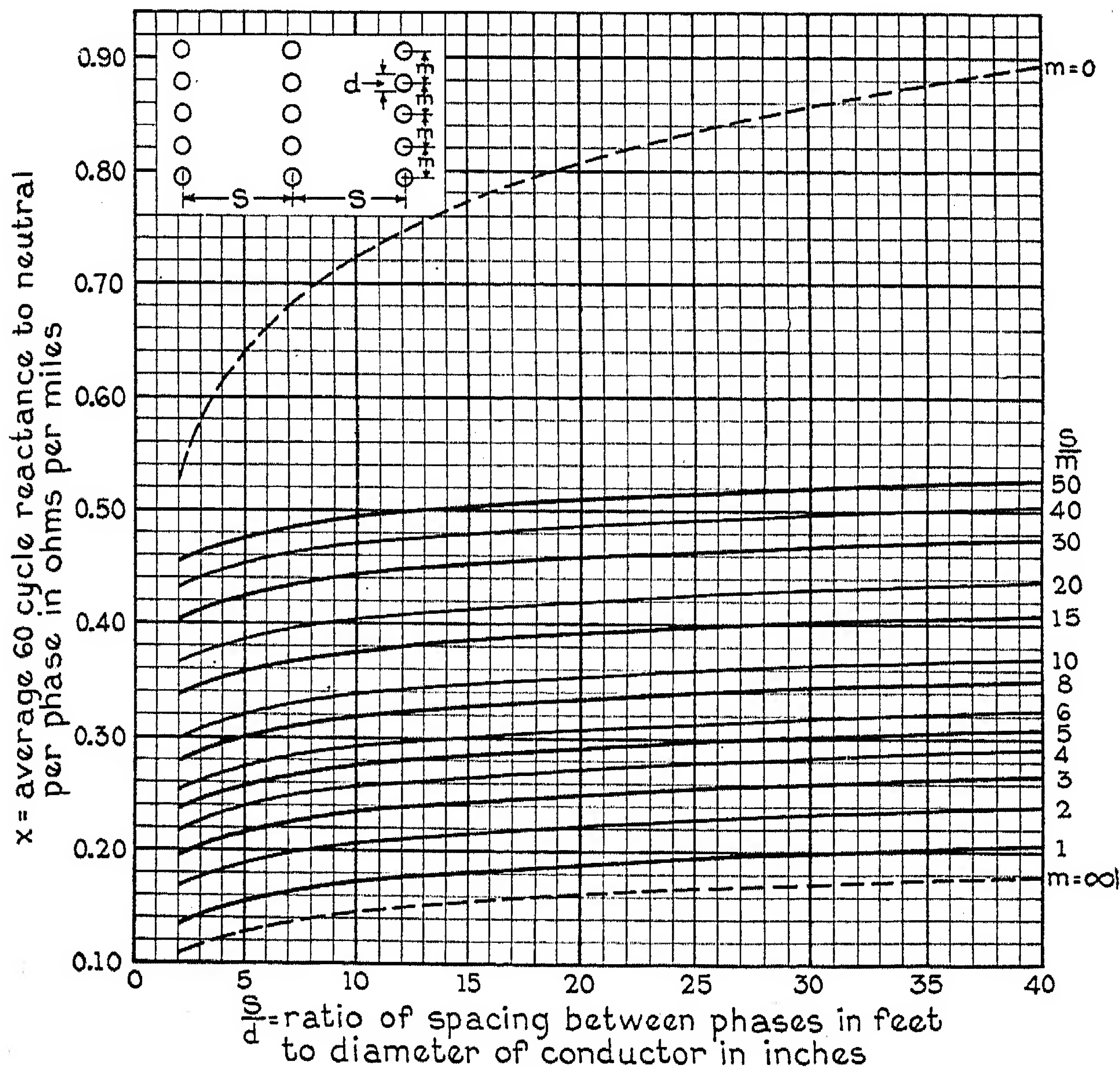

Fig. 7—Five Conductors per Phase Arranged Vertically

60-cycle reactance as affected by S/d and S/m
S = horizontal distance between adjacent phases in feet
m = distance between adjacent conductors of the same phase in feet
d = diameter of the conductor in inches

In Fig. 8 the lower dotted curve gives the 60-cycle capacity susceptance to neutral of the single-conductor circuit. The upper dotted curve gives the upper limit of the capacity susceptance per phase as the spacing between the conductor of the phase is indefinitely increased. It is evident that the capacity susceptance per phase for the multiple-conductor circuit is greater than that of the single-conductor circuit for the same S/d, while the capacity susceptance per conductor is less.

Diameter to Avoid Corona

Fig. 9 gives the approximate diameter required to avoid corona at a given voltage in terms of x_m, the minimum 60-cycle reactance of any conductor. Since the reactance per conductor is greater in the multiple- than in the single-conductor circuit, from Fig. 9 it follows that a smaller diameter is required to avoid corona in the multiple- than in the single-conductor circuit. With the same conductor diameter, the multiple-conductor circuits can be operated at a higher voltage without corona than the single-conductor circuit.

Charging Current and Rise in Voltage at No-Load

At a given voltage and frequency the charging currents on the multiple- and single-conductor circuits are directly proportional to their respective capacitances to neutral per phase. Since the larger current flows through the smaller inductance and vice versa, the rise in voltage at no-load is practically the same for both the multiple- and single-conductor circuits. These statements follow from the transmission line equations[5] (**19**) and (**20**) under no-load conditions, *i.e.* $I_r = 0$.

$$e_s = e_r \cosh \sqrt{ZY} + I_r Z \left(\frac{\sinh \sqrt{ZY}}{\sqrt{ZY}} \right) \quad (19)$$

$$I_s = I_r \cosh \sqrt{ZY} + e_r Y \left(\frac{\sinh \sqrt{ZY}}{\sqrt{ZY}} \right) \quad (20)$$

where

e_s and e_r = voltage to neutral at sending and receiving end of line respectively in volts.

I_s and I_r = the corresponding phase current in amperes.

$Z = l\,(r + jx) = l\,(r + j\,2\pi f L)$ = total impedance of one phase to neutral in ohms.

$Y = l\,(g + jb) = l\,(o + j\,2\pi f C)$ = total admittance of one phase to neutral in mhos.

r, L, g and C = resistance, inductance, leakance and capacitance respectively to neutral per mile.

l = length of line in miles.

Replacing the hyperbolic terms in (**19**) and (**20**) by their equivalent series, and setting $I_r = 0$, at no load

$$e_g = e_r \left(1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \ldots\ldots \right) \quad (21)$$

$$I_g = e_r Y \left(1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \ldots\ldots \right) \quad (22)$$

For single-conductor circuits the product of inductance and capacitance varies but slightly throughout the practical range of conductor diameters and spacings. Therefore, for a given frequency and length of line, the terms in parenthesis in (**21**) and (**22**) are substantially constant. The effect of resistance, and variations in the product of inductance and capacitance, on the hyperbolic terms used in transmission line calculation are shown graphically in "Simplified Transmission Line Calculations" by the author, *General Electric Review*, May 1926. In all practical cases these effects are small.

In multiple-conductor circuits the inductance per phase is decreased and the capacitance increased over that of the single-conductor circuit in approximately the same ratio, therefore the product of inductance and capacitance will not be greatly changed. This product, in general, will be slightly less for multiple- than for single-conductor circuits, but it will not be more than 5 per cent less. The effect of this variation on the terms in parenthesis will be insignificant except for very long lines. It is therefore evident from (**22**) that the charging current varies directly with the capacitance, and from

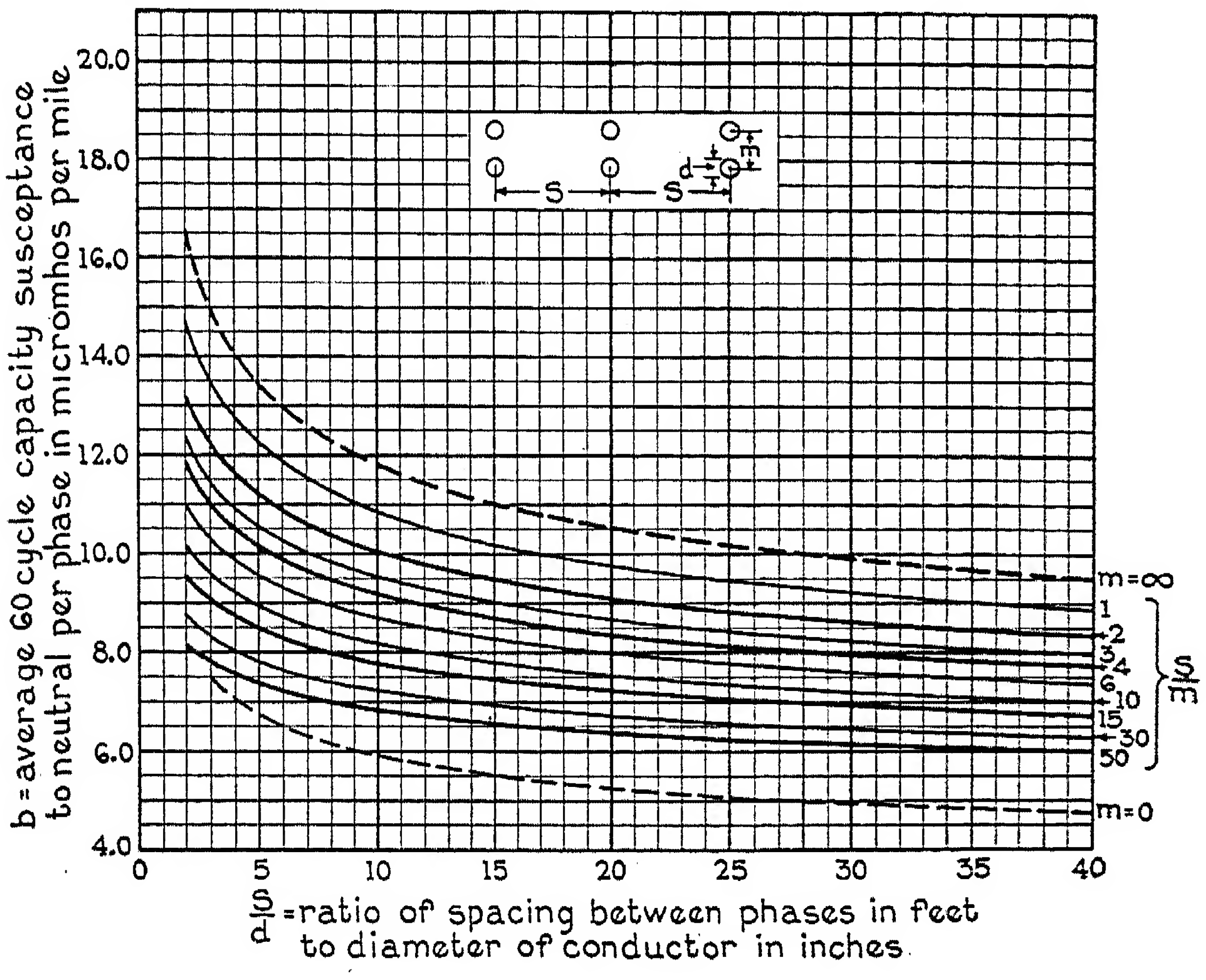

Fig. 8—Two Conductors per Phase Arranged Vertically

60-cycle capacity susceptance as affected by S/d and S/m

S = horizontal distance between adjacent phases in feet

m = distance between conductors of the same phase in feet

d = diameter of the conductor in inches

(**21**) that the rise in voltage at no-load for the single- and multiple-conductor circuits is substantially the same.

Power Transmitted Under Given Terminal Conditions

For the same receiver voltage, equation (**19**) shows that I_r may be increased in the same ratio that Z is decreased without affecting e_g, since the hyperbolic terms are substantially constant for a given frequency and length of line for both single- and multiple-conductor circuits. It follows, therefore, that when resistance is small, or when the ratio of resistance to reactance is the same for both circuits, the power which can be transmitted at a specified voltage drop, receiver voltage, and power factor, varies inversely as the respective reactances of the single- and multiple-conductor circuits.

For unequal ratios of resistance to reactance, this simple relation will hold only approximately, but in any case it may be used to advantage in estimating the

power which can be transmitted over a multiple-conductor circuit. It may also be used to determine approximately the spacings required with a proposed arrangement of conductors to deliver a given amount of power under specified terminal conditions.

For example: Suppose it is known that 150,000 kw. can be delivered over a certain 230-kv. single-conductor line with given terminal voltages and receiver-end power factor. The reactance of the line is 0.8 ohms per mile.

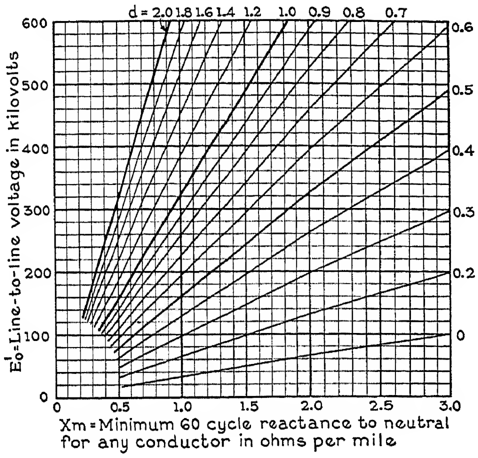

FIG. 9—E_0', FIRST APPROXIMATION OF CORONA STARTING VOLTAGE AS AFFECTED BY DIAMETER OF CONDUCTOR AND MINIMUM REACTANCE PER MILE AT 60 CYCLES

d = diameter of conductor in inches

$E_0 = E_0'/D$, where $D = 1 + k \dfrac{d}{m}$

It is required to transmit 210,000 kw. at substantially the same terminal conditions over a circuit of the same length having two conductors per phase, vertically arranged.

To increase the power in the ratio 210/150 the reactance must be reduced to approximately $0.8 \times 150/210 = 0.571$ ohms per mile. With two conductors per phase the average reactance per conductor, x_c, will be $2 \times 0.571 = 1.143$ ohms per mile. From Table II $x_k = 0.056$, and from (**18**) $x_m = x_c - x_k = 1.087$ ohms per mile. From Fig. 9 the diameter to avoid corona at 230 kv. must be greater than 0.66 inches, how much greater depending upon the ratio d/m.

Fig. 1 shows all possible combinations of the ratios S/d and S/m which will give the required reactance. The choice of spacings between phases and between conductors of the same phase is limited by mechanical considerations; consequently, it may not be possible to use spacings which would give the best results electrically. The following combinations of S, m and d are among those which will give a reactance of approximately 0.571 ohms per mile, and have a corona starting voltage above 230 kv., the operating voltage.

S-ft.	m-ft.	d-inches	S/d	S/m
15	1.25	0.728	20.6	12.00
20	2.00	0.806	24.8	10.00
25	3.00	0.855	29.3	8.3

STABILITY

The methods now in use for determining the stability limits of systems composed of single-conductor circuits may be applied with equal rigor to systems containing one or more multiple-conductor circuits.

A multiple-conductor circuit can be designed to carry more power than a single-conductor circuit of the same length and operating voltage. Accordingly, fewer multiple- than single-conductor circuits are required to transmit a given amount of power from a generating plant to its load center under specified terminal conditions.

The relative stability of a proposed system with multiple- or single-conductor circuits would depend upon whether the operating conditions are to be the same for the two schemes with all lines in service or in the emergency conditions of one line out of service. When the two systems are designed to operate under the same terminal conditions with all lines in service, the total reactance and capacity susceptance between line terminals will be substantially the same whether multiple- or single-conductor circuits are employed. Consequently, under normal operating conditions the steady-state power limit and margin of stability would be the same for either arrangement. With a line out of service, the steady-state power limit with multiple-conductor circuits would be less than with single-conductor circuits. Should the systems be designed to operate under the same terminal conditions during emergency conditions, the relative steady-state stability with multiple-conductor circuits would be increased.

A fault on a conductor of a multiple-conductor circuit will be a less severe shock to the system than one on a

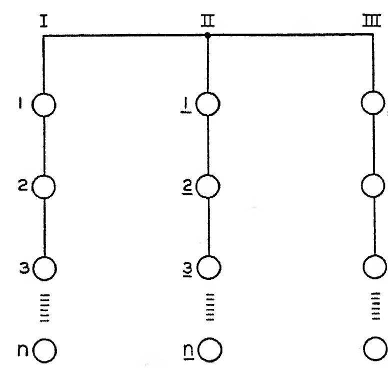

FIG. 10—THREE-PHASE MULTIPLE-CONDUCTOR CIRCUIT HAVING n CONDUCTORS PER PHASE

conductor of a single-conductor circuit. But after the fault has been cleared, the loss of a multiple-conductor circuit would reduce the synchronizing torque between generator and load to a greater extent than the loss of a single-conductor circuit. The stability of the two systems under transients conditions would depend upon the location and duration of the fault, the number of conductors involved and the relative amounts of power carried by a multiple- and a single-conductor circuit.

ACKNOWLEDGMENT

The author wishes to express her appreciation of the assistance given by Mr. R. N. Slinger and Mr. M. A. DeFerranti in the preparation of the curves.

Appendix A

INDUCTANCE AND 60-CYCLE REACTANCE OF MULTIPLE-CONDUCTOR CIRCUITS

When a three-phase multiple-conductor transmission line is completely transposed so that each conductor occupies each of the tower positions, or its equivalent, for equal distances, the phases being rotated in cyclic order, the total inductance to neutral of any conductor will be the sum of the inductances in each position.

Let

n = number of conductors per phase.
l = total length of each conductor.
$l/3n$ = length of conductor in each position.
I = current in phase A.
I/n = current in one conductor of phase A.

$$\frac{I}{n}\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)=\frac{I}{n}a^2$$

= current in one conductor of phase B

$$\frac{I}{n}\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)=\frac{I}{n}a$$

= current in one conductor of phase C

The phase positions on the tower will be designated by I, II, and III, and the conductor positions in the phase positions by 1, 2 . . . n, *1, 2* . . . *n*, and **1, 2** . . . **n** respectively. Fig. 10.*

Starting with phase A in position I, the conductor of phase A which first occupies the position I-1 will in turn occupy each of the $3n$ positions for equal distances, $l/3n$.

For sine waves of current of frequency f, the total inductive voltage drop in any conductor will be the sum of the voltage drops in the conductor for the $3n$ positions and may be expressed in terms of self-inductance, L_1, and mutual inductance, M.

With the conductor in position I-1

$$V_{I-1}=2\pi f\left[\frac{I}{n}(L_1+M_{12}+\ldots+M_{1n})+\frac{I}{n}a^2(M_{\mathit{11}}+M_{12}+\ldots+M_{1n})+\frac{I}{n}a(M_{\mathbf{11}}+M_{\mathbf{12}}+\ldots+M_{\mathbf{1n}})\right]\quad\textbf{(1a)}$$

There will be a total of n equations in (**1a**), one for each of the conductor positions in phase position I.

*The conductors may have any desired arrangement in the phase.

There will also be n equations when phase A occupies phase position II, and likewise n equations for position III. One equation for position II is given by (**2a**) and one for position III by (**3a**)

$$V_{II-\mathit{1}}=2\pi f\left[\frac{I}{n}(L_1+M_{\mathit{12}}+\ldots M_{\mathit{1n}})+\frac{I}{n}a^2(M_{\mathit{11}}+M_{\mathbf{12}}+\ldots M_{\mathit{1}\mathbf{n}})+\frac{I}{n}a(M_{\mathit{11}}+M_{12}+\ldots M_{\mathit{1}\mathrm{n}})\right]\quad\textbf{(2a)}$$

$$V_{III-\mathbf{1}}=2\pi f\left[\frac{I}{n}(L_1+M_{\mathbf{12}}+\ldots M_{\mathbf{1n}})+\frac{I}{n}a^2(M_{11}+M_{12}+\ldots M_{1n})+\frac{I}{n}a(M_{\mathbf{1}\mathit{1}}+M_{\mathbf{1}2}+\ldots M_{\mathbf{1}n})\right]\quad\textbf{(3a)}$$

Replacing a by

$$\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right),\ a^2\text{ by}\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)$$

and adding the voltage drops in the $3n$ positions, the total inductive voltage drop, V, is obtained.

$$V=j2\pi f\frac{I}{n}\Big[3nL_1+2(M_{12}+\ldots M_{1n}+\ldots+M_{\mathit{12}}+\ldots M_{\mathit{1n}}+\ldots M_{\mathbf{12}}+M_{\mathbf{1n}}+\ldots)-(M_{1\mathit{1}}+\ldots M_{1\mathit{n}}+\ldots+M_{\mathit{11}}+\ldots M_{\mathit{1}\mathbf{n}}+\ldots M_{\mathbf{1}1}+\ldots M_{\mathbf{1}n}+\ldots)+j\frac{\sqrt{3}}{2}(0)\Big]\quad\textbf{(4a)}$$

From equations (**1a**), (**2a**), (**3a**), (**4a**) and Fig. 10 it may be seen that there will be $\dfrac{n(n-1)}{2}$ mutuals between conductors in the same phase position, (some of which may be equal) and a total of $\dfrac{3n(n-1)}{2}$ for the three phase positions. The number of mutuals between conductors of different phases will be $3n^2$.

From equation (**4a**) the inductance per conductor, L_t, may be obtained by dividing through by $j2\pi f\dfrac{I}{n}$.

$$L_t=\frac{V}{j2\pi f\dfrac{I}{n}}=\Big[3nL_1+2\sum\left(M_{aa},\ \frac{3n(n-1)}{2}\text{ terms}\right)$$

$$- \sum (M_{ab}, 3n^2 \text{ terms}) \Big] \qquad (5a)$$

where M_{aa}, M_{ab} represent the mutual inductances between conductors of the same and different phases respectively.

The following formulas in absolute units for self and mutual inductance of non-magnetic wires in air or other non-magnetic medium follow from those given in the "Bulletin of the Bureau of Standards," Vol. 8, No. 1, p. 150-151, when the length, l', is great in comparison with the diameter, d, and the distance between wires, S:

$$L_s = 2l' \left[\log_e \frac{4l'}{d} - \frac{3}{4} \right] = \text{self-inductance} \qquad (6a)$$

$$M = 2l' \left[\log_e \frac{2l'}{S} - 1 \right] = \text{mutual-inductance} \qquad (7a)$$

Replacing l' in (**6a**) and (**7a**) by $l/3n$, the length of conductor in each position, substituting (**6a**) and (**7a**) in (**5a**) and simplifying, the total inductance to neutral per conductor may be obtained in absolute units, thus:

$$L_t = l \left[2 \log_e \frac{2\,(S_{gm})^n}{d\,(m_{gm})^{n-1}} + \frac{1}{2} \right] \text{abhenrys} \qquad (8a)$$

where

S_{gm} = the geometric mean of the distances between conductors of different phases; or, the $3n^2$ root of the product of the $3n^2$ independent distances between conductors of different phases.

m_{gm} = the geometric mean of the distances between conductors of the same phase; or, the $3n(n-1)/2$ root of the product of the $3n(n-1)/2$ independent distances between conductor of the same phase.

d = diameter of conductors.

l = length of conductors.

Expressing (**8a**) in practical units, with S_{gm}, m_{gm} and d in the same units, the average inductance to neutral per conductor is

$$L_c = 0.08047 + 0.74113 \log_{10} \frac{2\,(S_{gm})^n}{d\,(m_{gm})^{n-1}}$$

millihenrys per mile (**9a**)

The average 60-cycle reactance to neutral per conductor will be

$$x_c = 0.03034 + 0.27941 \log_{10} \frac{2\,(S_{gm})^n}{d\,(m_{gm})^{n-1}} \qquad (10a)$$

MINIMUM 60-CYCLE REACTANCE

In a completely transposed line the inductance of a conductor will vary with its position on the tower. Since each conductor occupies each tower positions, the minimum, as well as the average inductance, of all conductors will be the same. The inductance of the conductor will be smaller when it is in the center phase position than when in either of the two outside positions. It will be a minimum when the conductor is in the center phase position and in that conductor position which has the greatest geometric mean distance to the other conductors of the phase. If such a position is designated II-*1*, the inductance for the conductor in this position may be obtained from (**2a**). When the conductor in II-*1* is symmetrically located with reference to the conductors of the other two phase, the j term of the inductance will disappear; in any case it will be small and will therefore be neglected.

The minimum 60-cycle reactance to neutral of any conductor of a perfectly transposed multiple conductor may be written

$$x_m = 0.03034 + 0.27941 \log_{10} \frac{2\sqrt{(S_{11} \,.\, S_{12} \ldots S_{1n})(S_{11} \,.\, S_{12} \ldots S_{1n})}}{d\,(m_{12} \ldots m_{1n})} \text{ ohms per mile} \qquad (11a)$$

An equation giving, x_k, the difference between the average and minimum 60-cycle reactance in ohms per mile for any conductor, may be obtained by subtracting (**11a**) from (**10a**),

$$x_k = x_c - x_m = 0.27941 \log_{10} \frac{(S_{gm})^n\,(m_{12} \ldots m_{1n})}{(m_{gm})^{n-1}\sqrt{(S_{11} \,.\, S_{12} \ldots S_{1n})(S_{11} \,.\, S_{12} \ldots S_{1n})}} \qquad (12a)$$

When the ratio S/m is 5 or more, the ratio of $(S_{gm})^n$ to the radical in the denominator of (**12a**) approaches $(\sqrt[3]{2}S)^n/S^n = (\sqrt[3]{2})^n$, and (**12a**) becomes approximately

$$x_k = x_c - m_m = 0.27941 \log_{10} \frac{(\sqrt[3]{2})^n\,[m_{12} \ldots m_{1n}]}{(m_{gm})^{n-1}} \qquad (13a)$$

Appendix B

AVERAGE 60-CYCLE REACTANCE AND CAPACITY SUSCEPTANCE

Equations (**4**) and (**10**) give respectively the average 60-cycle reactance, x, and capacity susceptance, b, to neutral per mile for each phase of a completely transposed multiple-conductor circuit. Seven special symmetrical arrangements of conductors in multiple-conductor circuits are shown in the inserts in Figs 1 to 7. For these arrangements, when S_{gm} is expressed in terms S and m, and m_{gm} in terms of m, x and b will be expressed in terms of the two ratios S/d and S/m.

I. TWO CONDUCTORS PER PHASE

a. Vertical Arrangement of Conductors in the Same Phase. From Fig. 1.

$$(S_{gm})^n = (S_{gm})^2 = \left(\sqrt[12]{S^4\,(2S)^2\,(S^2 + m^2)^2\,(\overline{2S^2 + m^2})}\right)^2$$

$$= \sqrt[3]{4}\,S^2 \sqrt[6]{\left[1+\left(\frac{m}{S}\right)^2\right]^2\left[1+\left(\frac{m}{2S}\right)^2\right]} \qquad (1b)$$

$$(m_{gm})^{n-1} = m \qquad (2b)$$

Replacing $(S_{gm})^n$ and $(m_{gm})^{n-1}$ in (**4**) and (**10**) by their values in terms of S and m from (**1b**) and (**2b**), the average 60-cycle reactance and capacity susceptance to neutral per mile for each phase are, respectively,

$$x = \frac{2\pi f L}{n}$$

$$= \frac{1}{2}\left\{0.03034 + 0.27941 \log_{10} 24 \sqrt[3]{2}\,\frac{S}{d}\right\}$$

$$+ 0.1397 \log_{10} \frac{\sqrt[3]{2}\,\dfrac{S}{m}}{\sqrt[6]{\left[1+\left(\dfrac{m}{S}\right)^2\right]^2\left[1+\left(\dfrac{m}{2S}\right)^2\right]}}$$

ohms per mile (**3b**)

$$b = 2\pi f C n$$

$$= \frac{29.28}{\log_{10} 24 \sqrt[3]{2}\,\dfrac{S}{d} + \log_{10} \sqrt[3]{2}\,\dfrac{S}{m} \times \sqrt[6]{\left[1+\left(\dfrac{m}{S}\right)^2\right]^2\left[1+\left(\dfrac{m}{2S}\right)^2\right]}}$$

micromhos per mile (**4b**)

For the other six cases the average 60-cycle reactance equations only will be given.

b. Horizontal Arrangement of Conductors in the Same Phase, Fig. 2.

$$x = \frac{1}{2}\left\{0.03034 + 0.27941 \log_{10} 24 \sqrt[3]{2}\,\frac{S}{d}\right\}$$

$$+ 0.1397 \log_{10} \frac{\sqrt[3]{2}\,\dfrac{S}{m}}{\sqrt[6]{\left[1-\left(\dfrac{m}{S}\right)^2\right]^2\left[1-\left(\dfrac{m}{2S}\right)^2\right]}}$$

ohms per mile (**5b**)

II. Three Conductors per Phase

a. Vertical Arrangement of Conductors in the Same Phase, Fig. 3.

$$x = \frac{1}{3}\left\{0.03034 + 0.27941 \log_{10} 24 \sqrt[3]{2}\,\frac{S}{d}\right\}$$

$$+ 0.09314 \log_{10} \frac{\left(\dfrac{S}{m}\right)^2}{\sqrt[9]{\left[1+\left(\dfrac{m}{S}\right)^2\right]^5\left[1+\left(\dfrac{m}{2S}\right)^2\right]^2\left[1+\left(\dfrac{2m}{S}\right)^2\right]^2}}$$

ohms per mile (**7b**)

b. Conductors of the Same Phase at the Corners of an Equivalent Triangle. Fig. 4.

$$x = \frac{1}{3}\left\{0.03034 + 0.27941 \log_{10} 24 \sqrt[3]{2}\,\frac{S}{d}\right\} + 0.09314 \log_{10} \frac{\sqrt[3]{4}\left(\dfrac{S}{m}\right)^2}{\sqrt[9]{\left[1-\left(\dfrac{m}{S}\right)^2\right]^2\left[1-\left(\dfrac{m}{2S}\right)^2\right]\left[1+\left(\dfrac{m}{S}\right)^2+\left(\dfrac{m}{S}\right)^4\right]^2\left[1+\left(\dfrac{m}{2S}\right)^2+\left(\dfrac{m}{2S}\right)^4\right]}}$$

ohms per mile (**8b**)

III. Four Conductors per Phase

a. Vertical Arrangement of Conductors in the Same Phase, Fig. 5.

$$x = \frac{1}{4}\left\{0.03034 + 0.27941 \log_{10} 24 \sqrt[3]{2}\,\frac{S}{d}\right\} + 0.06985 \log_{10} \frac{\dfrac{(S/m)^3}{\sqrt{3}}}{\sqrt[12]{\left[1+\left(\dfrac{m}{S}\right)^2\right]^8\left[1+\left(\dfrac{2m}{S}\right)^2\right]^4\left[1+\left(\dfrac{3m}{S}\right)^2\right]^2\left[1+\left(\dfrac{m}{2S}\right)^2\right]^3\left[1+\left(\dfrac{3m}{2S}\right)^2\right]}}$$

ohms per mile (**9b**)

b. Conductors of the Same Phase at the Corners of a Square. Fig. 6.

$$x = \frac{1}{4}\left\{0.03034 + 0.27941 \log_{10} 24 \sqrt[3]{2}\,\frac{S}{d}\right\} + 0.06985 \log_{10} \frac{\sqrt{2}\,(S/m)^3}{\sqrt[12]{\left[1-\left(\dfrac{m}{S}\right)^4\right]^4\left[1-\left(\dfrac{m}{2S}\right)^4\right]^2\left[1+4\left(\dfrac{m}{S}\right)^4\right]^2\left[1+\dfrac{1}{4}\left(\dfrac{m}{S}\right)^4\right]}}$$

ohms per mile (**10b**)

IV. Five Conductors per Phase

a. Vertical Arrangement of Conductors in the Same Phase, Fig. 7.

$$x = \frac{1}{5}\left\{0.03034 + 0.27941 \log_{10} 24\sqrt[3]{2}\,\frac{S}{d}\right\} + 0.05588 \log_{10} \frac{(S/m)^4}{\sqrt[3]{4}\,\sqrt[5]{3^4}\,\sqrt[15]{\left[1+\left(\frac{m}{S}\right)^2\right]^{11}\left[1+\left(\frac{2m}{S}\right)^2\right]^{7}\left[1+\left(\frac{3m}{S}\right)^2\right]^{4}\left[1+\left(\frac{4m}{S}\right)^2\right]^{2}\left[1+\left(\frac{m}{2S}\right)^2\right]^{4}\left[1+\left(\frac{3m}{2S}\right)^2\right]^{2}}}$$

ohms per mile **(11b)**

Appendix C

Determination of the Disruptive Critical Voltage of a Multiple-Conductor Transmission Line

By S. B. Crary

For a transmission circuit which has one conductor per phase, the assumption that the potential gradient is uniform around the surface of the conductor is justifiable, when the ratios of diameters of conductors to spacing between phases, $\frac{d}{S}$, is small. However, a conductor of a multiple-conductor circuit may be located relatively close to the other conductors of the same phase. An appreciable field distortion may result which will lower the disruptive critical voltage. The following analysis determines the disruptive critical voltage, assuming that the field due to the conductors of the other phases is negligible and that the field produced by the other conductors of the same phase may be considered uniform in the region of the conductor under consideration.* By comparison with an exact solution for two equally charged conductors, developed by Dr. H. Poritsky, the latter assumption was found to be essentially correct for a ratio of spacing to diameter of conductor as low as 5 to 1.

Let

E_{01} = the field produced by the other conductors of the same phase in the region of conductor 1. Conductor 1 may be any conductor in any phase.

$$\therefore E_{01} = \sqrt{\overline{\Sigma E_x}^2 + \overline{\Sigma E_y}^2} \qquad \textbf{(1c)}$$

The intensity at a distance m from a line charge of Q per unit length is

$$E = \frac{2Q}{m} \qquad \textbf{(2c)}$$

Equation (**1c**), therefore, becomes

$$E_{01} = 2Q\sqrt{\left[\sum \frac{1}{m_{1n}} \cos\theta_{1n}\right]^2 + \left[\sum \frac{1}{m_{1n}} \sin\theta_{1n}\right]^2} \qquad \textbf{(3c)}$$

*Suggested by M. L. Henderson.

where

m_{1n} = the distance between conductor 1 and the nth conductor in the same phase.

θ_{1n} = the angle between a line joining the centers of conductors 1 and n and the reference axis.

At the surface of a conducting cylinder in a uniform field the maximum field intensity is $2E_{01}$, and the total maximum intensity at the surface when the conductor is charged is obtained by adding to $2E_{01}$, $\frac{2Q}{r}$.† Therefore the maximum intensity is

$$E_{m1} = 2E_{01} + \frac{2Q}{r} \qquad \textbf{(4c)}$$

where

r = the radius of the conductor.

It will be assumed that the charge Q per unit of length is the same on all the conductors of the same phase.

Substituting (**3c**) in (**4c**) and replacing r by $\frac{d}{2}$ equation (**5c**) is obtained.

$$E_{m1} = 4Q\left\{\sqrt{\left[\sum \frac{1}{m_{1n}} \cos\theta_{1n}\right]^2 + \left[\sum \frac{1}{m_{1n}} \sin\theta_{1n}\right]^2} + \frac{1}{d}\right\} \qquad \textbf{(5c)}$$

If the phase positions on the tower are designated by I, II and III and the conductor positions in the phases by 1, 2, — — n, *1*, *2*, — — *n*, **1**, **2**, — — **n** respectively, Fig. 10, the potential of conductor 1 in phase I will be:‡

$$\begin{aligned} V_1 = {} & P_{11} Q_1 + P_{12} Q_2 \text{------} + P_{1n} Q_n \\ & + P_{1\mathit{1}} Q_{\mathit{1}} + P_{1\mathit{2}} Q_{\mathit{2}} \text{------} + P_{1\mathit{n}} Q_{\mathit{n}} \\ & + P_{1\mathbf{1}} Q_{\mathbf{1}} + P_{1\mathbf{2}} Q_{\mathbf{2}} \text{------} + P_{1\mathbf{n}} Q_{\mathbf{n}} \end{aligned} \qquad \textbf{(6c)}$$

When the ratio of $\frac{2H}{S}$ is large (H = height above ground) the coefficients become

$$P_{11} = 2\log_e \frac{4H}{d}$$

†Page and Adams, "Principles of Electricity," pp. 97-98.

‡It is apparent that the equations developed are general and apply to any arrangement of conductors and phases. However, a specific arrangement as that shown in Fig. 10 and a particular conductor (1) in a particular phase (I) has been chosen for clearness. The equations can be applied when conductor 1 is any conductor and phase I is any phase.

$$P_{1n} = 2 \log_e \frac{2H}{m_{1n}}$$

$$P_{1n} = 2 \log_e \frac{2H}{S_{1n}}$$

$$P_{1n} = 2 \log_e \frac{2H}{S_{1n}}$$

For a three-phase system the charge on the other two-phase conductors is $-nQ$ per unit of length when the charge is nQ per unit of length on phase 1. Assuming that the charge on each of the other phase conductors is $-\frac{Q}{2}$ when the charge on conductor 1 is Q, the potential of conductor 1 of phase 1 becomes

$$V_1 = 2Q \log_e \frac{2\sqrt{(S_{11}\, S_{12} \ldots S_{1n})\,(S_{11} \,.\, S_{12} \ldots S_{1n})}}{d\,(m_{12} \ldots \ldots m_{1n})} \tag{7c}$$

Substituting the value of Q determined from equation (**5c**) in equation (**7c**) and 21.1 kv/cm.* for E_{m1} the following expression is obtained for the disruptive critical voltage in effective kv. to neutral of conductor 1:

$$V_1 = \frac{21.1 \log_e \dfrac{2\sqrt{(S_{11}\, S_{12} \ldots S_{1n})\,(S_{11} \,.\, S_{12} \ldots S_{1n})}}{d\,(m_{12} \ldots \ldots m_{1n})}}{2\left\{\sqrt{\left[\sum \frac{1}{m_{1n}} \cos\theta_{1n}\right]^2 + \left[\sum \frac{1}{m_{1n}} \sin\theta_{1n}\right]^2} + \frac{1}{d}\right\}} \tag{8c}$$

In any arrangement (on the basis of the preceding assumptions) the conductors of the center phase have a lower disruptive critical voltage than those of the two outside phases, as the numerator of the argument of the logarithm of equation (**8c**) has a smaller value for the center phase conductors than it has for the outside phase conductors. Further from an analysis of this equation, it is apparent that a certain conductor of the center phase may have the lowest critical voltage depending upon the arrangement of the conductors of that phase. If the arrangement of the conductors in a given phase is symmetrical, however, the disruptive critical voltage will be the same for all the conductors of that particular phase.

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Discussion

DOUBLE CONDUCTORS FOR TRANSMISSION LINES

(DWIGHT AND FARMER)

THREE-PHASE MULTIPLE-CONDUCTOR CIRCUITS

(CLARKE)

Hillel Poritsky: The case of equal charges of opposite sign is a familiar one from two-phase lines or from considerations of the cylindrical condenser. As is well known, the field may be expressed in terms of logarithmic functions, and yields circles both for the lines of force and the equipotentials. On the other hand, it will be seen that the case of equal charges of the *same* sign requires doubly periodic or elliptic functions for its treatment.

This discussion is confined to the case of cylinders of the *same* radius a. Let Q be the charge on each cylinder per unit length of axis, and $2b$ the (shortest) distance between the cylindrical surfaces. In a plane perpendicular to the cylinder axes and cutting the cylindrical surfaces in circles C_1, C_2 place a coordinate system with the x-axis intersecting the cylinder axes and the origin half way between the center of circles C_1, C_2. The problem resolves itself into finding a harmonic function u (the potential) which takes on the same constant value along the circle C_1, C_2, and whose conjugate harmonic function v (or flux function) increases by $4\pi Q$ for a path which encircles either C_1 or C_2 alone in the positive sense; moreover, at infinity u is to become infinite like $2Q \log (x^2 + y^2)$. This problem is treated best by means of complex quantities and the theory of analytic functions.

In the x_1y-plane we introduce the complex variable

$$z = x + iy,\ i^2 = -1.$$

As is known the potential u and its conjugate harmonic function v define an analytic function,

$$w = w(z) = u + iv. \tag{1}$$

First we shall carry the problem from the z-plane to a z_1-plane, where

$$z_1 = x_1 + iy_1 = \log \frac{c+z}{c-z},\ c = \sqrt{b\,(b+2a)}\,. \tag{2}$$

This carries the circles C_1, C_2 into the straight lines

$$x_1 = \pm d, \tag{3}$$

where

$$d = \log \frac{\sqrt{b+2a} + \sqrt{b}}{\sqrt{b+2a} - \sqrt{b}} \qquad d > 0. \tag{4}$$

The region outside the circles is transformerd into the region between the lines (3) and the original field goes into a field which is periodic in the y_1-direction of period 2π. Since the potential u is constant along $x_1 = \pm d$, the field may be extended across each of these boundaries to a width $2d$ on each side, then extended across the new boundaries, etc., resulting finally in a doubly periodic field, the periods being $4d$ and $2\pi i$. This field consists of sinks of lines of force at $z_1 = \pi i$, the image of $z = \infty$, and of its congruent points, and of sources at $z_1 = 2d + \pi i$ and its congruent points. At each source $8\pi Q$ lines start, and an equal number ends at each sink.

The field in question is described by means of

$$w = u + iv = 4Q \log \left[\Theta_1(z_2)/\Theta_1\left(z_2 - \frac{1}{2}\right)\right], \tag{5}$$

where

$$z_2 = (z_1 - \pi i)/4d, \tag{6}$$

and Θ_1 is defined by means of

$$\Theta_1 (v,\omega) = 2 \sum_{n=0}^{\infty} (-1)^n q^{\left(n+\frac{1}{2}\right)^2} \sin [(2n+1)\pi v], \; q = e^{\pi i\omega} \tag{7}$$

and

$$\omega = 2\pi i/4d. \tag{8}$$

The variable z_2 is introduced merely as a change of scale, reducing the periods $4d$, $2\pi i$ of the z_1-plane to unity and ω (given by (8)) in the z_2-plane; moreover, the origin has been shifted to one of the sinks correspondingt o $z = \infty$. The function $\Theta_1 (z_2)$ (the second argument ω has not been indicated) has roots at $z_2 = m + n\omega$ (see "Functions of a Complex Variable," by Pierpont, pp. 428, 429), hence

$$\Theta_1 (z_2)/\Theta_1 \left(z_2 - \frac{1}{2} \right)$$

has roots and poles at the sinks and sources respectively of the above field; it may also be shown to be doubly periodic.

To obtain the field intensity in the z-plane we need to know dw/dz. This may be evaluated from the formula

$$\frac{dw}{dz} = \frac{dw}{dz_2} \cdot \frac{dz_2}{dz_1} \cdot \frac{dz_1}{dz}.$$

This maximum field intensity occurs at $z = b + 2a$ and comes out to be

$$2\pi Q \frac{\sqrt{b}}{ad\sqrt{b+2a}} \cdot \frac{1 + 3q^{1\cdot 2} - 5q^{2\cdot 3} - 7q^{3\cdot 4} + 9q^{4\cdot 5}}{1 - q^{1\cdot 2} - q^{2\cdot 3} + q^{3\cdot 4} + \ldots}$$

The case of two cylinders which touch one another may be handled either as a limiting case of the above by letting b approach zero, or may be treated directly by means of the transformation

$$z_1 = 1/z$$

in place of (2). The field, after reflection across the straight lines corresponding to C_1, C_2 and further reflection is now simply periodic and may be described by means of circular functions. Thus

$$w = 4Q \log \left[\sin z_2/\sin \left(z_2 - \frac{\pi}{2} \right) \right]$$

where $z_2 = \dfrac{\pi a}{2} z_1$,

and the maximum intensity is $\dfrac{Q\pi}{a}$

S. B. Crary: In determining the voltages of transmission lines which have only one conductor per phase, the assumption is usually made that the potential gradient around the surface of the conductor is uniform. This is justifiable when the ratios of diameter of conductor to spacing between phases is small, which is usually the case. However, a conductor of a multiple-conductor circuit may be located relatively close to the conductors of the same phase. An appreciable field distortion may result. This field distortion is produced by the proximity of conductors having charges of equal magnitude and of the same sign.

The preceding discussion, by Dr. H. Poritsky, describes the method in which he obtained the exact solution for the maximum potential gradient of two cylinders having charges equal in magnitude and of the same sign. This formula may be applied in order to determine the critical disruptive voltage of a double-conductor transmission circuit. The exact solution for more than two conductors would undoubtedly lead to a result more involved and difficult to apply.

In order to obtain a result of sufficient engineering accuracy for the practical cases formula (5c) given in Miss Clarke's paper was developed. This formula corrects for the field distortion due to the conductors of the same phase by making two assumptions that are applicable to practical multiple-conductor transmission circuits. These are:

1. The field distortion due to the conductors of the other phases and the ground plane is negligible.
2. The field produced by the other conductors of the same phase may be considered uniform in the region of the conductor under consideration.

The formula of Dr. Poritsky's provided an excellent means of determining the correctness of the second assumption. Fig. 1 shows graphically the relative accuracy obtained by using the approximate formula (5c) given in Miss Clarke's paper as compared with the result obtained by using Dr. Poritsky's formula. The approximate formula is seen to be essentially correct for a ratio of m/d as low as 5 to 1, which covers the practical range of spacings for multiple-conductor circuits.

The lower curve shows the relative gradient obtained when the field distortion is neglected. For a ratio of m to d of 5, an error of 15 per cent would be obtained if the field distortion were not considered.

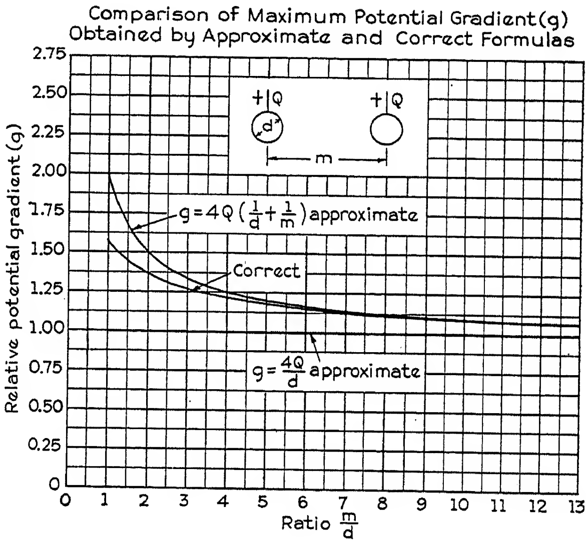

FIG. 1

Good agreement was found between the test results given in the text by F. W. Peek, Jr., "Dielectric Phenomena in High Voltage Engineering," Figs. 57 and 59, for the visual corona for two conductors at the same potential and for a triangular arrangement of conductors. Equation (8c) for the disruptive critical voltage was used, modified for single-phase and multiplied by the factor $\left(1 + \dfrac{0.301}{\sqrt{r}}\right)$ in order to determine the visual corona voltage.

W. W. Lewis: The General Electric Company has been studying the use of multiple conductors from time to time for a great many years. When the economical side is worked out, multiple conductors do not look so attractive. This is mainly because the large weight of conductors in this scheme requires heavy towers and short spans. In order to obtain continuously the theoretical advantages in the way of decreased reactance, increased capacitance and increased corona starting voltage, it is desirable to maintain as closely as possible the spacing between conductors in the same phase and in the different phases. This requires special insulating arrangements and supporting and separating yokes, which increase the expense of the installation.

About two years ago, we made a study of a transmission system, intended to deliver 500,000 kilowatts over a distance of 300 miles. Split conductors, 345-kv. and 230-kv. lines were considered and compared with conventional lines of the same voltage. The power limit of each arrangement was on the basis of steady state stability.

With the split conductor, 345-kv. line, two conductors were used per phase, each 1,192,500 cir. mils A.C.S.R. Various spacings between conductors and between phases and various lengths of span were considered and the most economical arrangement adopted. This line would carry 500,000 kw. The investment, including line, step-up and step-down transformers, synchronous condensers, switchgear and substations, was $48.40 per kw. and the yearly charge $7.90 per kw. Two conventional lines (one conductor per phase) for 345 kv., would carry 660,000 kw. and entail an investment of $49 per kw. and a yearly charge of $7.60 per kw.

Two split conductor 230-kv. lines would carry 470,000 kw. at an investment cost of $63.30 per kw. and a yearly charge of $10.70 per kw. Four conventional 230-kv. lines would carry 600,000 kw. at an investment cost of $64.20 per kw. and a yearly charge of $10.50 per kw.

From these figures it appears that at 345 kv., two conventional lines have about the same cost per kw. and same yearly charge as one split conductor line. The two conventional circuits have a greater total capacity and have the obvious advantage that the total flow of power is not dependent upon one tower line. At 230 kv., the cost per kw., and the yearly charge on two split conductor lines and four conventional lines is about a toss-up. However, the capacity of the two split conductor lines is somewhat less than required in this particular case; while the four conventional lines have a considerable margin in capacity.

Since the split conductor line is untried in practise and has certain constructional and operating disadvantages with no advantage in cost, it seems desirable to adhere to the conventional construction for the present.

W. S. Moody: The possibilities of using higher voltages than 220,000 are very welcome to all who have contributed to the gradual and steady rise in transmission voltages from 10,000 to 220,000 volts. For some 20 years this increase, as we all know, was roughly but quite regularly some 10,000 volts per year. Since this progress stopped some ten years ago, transformer engineers have been "chafing at the bit" like race horses before the barrier. The only comfort was in the fact that this restraint was the result of there being no more worlds to conquer, at present, rather than any inability to keep up the progress on the part of the engineers.

Higher voltages than 220,000 are *commercially* practical only when very large power is to be transmitted a long distance before distribution of any of this block of power begins. Such opportunities will doubtless come along soon after commercial conditions are again normal notwithstanding constant improvement in steam plants. Advantage should be taken of present conditions therefore to discuss and solve the problems involved so as to have them ready before the first opportunity to use them arrives.

H. B. Dwight: The question whether double conductors may sometimes be advantageous for transmission lines is most naturally investigated by comparing designs which use single- and double-conductor construction. Unless some very cogent reason is encountered, all other features of the alternative designs should be alike—they should have the same voltage, the same resistance per phase, that is, approximately the same conductor weight, etc. The influence of extraneous features should be avoided. The comparisons in our paper, which does not deal with record-breaking voltages, are on the above basis and it seems easy and natural to make them so.

With the comparisons so direct, the following conclusions for the range considered seem to stand forth:

a. There is approximately 20 per cent advantage in kw. rating in favor of double-conductor construction, when reactance determines the allowable load.

b. This advantage may or may not be neutralized by extra cost due to special mechanical features, depending particularly on whether provision is to be made for a large ice load.

c. Very little advantage as regards corona is obtained from double-conductor construction, if the disruptive critical voltage e_0 is used as the criterion.

If now hollow conductors are used, part of the advantage as regards reactance is taken away, and probably all of the advantage as regards corona, at present voltages. None of the papers at this meeting seems to have discussed definitely the question as to how thin and of how large diameter a hollow conductor may be, for successful use. Without an exact answer to this question, it is very difficult to make comparisons at extreme, record-breaking voltages.

However, it seems that a transmission line using single, hollow conductors could be built for ratings much beyond those now used, and in cases where this is so, comparisons of double-conductor lines should be made on the basis used in our paper, of equal voltage and conductor weight per phase. Comparisons become much easier when it is admitted for a certain case that a single-conductor line can be built to duplicate a proposed double-conductor line exactly except for the number of ohms of reactance.

The reactance seems to be the most important feature at present and it may, in favorable cases, make double-conductor construction advisable.

Solution of Circuits Subjected to Traveling Waves

BY HAROLD L. RORDEN*

Associate, A.I.E.E.

Synopsis.—The major purpose of this paper is to illustrate how accurately the physical assumptions made in applying conventional traveling wave theory are substantiated by test results.

Analytical solutions for traveling waves at a transition point, as given in "Traveling Waves on Transmission Systems," A.I.E.E. TRANS., *1931 (L. V. Bewley) are used in this analysis. These derivations involve the solution of circuits that may be used to represent equivalent terminal conditions of transmission systems, when simplifying assumptions are made, and when the free traveling wave can be expressed in exponential form. Discussion of types and specifications of such waves, attenuation and distortion, and the reflection and refraction operators, is limited to that essential for an understanding of the procedure. A method of calculating successive reflections on a short line, and a step-by-step solution involving a variable parameter at a transition point are included.*

* * * * *

GENERAL

THE conventional theory of traveling waves is based on a single-conductor circuit and supposes the transmission line to be ideal. That is, the line may be represented as a pure surge impedance $\sqrt{L/C}$, where L and C are to be determined on the assumption that ground is a true zero potential surface. The losses of the line are negligible for waves below the critical corona point. Coupling with adjacent conductors is ignored,

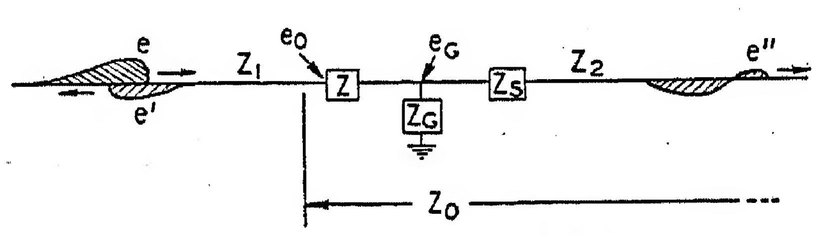

FIG. 1—EQUIVALENT CIRCUIT OF GENERAL NETWORK

and the waves are calculated by the simple single-conductor theory, *i. e.*, the associated electrostatic and electromagnetic fields exist between the conductor and its image an equal distance below the ground surface.

Comparative tests conducted to determine the relative error involved in the preceding assumptions illustrate several conclusive points:

1. Traveling waves on single-conductor circuits, if below the corona limit, can be calculated with excellent engineering accuracy when the parameters of the transition point are known.
2. Successive reflections can be calculated accurately by means of superposition, and the calculation facilitated by lattices.
3. Step-by-step calculations are applicable to traveling wave problems involving transition points with variable parameters.

The effect of traveling waves on terminal apparatus is usually most easily solved by means of Heaviside's operational calculus. Solutions have been given for various terminal apparatus that may be encountered in service, by considering them to be simple lumped impedances. These are intended to approximate the general conditions of a transmission system, which may be reduced to the equivalent circuit of Fig. 1. In Fig. 1, Z_1 represents the surge impedance of the line on which the transient wave originates, Z, Z_s, and Z_g, are terminal impedances, and Z_2 the surge impedance of all outgoing lines in parallel.

With the cathode ray oscillograph valuable information has been obtained relative to the wave shape and other peculiarities of traveling waves. Many irregularities exist in these waves, but generally they may be reproduced to a fair enough degree of accuracy so that their effect may be determined both analytically and experimentally.

WAVE SHAPES

Transients of the order of lightning waves may be approximately represented by the difference of two

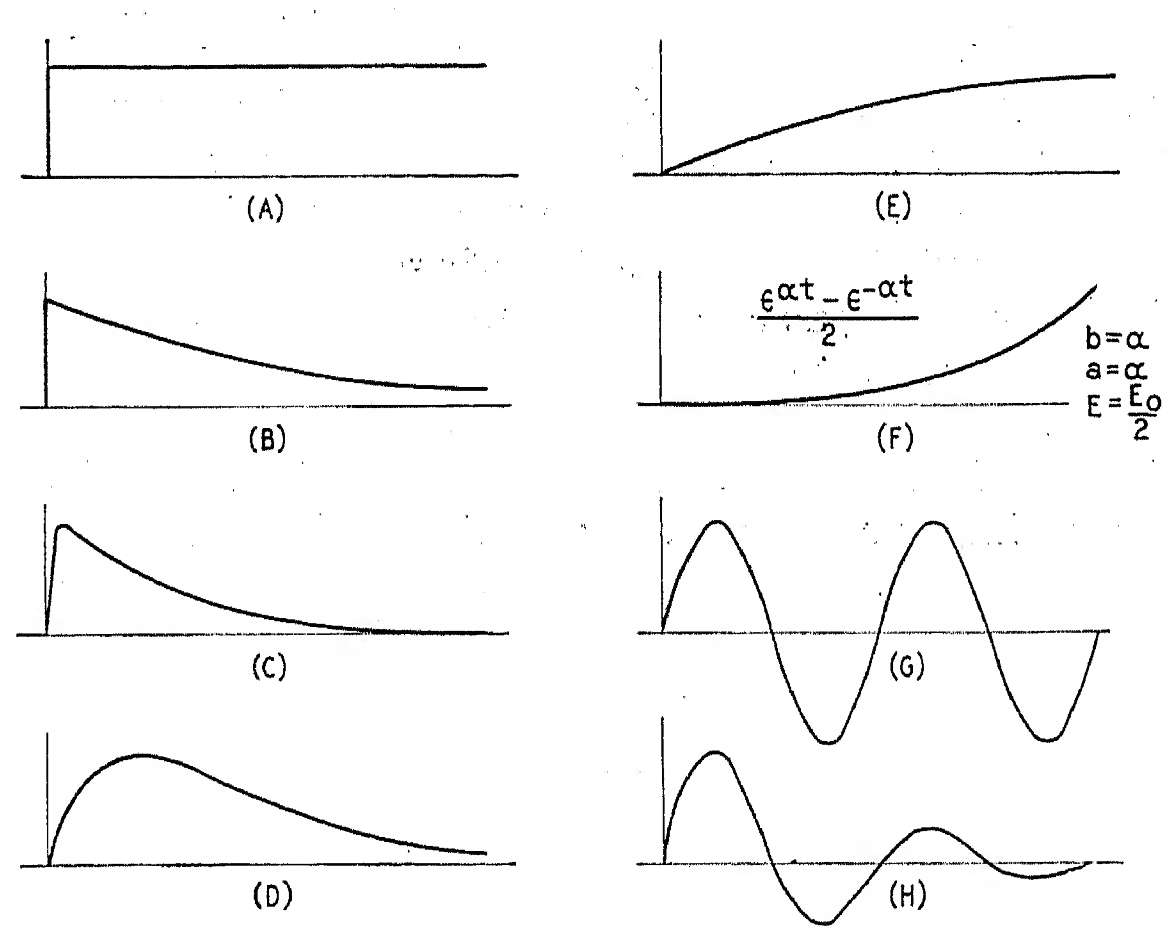

FIG. 2—WAVE SHAPES GIVEN BY $e = E\,(\epsilon^{-at} - \epsilon^{-bt})$

exponentials, merely by a proper adjustment of parameters. This is of considerable importance in effecting the general solution of traveling waves. Fig. 2 illustrates different types of waves that may be expressed as the difference of two exponential terms, of the form $e = E\,(\epsilon^{-at} - \epsilon^{-bt})$. The parameters a and b may be adjusted to give any of the wave shapes shown in Fig. 2. Probably the most common of these waves are of the

*Power Transformer Dept., General Electric Co., Pittsfield, Mass.

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form illustrated by C and D of Fig. 2, where both a and b are real and positive, and they are the simplest to duplicate in the laboratory.

A set of curves has been devised by Mr. T. Brownlee whereby a wave of the exponential form in which a and b are real and positive can be defined completely, knowing only the time (t_1) for the wave to reach its crest, and the total time (t_2) to fall to half its crest voltage. These curves were plotted empirically on the assumption that for a given ratio of t_2/t_1, there would be a fixed value of b/a. However, the truth of this relation may be derived rationally. By differentiating the equation for e, and equating to zero for the maximum, it may be shown that the time to reach the crest is

$$t_1 = \frac{\log (b/a)}{b - a} \tag{1}$$

Therefore,

$$at_1 = \frac{\log (b/a)}{\dfrac{b}{a} - 1} = B \tag{2}$$

The crest voltage E_1 is given by

$$E_1 = E\,(\epsilon^{-at_1} - \epsilon^{-bt_1}) \tag{3}$$

$$\frac{E_1}{2} = E\,(\epsilon^{-at_2} - \epsilon^{-bt_2}) \tag{4}$$

Substituting (2) in (3), there is

$$\frac{E_1}{E} = \epsilon^{-B} - \epsilon^{-(b/a)B} = R \tag{5}$$

The main function of the parameter b is to determine the front of the wave, and for all excepting the cases where t_2 approaches t_1, it is so large compared to a that $\epsilon^{-bt_2} \cong 0$.

Solving (4) for at_2 and combining with (1), there results

$$t_2/t_1 = \frac{\log \dfrac{2}{R}}{B} \tag{6}$$

from which it follows that t_2/t_1 is a function of (b/a), since both R and B have been shown to be functions of (b/a). Thus the three expressions at_1, E_1/E, and t_2/t_1 may be plotted against the common term b/a. Fig. 3 shows such a set of curves, from which a wave can be completely specified. As an example, let us suppose the record of a lightning wave shows it to reach its crest in 3 microseconds and to fall to half-crest value in 21 microseconds. Then $t_2/t_1 = 7$, and from Fig. 3, $b/a = 28.5$. For this value of b/a, $at_1 = 0.122$ and $E_1/E = 0.852$.

Solving,

$$a = 0.041,\ b = 1.15$$

The wave is then completely defined as

$$e = E\,(\epsilon^{-.041t} - \epsilon^{-1.15t})$$

where the crest voltage will reach 85.2 per cent of E, and t is measured in microseconds. The ratio E_1/E is particularly valuable in laboratory studies where capacitors are used as a lightning generator, charged to a voltage E. Otherwise it is necessary to determine the effect of waves on terminal impedances, as will be shown later. From the incident wave the resultant stresses on terminal apparatus may be completely determined, as will also be shown later.

The curves of t_2/t_1 in Fig. 3 are plotted on the assumption that $\epsilon^{-bt_2} = 0$. The error thus involved becomes appreciable when b/a is less than 2.5. By expressing equation (4) in the form

$$\frac{E_1}{2E} = \epsilon^{-B(t_2/t_1)} - \epsilon^{-B(b/a)(t_2/t_1)}$$

and plotting a family of curves for the lower values of t_2/t_1 together with $R/2$ from equation (5), it may be shown rationally that t_2/t_1 remains a function of b/a throughout its lower range. To extend the curves of Fig. 3 to the lower limit where $b = a$ involves a trial

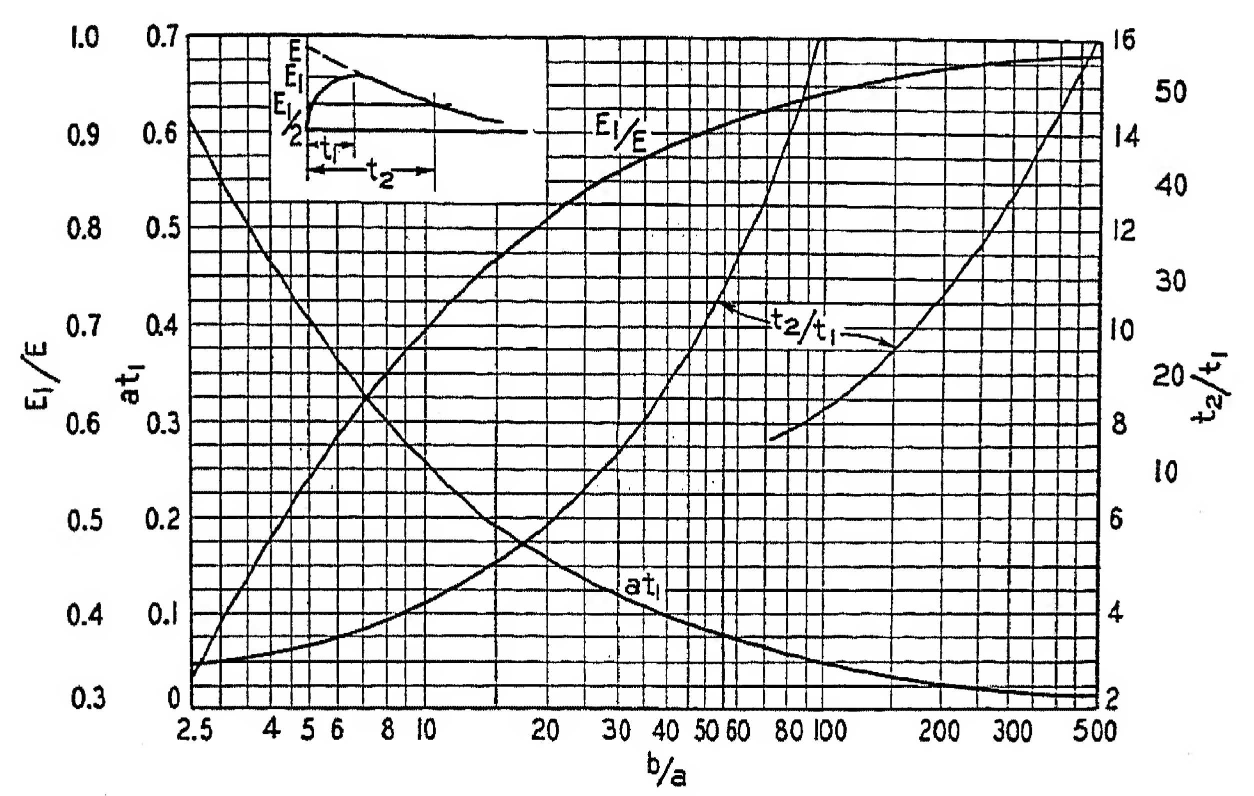

FIG. 3—GRAPHICAL DETERMINATION OF WAVES EXPRESSED BY $e = E\,(\epsilon^{-at} - \epsilon^{-bt})$

and error method, and does not extend the lower limit of t_2/t_1 appreciably.

GENERAL SOLUTION

By considering terminal impedances to consist of lumped constants, the determination of voltages and waves resulting from an incoming free traveling wave can be effected in terms of the reflection and refraction operators. With the symbols as defined in Fig. 1, these operators are

$$\left(\frac{Z_o - Z_1}{Z_o + Z_1}\right) = \text{reflection operator.}$$

$$\left(\frac{Z_G}{Z_G + Z_S + Z_2}\right)\left(\frac{2Z_2}{Z_o + Z_1}\right) = \text{refraction operator.}$$

All impedances are expressed in operational form, the subscript (p) being dropped for simplicity, and they derive their respective voltages when applied to the incident wave. The total voltage e_o at the transition point (Fig. 1) is the sum of the incident and the reflected waves ($e_o = e + e'$) for which the operator is $2Z_o/(Z_o + Z_1)$.

TABLE I—CIRCUITS OF FIG. 4

$e_o = e + e'$

	e'	α	β	A
A.......	e			
B.......	$\frac{R-Z}{R+Z}\,e$			
C........	$-e$			
D.......	I, III	$\frac{Z}{L}$	$\frac{Z}{L}$	1
E.......	I, III	$\frac{1}{CZ}$	$\frac{1}{CZ}$	-1
F........	I, III	$\frac{ZR}{L(R-Z)}$	$\frac{ZR}{L(R+Z)}$	β/α
G........	I, III	$\frac{R-Z}{ZRC}$	$\frac{R+Z}{ZRC}$	-1
H.......	II, IV	$-\frac{1}{2CZ}$	$\frac{1}{2CZ}$	-1
I........	II, IV	$\frac{Z-R}{2RCZ}$	$\frac{Z+R}{2RCZ}$	-1
J........	$e - Zi_0$			
K.......	I, III	$\frac{1}{C(Z-R)}$	$\frac{1}{C(Z+R)}$	$-\beta/\alpha$
L........	I, III	$\frac{Z-R}{L}$	$\frac{Z+R}{L}$	1
M.......	II, IV	$\frac{-Z}{2L}$	$\frac{Z}{2L}$	1
N.......	II, IV	$\frac{R-Z}{2L}$	$\frac{R+Z}{2L}$	1
O..........	Same as N before gap sparkover Same as C after gap sparkover			

By the refracted wave is meant the voltage, as a function of time, that passes onto the line Z_2, and is equal to e''.

All of the circuits in Figs. 4, 5 and 6 may be solved by one of two general equations,[1] the one to apply depending on whether or not the impedances are such as to make oscillations possible. For an incident wave of the form $e = E\epsilon^{-at}$ these expressions are

$$AE\left[\frac{a+\alpha}{a-\beta}\,\epsilon^{-at} - \frac{\alpha+\beta}{a-\beta}\,\epsilon^{-\beta t}\right] \text{ for the non-oscillatory circuits} \quad \text{(I)}$$

and

$$AE\left\{\frac{\omega_o^2 - 2a\alpha + a^2}{\omega_o^2 - 2a\beta + a^2}\,\epsilon^{-at}\right.$$

$$\left. + \frac{2(\alpha-\beta)\,\epsilon^{-\beta t}}{\omega(\omega_o^2 - 2a\beta + a^2)}\left[(\omega_o^2 - a\beta)\sin\omega t + a\omega\cos\omega t\right]\right\}$$

for the oscillatory circuits **(II)**

where $\omega_o^2 = 1/LC$ and $\omega^2 = \omega_o^2 - \beta^2$, for the oscillatory circuits. α, β, and the amplitude factor A, are functions of the impedances. In an earlier publication[2] it was shown that for an incident wave containing two

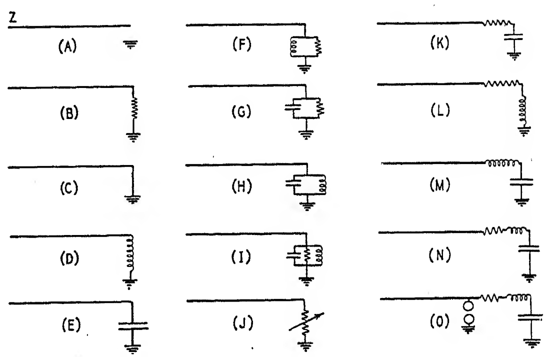

FIG. 4—TERMINAL IMPEDANCES

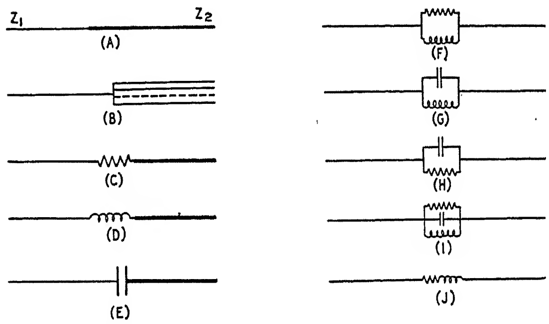

FIG. 5—JUNCTIONS BETWEEN CIRCUITS

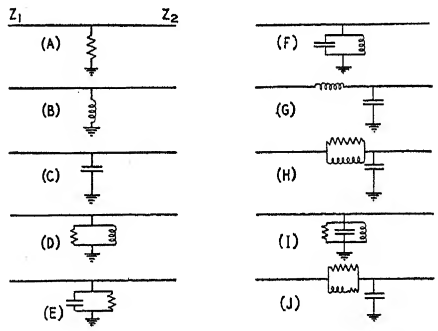

FIG. 6—JUNCTIONS BETWEEN CIRCUITS

or more exponential terms, the above expressions could readily be expanded to correspond. Thus for an incident wave of the form $e = E(\epsilon^{-at} - \epsilon^{-bt})$ the expressions (I) and (II) become

$$AE\left[\frac{a+\alpha}{a-\beta}\,\epsilon^{-at} - \frac{b+\alpha}{b-\beta}\,\epsilon^{-bt} + \frac{(\alpha+\beta)(a-b)}{(a-\beta)(b-\beta)}\,\epsilon^{-\beta t}\right] \quad \text{(III)}$$

1. For references see Bibliography.

and

$$AE \left\{ \frac{\omega_o^2 - 2a\alpha + a^2}{\omega_o^2 - 2a\beta + a^2}\, \epsilon^{-at} - \frac{\omega_o^2 - 2b\alpha + b^2}{\omega_o^2 - 2b\beta + b^2}\, \epsilon^{-bt} \right.$$

$$+ \frac{2(\alpha - \beta)\, \epsilon^{-\beta t}}{\omega(\omega_o^2 - 2a\beta + a^2)} [(\omega_o^2 - a\beta) \sin \omega t + a\omega \cos \omega t]$$

$$\left. - \frac{2(\alpha - \beta)\, \epsilon^{-\beta t}}{\omega(\omega_o^2 - 2b\beta + b^2)} [(\omega_o^2 - b\beta) \sin \omega t + b\omega \cos \omega t] \right\} \quad \text{(IV)}$$

The values of ω_o and ω are as defined for (**I**) and (**II**). In (**II**) and (**IV**) where the combination of L and C is such that the circuit is more than critically damped, $\omega = j\sqrt{\beta^2 - \omega_o^2}$ and the resultant expression reduces by the familiar substitution of hyperbolic functions to a real term.

TABLE II—CIRCUITS OF FIG. 5

$e_o = e + e' \qquad e'' = \frac{Z_2}{Z_1}(e - e')$

	e'	α	β	A
A..	$\frac{Z_2 - Z_1}{Z_2 + Z_1}\, e$			
B..	$\frac{Z_t - Z_1}{Z_t + Z_1}\, e$	Z_t = Surge impedance of all outgoing lines in parallel		
C..	$\frac{Z_2 - Z_1 + R}{Z_2 + Z_1 + R}\, e$			
D..	I, III	$\frac{Z_1 - Z_2}{L}$	$\frac{Z_1 + Z_2}{L}$	1
E..	I, III	$\frac{1}{C(Z_1 - Z_2)}$	$\frac{1}{C(Z_1 + Z_2)}$	β/α
F..	I, III	$\frac{R(Z_1 - Z_2)}{L(R - Z_1 + Z_2)}$	$\frac{R(Z_1 + Z_2)}{L(R + Z_1 + Z_2)}$	$\frac{R - Z_1 + Z_2}{R + Z_1 + Z_2}$
G..	II, IV	$\frac{1}{2C(Z_2 - Z_1)}$	$\frac{1}{2C(Z_2 + Z_1)}$	β/α
H..	I, III	$\frac{R - Z_1 + Z_2}{RC(Z_1 - Z_2)}$	$\frac{R + Z_1 + Z_2}{RC(Z_1 + Z_2)}$	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$
I..	II, IV	$\frac{Z_2 - Z_1 + R}{2RC(Z_2 - Z_1)}$	$\frac{Z_2 + Z_1 + R}{2RC(Z_2 + Z_1)}$	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$
J.	I, III	$\frac{Z_1 - Z_2 + R}{L}$	$\frac{Z_1 + Z_2 + R}{L}$	1

Most of the circuits shown in Figs. 4, 5, and 6 have been calculated in this analysis. In the circuits of Fig. 4 where Z_2 becomes infinite, there is no transmitted wave, and since the relation $e_o = e + e'$ is general, the complete solution is obtained from the incident wave by calculating for the reflected wave only. In Fig. 5, Z_G becomes infinite and the refraction operator is given by

TABLE III—CIRCUITS OF FIG. 6

$e_o = e'' = e + e'$ (Exceptions, G, H, J)

	e'	α	β	A
A..	$\frac{Z_2R - Z_1R - Z_1Z_2}{Z_2R + Z_1R + Z_1Z_2}\, e$			
B.......	I, III	$\frac{Z_1Z_2}{L(Z_2 - Z_1)}$	$\frac{Z_1Z_2}{L(Z_2 + Z_1)}$	β/α
C.......	I, III	$\frac{Z_2 - Z_1}{Z_1Z_2C}$	$\frac{Z_2 + Z_1}{Z_1Z_2C}$	−1
D.......	I, III	$\frac{RZ_1Z_2}{L(RZ_2 - RZ_1 - Z_1Z_2)}$	$\frac{RZ_1Z_2}{L(RZ_2 + RZ_1 + Z_1Z_2)}$	β/α
E.......	I, III	$\frac{RZ_2 - RZ_1 - Z_1Z_2}{Z_1Z_2RC}$	$\frac{RZ_2 + RZ_1 + Z_1Z_2}{Z_1Z_2RC}$	−1
F.......	II, IV	$\frac{Z_1 - Z_2}{2Z_1Z_2C}$	$\frac{Z_1 + Z_2}{2Z_1Z_2C}$	−1
I........	II, IV	$\frac{Z_1R - Z_2R + Z_1Z_2}{2Z_1Z_2RC}$	$\frac{Z_1R + Z_2R + Z_1Z_2}{2Z_1Z_2RC}$	−1
G H....... J	II, IV	See Bibliography, Reference 2		

$2Z_2/(Z_o + Z_1)$. The refracted wave then is given by $e'' = (Z_2/Z_1)(e - e')$. Thus e_o, e', and e'', may all be obtained directly from e by obtaining the solution for e' only. In Fig. 6 for all circuits excepting (G), (H), and (J), Z and Z_s become zero, $e'' = e_o = e + e'$, and the complete solution is again obtained by solving for e'. For the circuits of Fig. 6 (G), (H), and (J), the complete solution becomes somewhat more lengthy.

Since, as has been shown, the complete solutions may be obtained by solving for the reflected wave only, it is necessary to apply the parameters to the general equation only for this one solution. α, β, and A for the circuits of Figs. 4, 5, and 6 are given in Tables I, II, and III, by means of which the circuits may be completely solved for any incident wave expressed in the exponential form.

Experimental Line—Surge Impedance

For test purposes, a transmission line about two miles long (11½ microseconds) was used, this line consisting of two parallel conductors spaced eight feet apart and forming a loop so that both ends terminated at the laboratory where both the lightning generator and the oscillograph were located. For the circuits of Fig. 5 both lines were at first used, with the result that waves were badly distorted due to mutual effects between the two lines. Since it is not the purpose here to investigate the effects of mutually coupled circuits, the outgoing line from the terminal impedance was replaced by a resistance equivalent to the line surge impedance. That this is an exact substitution may be shown mathematically, and is verified by the results obtained.

In Fig. 7A the oscillograms e_1 and e are the waves recorded by the cathode ray oscillograph at both ends of the line, when a resistance $R = Z$ was connected to ground as is illustrated. The magnitude of the line surge impedance Z may be determined in several ways. Since inserting a resistance $R = Z$ at the far end of the line makes it equivalent to approximating a line of infinite length, R may be adjusted until there are no reflections, as observed with the oscillograph. A more accurate method is to send a wave of fast front on the line, regardless of the impedance at the far end, and obtain a record of the wave at the impulse generator. Then the surge impedance may be very accurately calculated from the equation representing the wave. As an illustration let us observe the wave in Fig. 7A. In the sketch directly below the oscillogram the solid line e_1 is a reproduction of the wave at the generator. It is readily seen that for points well down on the wave tail, the equation is given very exactly by $e = E\,\epsilon^{-at}$, where $a = 1/ZC$, since the circuit is that of a capacitance discharging into an equivalent resistance. Thus by arbitrarily choosing two points t_1 and t_2 on the tail, the surge impedance may be obtained from the relation

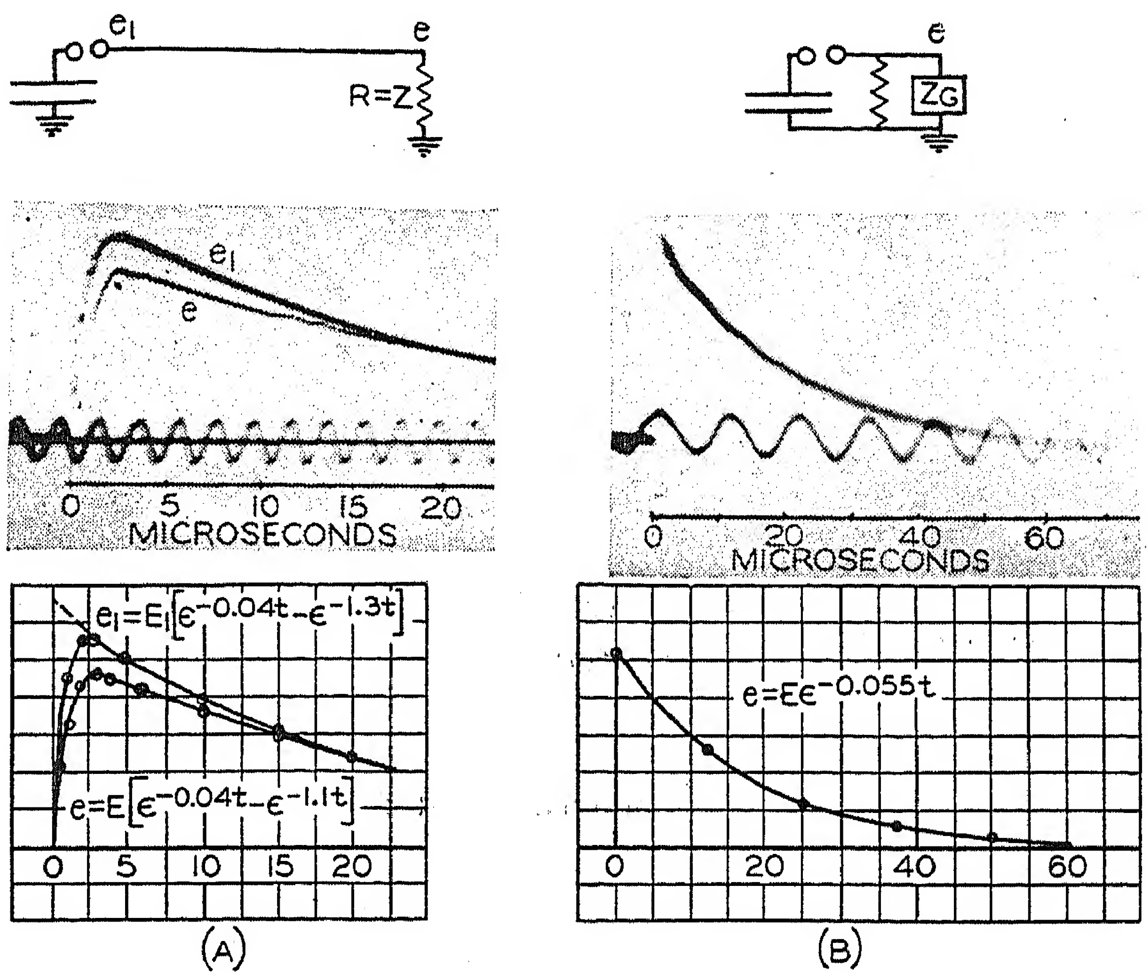

FIG. 7—INCIDENT WAVES

$$Z = \frac{t_2 - t_1}{C \log \dfrac{e_1}{e_2}}$$

where C is the capacitance of the impulse generator. The surge impedance of the line used in these tests is 510 ohms, obtained by this method. From this result E may be found by substituting $a = 1/ZC$ in the single exponential form for the incident wave, or it may be found directly from the curves of Fig. 3 by scaling the time to crest and time to half voltage. The value of E thus obtained is indicated by the broken line of Fig. 7A, and the complete equation of the wave, as obtained by the curves of Fig. 3 plot the points in the circles.

Where an oscillograph is not available, the surge impedance of a line may be approximated experimentally, but the method is not as accurate as the preceding one. This is done by sending a wave on the line from an impulse generator, measuring the voltage, and then substituting a resistance for the line, adjusting it until the measured voltage is the same as that obtained on the line. If the average height h of the line is known, and also the radius r of the conductor, the surge impedance may be found from the relation $Z = 60 \log (2h/r)$. Values thus obtained are found to check very well with those determined experimentally.

INCIDENT WAVE

The complete equation of a wave may be obtained readily by the method of superposition as well as by the curves of Fig. 3. Consider the complete wave to be made up of its component parts ϵ^{-at} and $-\ \epsilon^{-bt}$, plotting approximately into the wave tail and wave front respectively. Then a may be determined from the first term by the method described above, by arbitrarily choosing two points on the tail of the wave, and b may be found by treating the second term similarly.

The difference between the two waves of Fig. 7A is a measure of the voltage losses sustained by the wave in traveling about two miles. The lightning generator was charged to about 75 kilovolts. It will be observed that very little change has resulted in the time to reach crest voltage, but that the amplitude has dropped 17 per cent, and that the wave tail has lengthened. This phenomenon is not unusual, but it serves to illustrate what differences might be encountered if these losses were not considered in the mathematical analysis.

In this investigation the incident wave is that obtained directly at the terminal impedance when that impedance consists only of a resistance $R = Z$ to ground. But all oscillograms were not taken at the same time, and therefore the incident wave, although sent from the same generator and maintaining as nearly as possible the same voltage, has some slight differences. This, also, is not unusual.

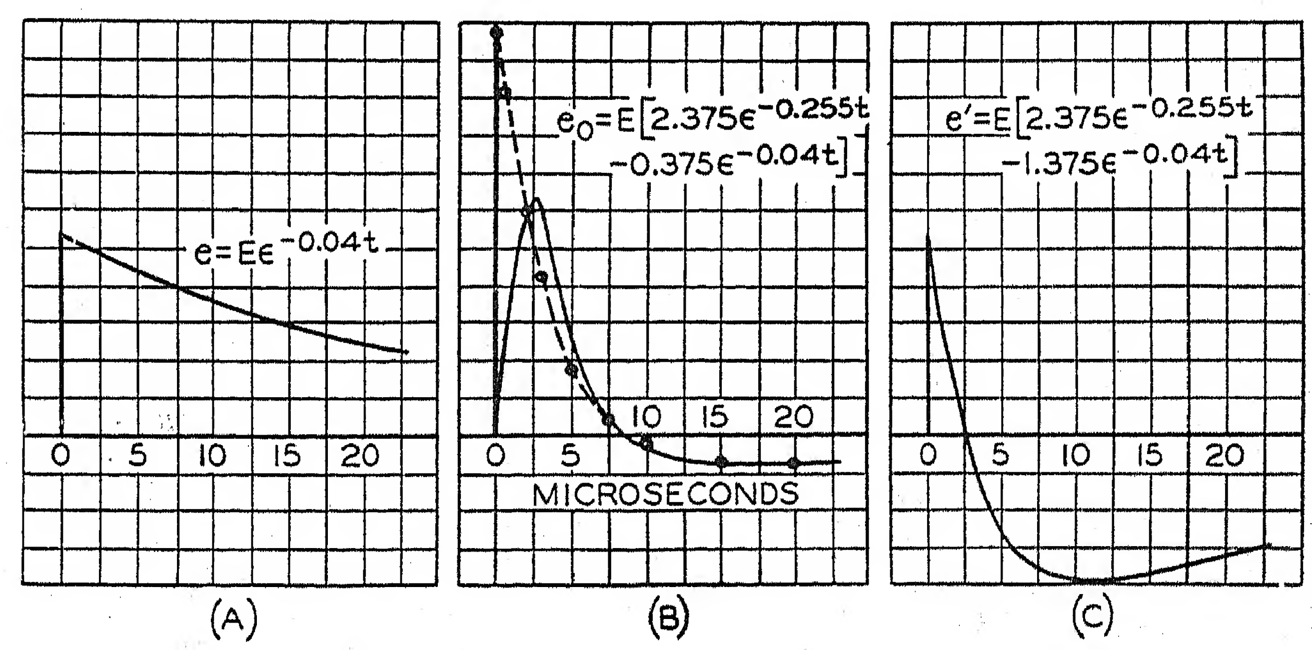

FIG. 8—ERROR DUE TO NEGLECTING WAVE FRONT

(See Fig. 10A)

EXPERIMENTAL CORRELATION

How precisely the mathematical solution of circuits involving necessary simplifying assumptions may agree

with oscillograms is demonstrated in Figs. 9 to 17 inclusive. The labor of calculating the waves obtained at the terminal impedances would become greatly simplified if the incident wave could be considered of the single exponential form, *i. e.*, if the wave front were neglected. This is obvious from an inspection of the expressions (**I**), (**II**), (**III**), (**IV**). The labor is particularly tedious for the oscillatory cases. Whether or not the front can be neglected depends on the relative time constants of the circuit. In Fig. 8 is shown a case in which neglect of the front leads to serious error. Fig. 8A is the assumed form of incident wave, neglecting the front, where the actual wave is that of Fig. 7A. The solid line of Fig. 8B is a plot of the oscillogram of Fig. 10A and is the voltage wave e_o obtained at the terminal impedance when it consisted of an inductance of 0.002

to the oscillographic deflection of the incident wave, rather than being given in kilovolts. For Figs. 9 to 13, the waves were sent over the line previously discussed, with an initial potential of about 75 kilovolts and the wave form e of Fig. 7A is taken as the incident free traveling wave. For Fig. 14 the waves were applied directly from the impulse generator, and the voltage e_o is of the form given in Fig. 7B. The waves of Fig. 14 have been published,[2] but since they serve as illustrations of several of the circuits of Fig. 6 they are repeated here. They serve the further purpose of illustrating the correlation that may be obtained when neglecting the front, with a voltage wave approximating as nearly as possible the single exponential form.

Points of interest of Figs. 9 to 14 will be discussed in order. For all the oscillograms and calculations the

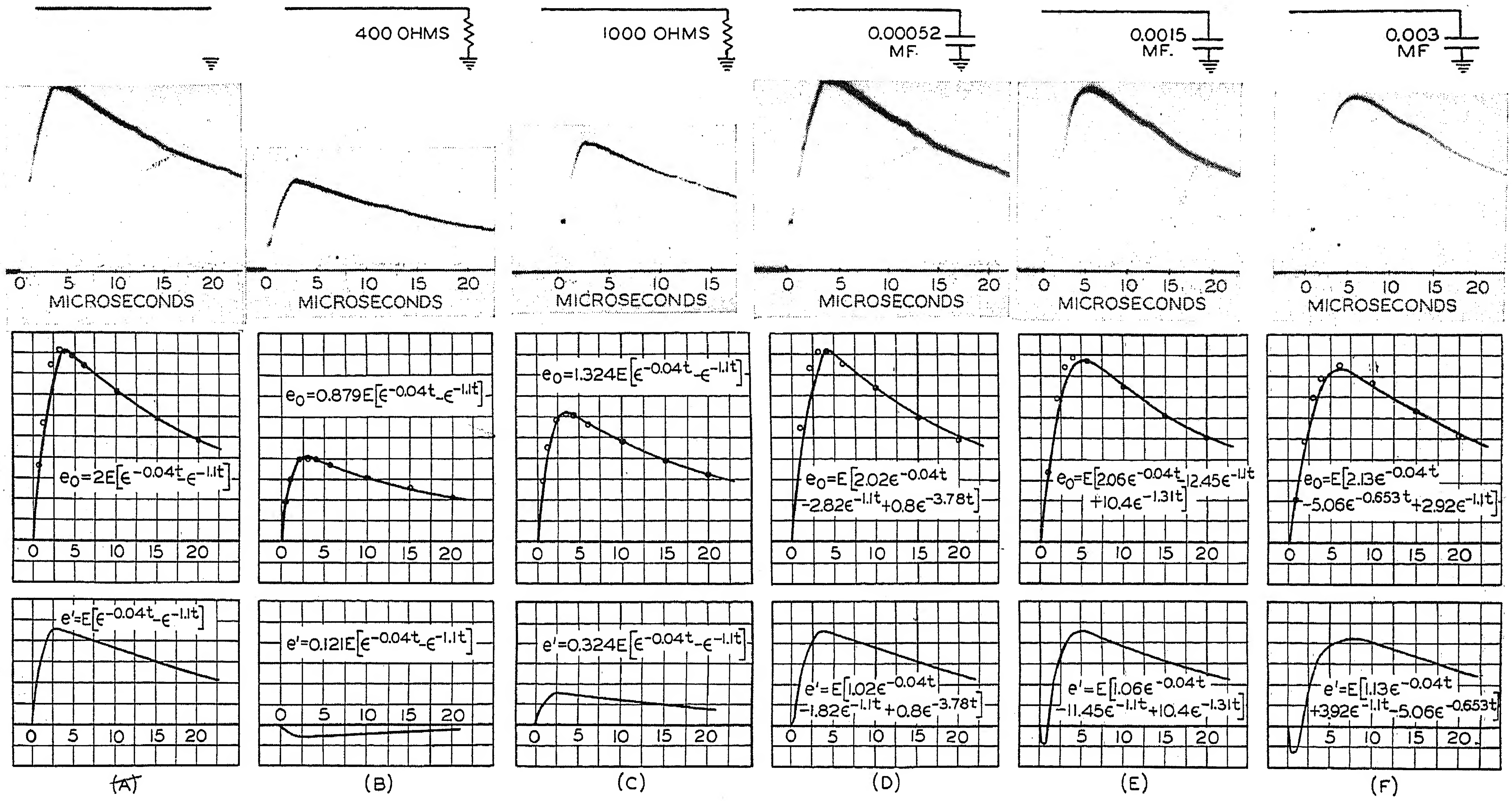

FIG. 9—CIRCUITS OF FIG. 4

$e = E\,(\epsilon^{-0.04t} = \epsilon^{-1.1t}) \qquad e_0 = e + e'$

henrys. The broken line joining the circles of Fig. 8B represents the calculated voltage, for which the actual voltage crest is in error 57 per cent. Fig. 8C is the reflected wave e', calculated only, such that $e' = e_o - e$.

In these tests no extraneous constants were in the circuit to produce a slow front wave, *i. e.*, the front deviated from reaching crest in zero time due only to physical limitations of the circuit. It would seem that in a study of traveling waves generally the front can not be neglected without involving a high degree of error.

The Figs. 9 to 14 inclusive illustrate the experimental and mathematical solutions of most of the circuits of Figs. 4, 5, and 6, for the specific impedances as given. The voltage scale of the plotted waves is comparable

constants used in the impedances were as listed herewith, unless otherwise noted in the figures:

$$R = 1{,}000 \text{ ohms}$$
$$L = 0.002 \text{ henrys}$$
$$C = 0.0015 \times 10^{-6} \text{ farads}$$

For each case the circuit is shown at the top of the column, then the oscillogram of e_o, followed by the replot of the oscillogram and the calculated points (circles). The reflected wave e' is plotted from calculated values only, since the oscillographic record of this could only be obtained in these tests as the difference between e_o and e. However, they deviate from this only by the difference found in e_o, since the calculated incident wave checks the oscillogram very precisely. Next in the

column, where they exist, are given the oscillogram and calculations for the refracted wave e'', excepting where $e'' = e_o$.

FIG. 9—CIRCUITS OF FIG. 4

A. By superposition $e_o = 2e$ where the line is open at the far end, and the reflected wave $e' = e$, the incident wave.

B and C. With a resistance R less than Z the reflected wave is negative, and with R greater than Z the reflection is positive. These two waves were taken primarily to check the magnitude of the surge impedance obtained by the method previously described. From these two oscillograms, and the tentative value of 510 ohms for Z, the incident wave was calculated. Then with $R = 510$ ohms the wave e of Fig. 7A was obtained and found to check the calculation precisely.

D, E and F. These cases illustrate the effect of capacitances of different magnitudes on e_o and e'. The capacitance of 0.00052 μf. differs but little from an open circuit. As the capacitance is increased the negative loop of the reflected wave becomes larger, illustrating the well known principle that a capacitance acts as a short circuit for the first instant of time, and finally as an open circuit. The case of Fig. 4C is not illustrated since obviously $e_o = o$ and $e' = -e$. This represents an infinite capacitance, and the entire reflected wave is negative.

FIG. 10—CIRCUITS OF FIG. 4

A. This is a more exact solution of the circuit of Fig. 4D than that given in Fig. 8. By more exactly approximating the incident wave the mathematical and experimental results can be made to check to a surprising degree of accuracy. On this wave the only discrepancy to be found is a slight variation in the front. e' is the inverse of the form of Fig. 9F, showing the inductance to act as an open circuit for the initial, and a closed circuit for the final period of time.

B and C. These figures illustrate the effect of an inductance and capacitance respectively shunted by a resistance. e' illustrates the two opposite forms of reflected waves again.

D. This figure is the case of a combination impedance the equation for which comes under the oscillatory form, but is more than critically damped. e' illustrates the predominating effect of the capacitance in the initial stage and the inductance in the final stage.

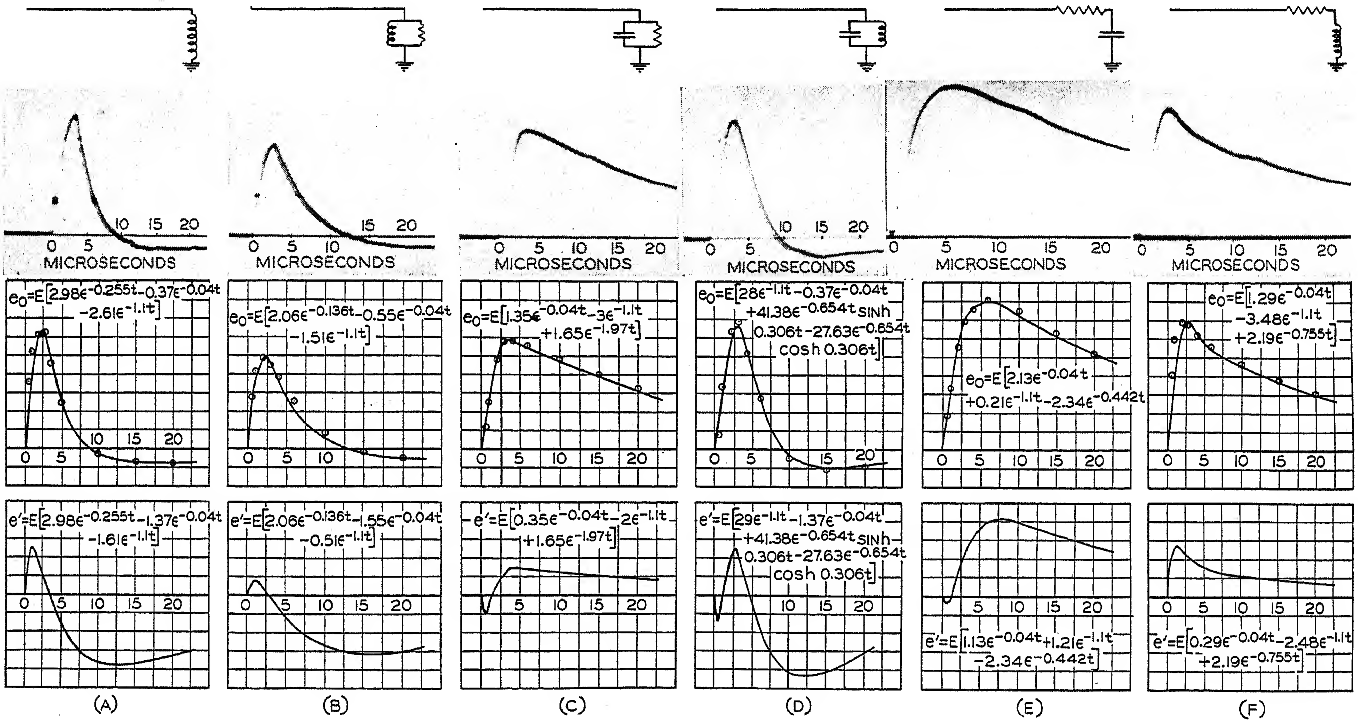

FIG. 10—CIRCUITS OF FIG. 4

$e = E(\epsilon^{-0.04t} - \epsilon^{-1.1t})$ $e_0 = e + e'$

E and F—are illustrations of Fig. 4K and L respectively.

FIG. 11—CIRCUITS OF FIGS. 4 AND 4O

This figure probably represents the most practical application to circuits found in service. It represents a terminal impedance consisting of a transformer at the end of a line and protected by a current limiting reactor. The series resistance illustrated is inherent in the reactor. The voltage scale of the oscillograms is reduced by a ratio of 1.73 to those of the preceding figures in order to obtain the complete record on the film. The sketches are plotted to the same voltage scale as the oscillograms for comparative purposes, being therefore reduced by the above ratio from the incident wave of Fig. 7A.

A. Illustrates the full wave impressed on the impedance. e_o is obtained as before, for the oscillatory case, and rises to 204 per cent of the crest value of the incident wave. The reflected wave has a crest of 124 per cent of the incident wave. The voltage across the capacitance e_c rises to 130 per cent of the incident wave and is not directly obtainable by the general equations, excepting that it may be considered a special case of e'' of Fig. 6J. And if this is done the simplification be-

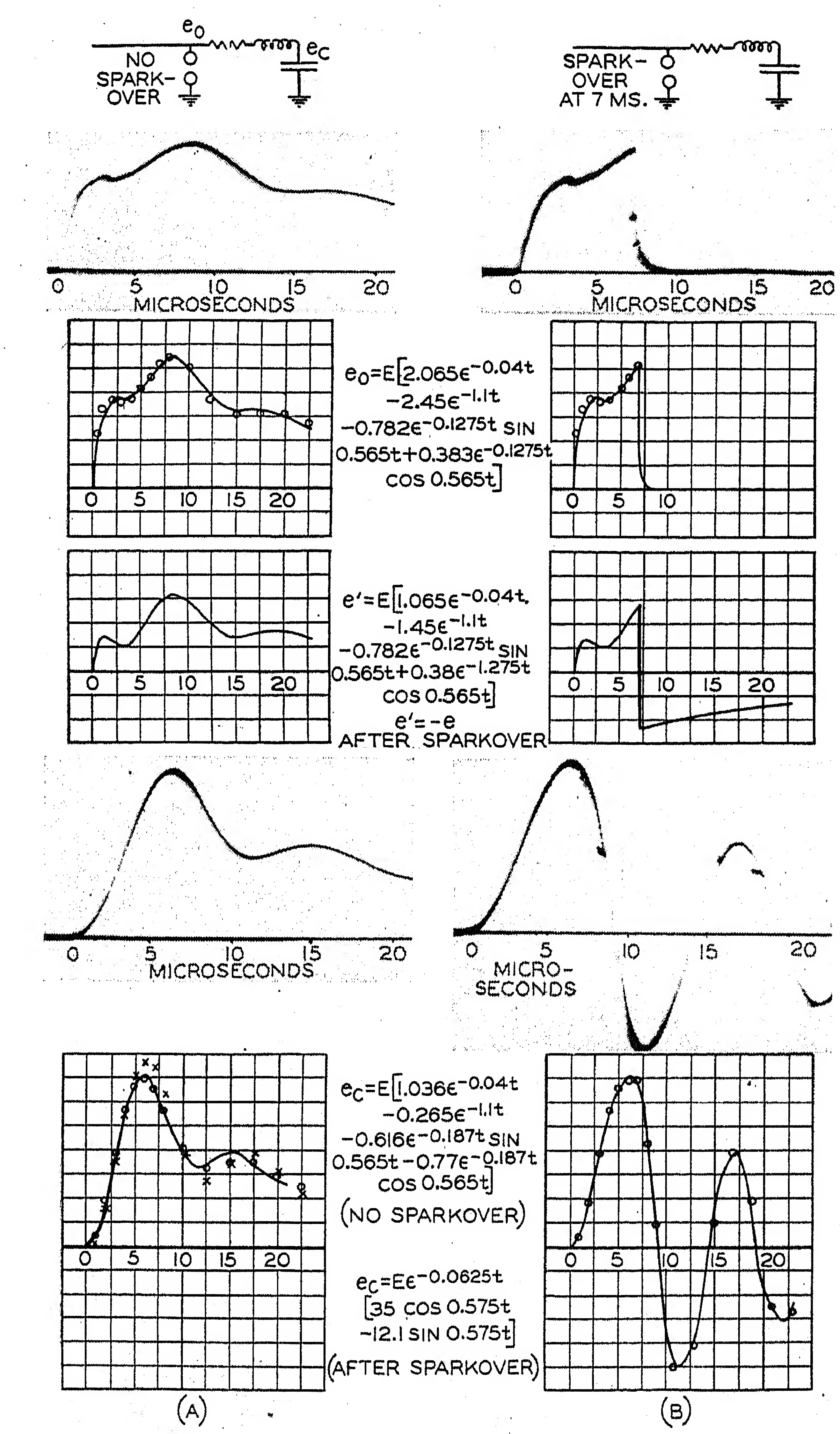

Fig. 11—Circuit of Fig. 4(O)

$e = E(\epsilon^{-0.04t} - \epsilon^{-1.1t}) \quad e_0 = e + e'$

comes so involved that it is more practical to derive the equation from the impedance operators. The calculated values of e_c become noticeably in error after about 12 microseconds, while the difference in e_o is not appreciable. In the test circuit, the series resistance was not inserted as is illustrated in the sketch, and e' and e_o were calculated as though the terminal impedance consisted of L and C only. The results of this calculation are the circles on the sketch of e_o, and the solid line of e'. The crosses on the sketch of e_c illustrate that voltage calculated by the same method. It is evident that the inherent resistance of the impedance combination has little effect on the wave on the line side, but can not be neglected for the voltage transmitted to the capacitance. By considering the reactor to have an inherent resistance of 250 ohms the circles on e_c are obtained, which seems near enough for practical purposes. The value of 250 ohms was obtained by considering the damping factor of the circuit to consist of the line surge impedance plus such a series resistance as to approximate the actual decrement of the damped oscillatory component of e_c.

Probably such a close approximation is not justified, since use of the voltage wave obtained by taking the series resistance equal to zero would be an approximation tending toward greater safety in protecting for such a voltage. And the magnitude of such a series resistance undoubtedly is not constant for all waves and circuits, making an approximation doubtful without very careful study.

B. This represents the same circuit as in Fig. 11A, but with a sparkover near the crest of the voltage wave on the line side of the impedance. All voltages are precisely the same as in 11A before sparkover, but present a different problem after sparkover. The voltage e_c is of particular interest since to obtain a reasonable check with the oscillogram the decrement factor could not be compared with that of the preceding case. Here the entire damping factor is inherent in the circuit and its constants after sparkover, since the line surge impedance does not enter into the circuit. In order to plot the circles on the sketch of e_c, this resistance, as determined by the same method of superposition of the wave components was again found to be 250 ohms. The calculation for e_c after sparkover is probably most easily derived by direct methods rather than by resorting to Heaviside's calculus. The axis of oscillation of e_c shifts to the zero axis after sparkover.

Since this case is of a nature common to transmission systems, it is of interest to include its derivation. The reflected wave at the line side of the impedance is obtained from the general equations, or it may be derived easily by the operational calculus. The impedance operator for Fig. 4N is

$$Z_o = R + pL + \frac{1}{pC}$$

The reflection operator is

$$\frac{LCp^2 + (RC - ZC)\,p + 1}{LCp^2 + (RC + ZC)\,p + 1}$$

which, when operating upon the voltage of the incident wave, derives the reflected wave. It is expedient to consider the incident wave of the form $e = E\epsilon^{-at}$, thereby obtaining equation (II). As previously mentioned, the solution for any other incident wave then follows di-

rectly. Substituting α and β as in Table I to reduce the expression to a familiar form, there results

$$e' = \frac{p^2 + 2\alpha p + \omega_o^2}{p^2 + 2\beta p + \omega_o^2} E\epsilon^{-at}$$

Applying the shifting theorem, this becomes

$$e' = E\epsilon^{-at}\frac{p^2 + 2(\alpha - a)p + (a^2 - 2\alpha a + \omega_o^2)}{p^2 + 2(\beta - a)p + (a^2 - 2\beta a + \omega_o^2)} \quad (1)$$

the solution for which reduces to the expression (II).

The voltage e_c of Fig. 11A is derived similarly but for which the impedance operator is

$$\frac{Z_c}{Z_o} = \frac{1}{LCp^2 + RCp + 1}$$

e_c after sparkover, in Fig. 11B is given by

$$e_c = \frac{1}{C}\int i\, dt$$

The current is determined by solving the differential equation for a closed circuit consisting of a capacitance, an inductance and a resistance in series, given by

$$\frac{1}{C}\int i\, dt + L\frac{di}{dt} + Ri = 0$$

After differentiating this may be written in the form

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right) i = 0$$

the solution for which is

$$i = A\epsilon^{(-R/2L + j\omega)t} + B\epsilon^{(-R/2L - j\omega)t}$$

This is reduced and substituted in the equation for e_c to give

$$e_c = \frac{1}{C}\int \epsilon^{-(R/2L)t}(A' \cos \omega t + B' \sin \omega t)\, dt$$

Integrating, and determining the constants from the condition that at the instant of sparkover $t = o$, $e_c = E_c$, and $i_c = (e - e')/Z = I$, the equation simplifies into

$$e_c = \epsilon^{-(R/2L)t}\left[E_c \cos \omega t + \left(\frac{RE_c}{2L\omega} + \frac{I}{\omega C}\right)\sin \omega t\right]$$

The magnitude of E_c and ZI have been scaled from the oscillograms for the solution of Fig. 11.

Fig. 12—Circuits of Fig. 5

These five cases are illustrative of a series impedance in a line. Since the checks obtained are equally as good as those of Figs. 9 and 10 the substitution of an equivalent resistance for a surge impedance as previously

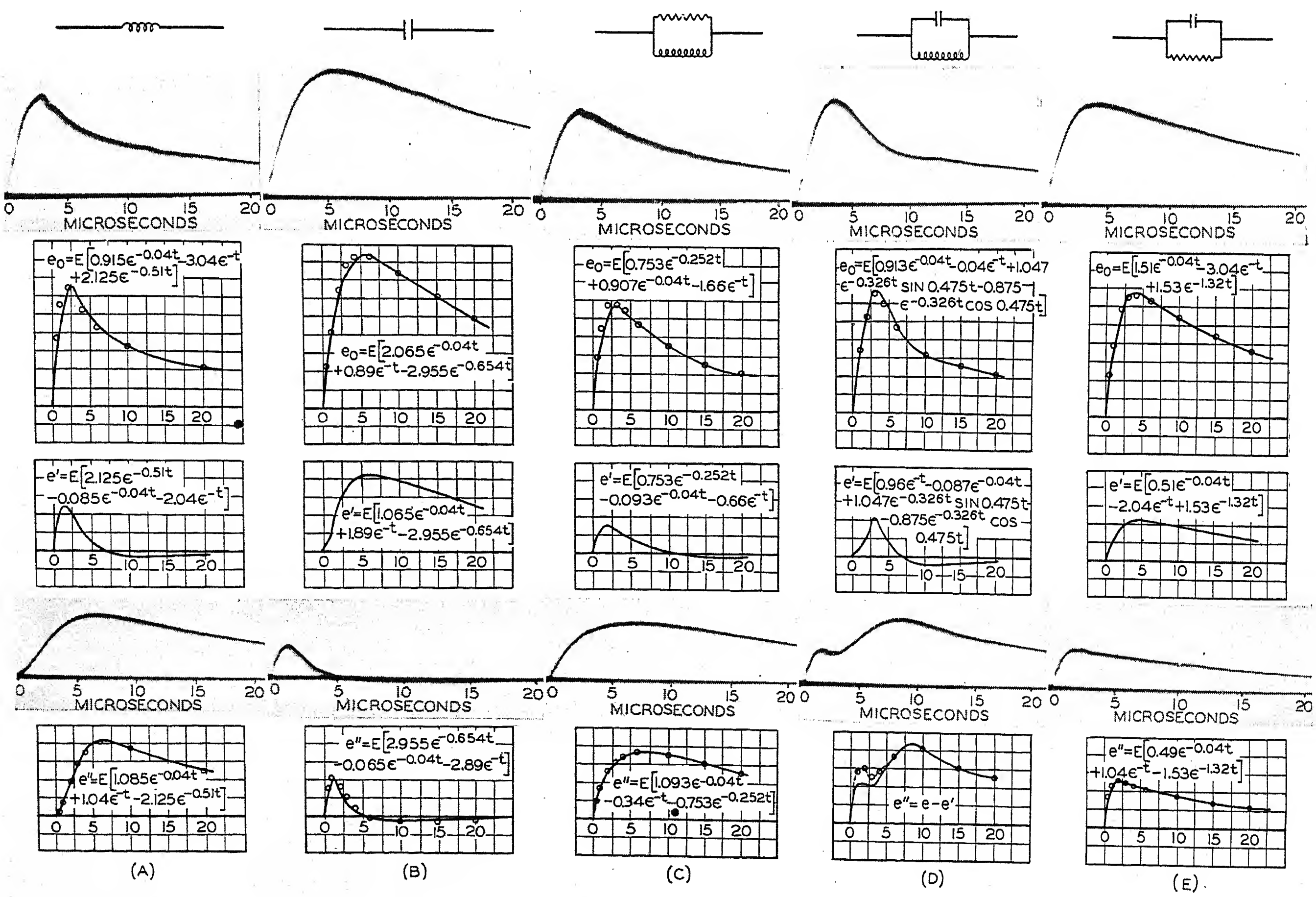

Fig. 12—Circuits of Fig. 5

$e = E(\epsilon^{-0.04t} - \epsilon^{-t}) \qquad e_0 = e + e'. \quad e'' = e - e'$

mentioned, is justified. For the figures shown, $Z_1 = Z_2 = 510$ ohms. The voltage deflection of the oscillograms is reduced by a ratio of 1.21 to that of the incident wave of Fig. 7A, but the sketches are plotted to the same scale as in Figs. 9 and 10 for comparative purposes.

Fig. 13—Circuits of Fig. 6

As in the preceding group the line Z_2 was replaced by a resistance of 510 ohms, so that $Z_1 = Z_2 = 510$ ohms. The reflected waves are predominately negative, but in general this depends on the ratio of Z_2 to Z_1, as well as on the magnitude of the terminal impedances. In (*E*) the general equation is again reduced from the complex form.

Fig. 14—Circuits of Fig. 6

The transmission line was not used in these tests, the impulse voltages being applied directly from the lightning generator to the impedances. In this way a wave of steeper front was obtainable, and the duration was controlled by means of a shunt resistance. The wave applied to the impedances was of the form shown in Fig. 7B but with different values of the parameter a. It is seen to have a front of sufficiently close to zero time to make such an assumption substantially correct. The fair check between the oscillograms and the calculated points further justifies the assumption, and is of particular interest to illustrate how closely a wave of zero time to reach crest may be approximated without serious error.

By using the single exponential form the labor involved in calculating the voltage is greatly reduced. In *A*, it is reasonable to conclude that the error could be accounted for in the inherent constants, the omission of which gives results on the side of safety. It is interesting to note the precise check in *B* where the oscillatory term becomes negligible and gives an abrupt change in the slope of the tail.

These circuits are special cases of Fig. 6, G, H and J, and the waves shown are discussed in more detail in their original publication.[2] The sketches of $e_o - e''$ represent the voltage across the inductance of a current limiting reactor. Since this is not a case of traveling waves, the incident wave becomes the total voltage at the reactor and there is no reflected wave.

Successive Reflections Between a Capacitance and a Resistance

The solution for waves traveling between a capacitance and a resistance shows that the effect of the line surge impedance is eliminated by successive reflections. Fig. 15 illustrates a line of length τ (time units) and of surge impedance Z, having an impulse generator at one end and a resistance at the other. The reflection operators at R and C are respectively,

$$a = \frac{R - Z}{R + Z}$$

and

$$b = \frac{1/pC - Z}{1/pC + Z} = \frac{\alpha - p}{\alpha + p} \text{ where } \alpha = 1/ZC$$

The solution of the system is readily understood from the lattice method of solving successive reflections.[3] Attenuation is included in the general derivation and is assumed to consist of an exponential decrement, given by $\epsilon^{-\beta t}$. The number of reflections is taken as $(n + 1)$ for simplicity in the form of the final equation. Then the total voltage at any point, x distant from C,

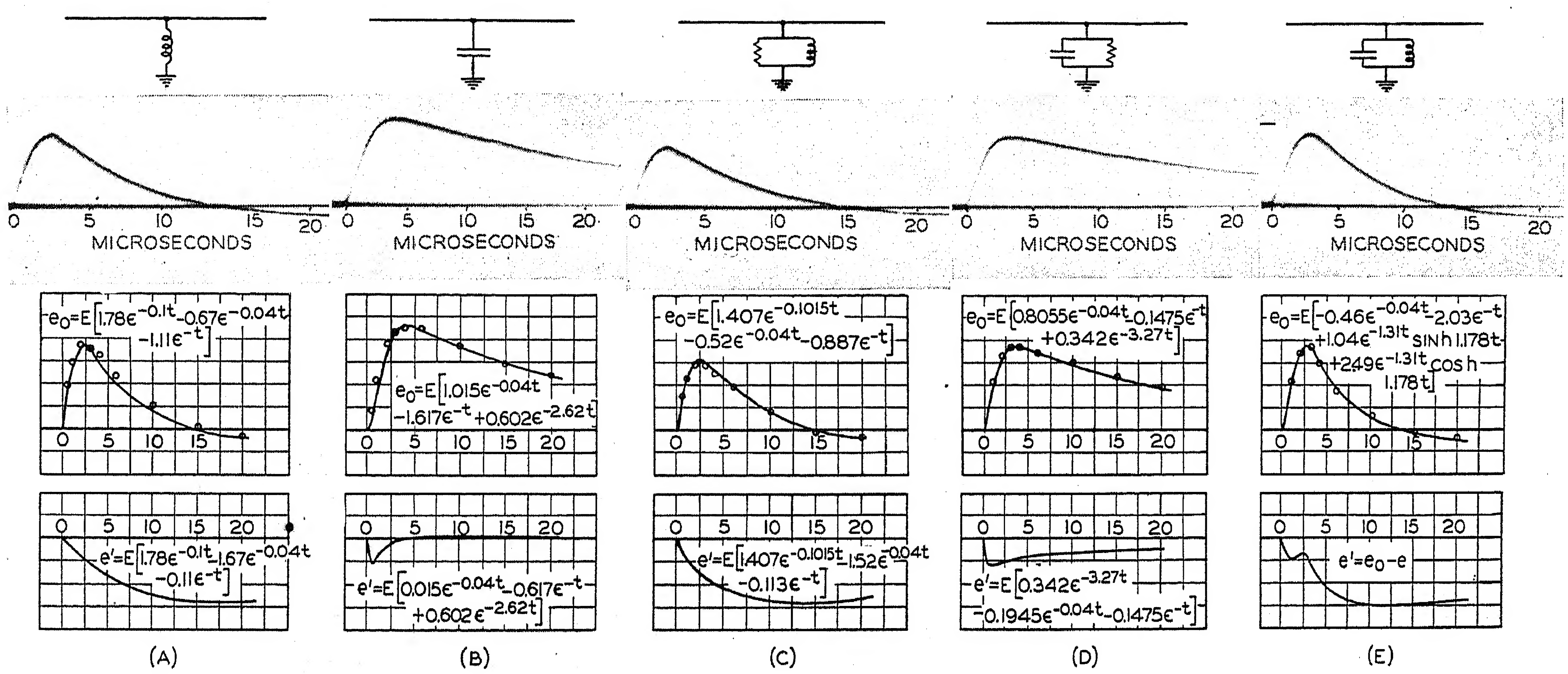

Fig. 13—Circuits of Fig. 6

$e = E(\epsilon^{-0.04t} - \epsilon^{-t}) \qquad e_0 = e'' = e + e'$

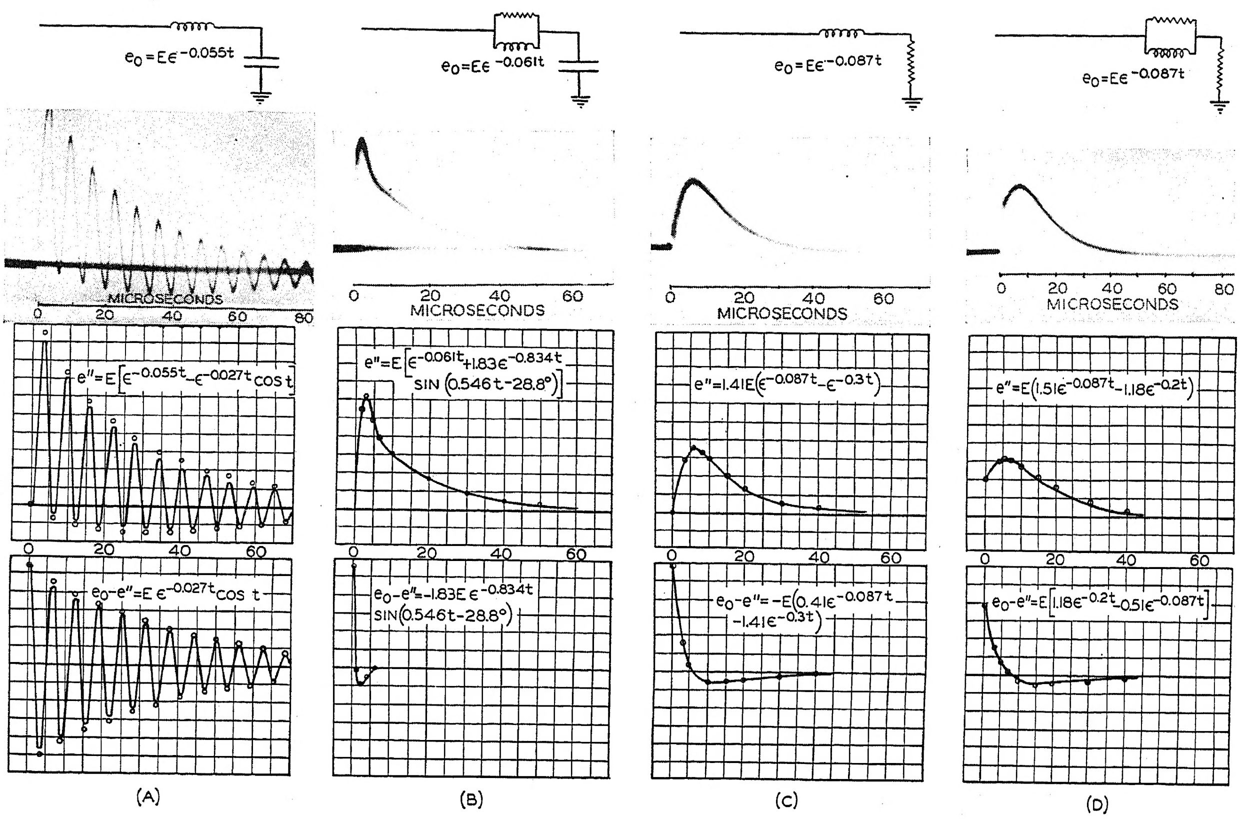

FIG. 14—CIRCUITS OF FIG. 6

(measured in time units) as derived from the lattice of Fig. 15, is

$$E_{(x)} = \Big[\epsilon^{-\beta x}{}_{(x)} + \epsilon^{-\beta(2\tau - x)}\, a_{(2\tau - x)} + \epsilon^{-\beta(2\tau + x)}\, (ab)_{(2\tau + x)}$$

$$+ \epsilon^{-\beta(4\tau - x)}\, (a^2 b)_{(4\tau - x)} + \epsilon^{-\beta(4\tau + x)}\, (a^2 b^2)_{(4\tau + x)} + \ldots \Big] f(t)$$

where $f(t)$ is the incident wave and the subscripts of the operators indicate the time at which they become effective. When $x = \tau$, this term reduces to

$$E_{(\tau)} = \Big[\epsilon^{-\beta\tau} (1 + a)_{(\tau)} + \epsilon^{-3\beta\tau}\, ab\, (1 + a)_{(3\tau)}$$

$$+ \epsilon^{-5\beta\tau}\, a^2 b^2 (1 + a)_{(5\tau)} + \ldots \Big] f(t)$$

of which the general term is

$$e_{(n+1)} = \Big[\epsilon^{-(2n+1)\beta\tau} (ab)^n (1 + a) \Big] f(t)$$

When operating on the incident wave this derives the voltage at R due to any reflection $(n + 1)$. Taking the incident wave of the form $e = E\epsilon^{-t/ZC} = E\epsilon^{-\alpha t}$, ignoring line distortion, and applying the shifting theorem, there results

$$e_{(n+1)} = E\,(1 + a)\, a^n\, \epsilon^{-(2n+1)\beta\tau}\, \epsilon^{-\alpha t}\, (2\alpha - p)^n \frac{1}{p^n} \quad (1)$$

The terms involving the operator expand by the binomial theorem into

$$\sum_{k=0}^{n} \frac{(-1)^k}{\underline{|k}} \frac{\underline{|n}}{\underline{|n-k}} (2\alpha)^{n-k}\, p^k\, \frac{t^n}{\underline{|n}},$$

since

$$\frac{1}{p^n} = \frac{t^n}{\underline{|n}}$$

But

$$p^k t^n = [n(n-1)(n-2) \ldots . (n-k+1)]\, t^{n-k} = \frac{\underline{|n}}{\underline{|n-k}}\, t^{n-k}$$

The voltage at R due to any one reflection $(n + 1)$ then is

$$e_{(n+1)} = E\,(1 + a)\, \frac{a^n}{\underline{|n}}\, \epsilon^{-\beta t}\, \epsilon^{-\alpha t'}$$

$$\sum_{k=0}^{n} \frac{(-1)^k}{\underline{|k}} \left(\frac{\underline{|n}}{\underline{|n-k}} \right)^2 (2\alpha t')^{n-k}$$

where t' indicates that the time function starts at $[t - (2n + 1)\,\tau]$ and t is counted from the instant the capacitance begins its initial discharge.

The total voltage at R at any reflection $(n + 1)$ is

$$E_{(n+1)} = \sum_{r=1}^{n+1} e_r\,(t')$$

where t' denotes the time functions to be displaced as defined above.

When the wave is shorter than 2τ, $e_{(n+1)}$ is the total voltage at R until the wave-length has been increased by successive reflections until it is greater than 2τ. If the wavelength is greater than twice the line length, the voltage at R increases in steps until the decrement is greater than $e_{(n+1)}$. For a wave very much longer than the line, the voltage increases in steps until the effect of successive reflections becomes negligible. Fig. 16 is an oscillogram of the voltage wave obtained at a resistor of 60 ohms, 600 ft. from the impulse generator, the capacitance of which was 0.75 μf. The line was the same as that used in the preceding tests, having a surge impedance of 510 ohms. Superimposed on the oscillo-

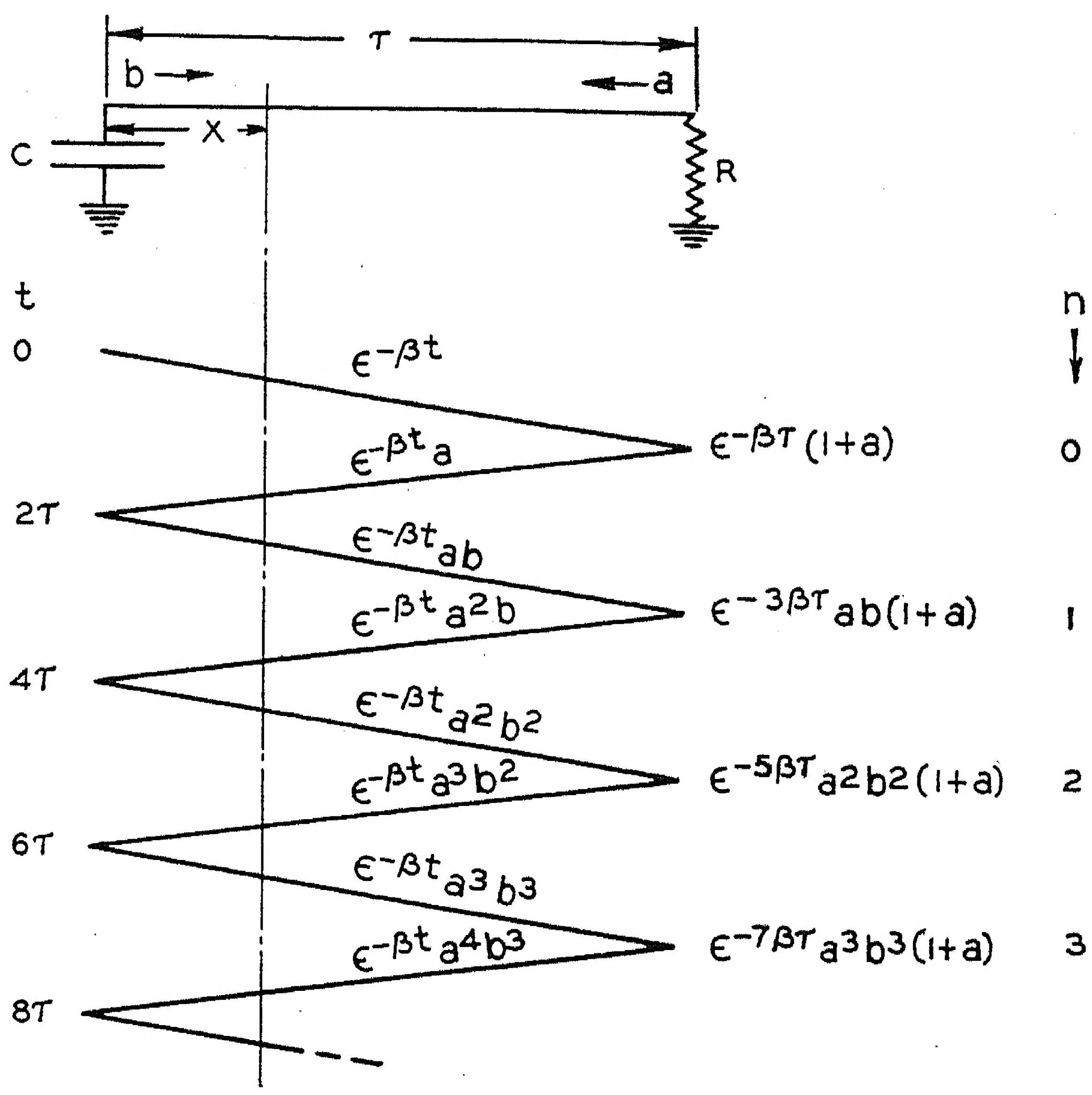

FIG. 15—SUCCESSIVE REFLECTIONS BETWEEN A CAPACITANCE AND A RESISTANCE OVER A TRANSMISSION LINE

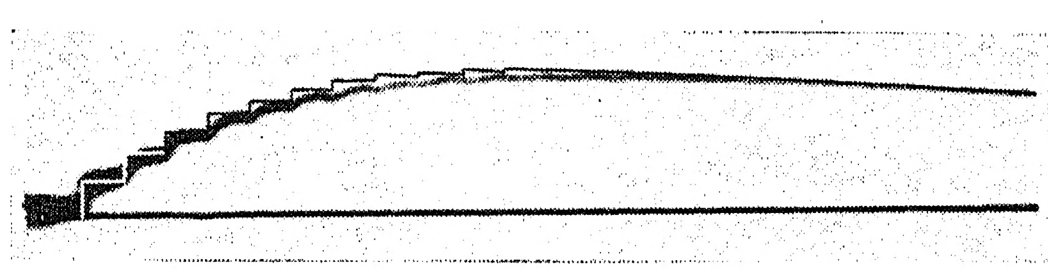

FIG. 16—RESULTANT VOLTAGE AT RESISTOR DUE TO SUCCESSIVE REFLECTIONS

gram is the calculated voltage wave, neglecting attenuation, for which $\beta = 0$. Reflections beyond the eleventh are neglected. The initial wave was 62 kilovolts, and the time between successive steps is 1.2 microseconds. It is evident from the oscillogram and from the equation, that the final voltage wave after the reflections become negligible, is independent of the line surge impedance.

STEP-BY-STEP SOLUTION FOR NEUTRAL GROUNDING IMPEDOR

The oscillogram of Fig. 17 illustrates voltage waves obtained at the line and neutral terminals of a transformer with thyrite as a neutral grounding impedor. Due to the variable characteristic of thyrite, the neutral voltage cannot be solved by any direct method as yet devised, but an approximate solution can be obtained by a step-by-step process, as derived from the differential equation. For the equivalent circuit of a transformer with an impedance consisting of R, L and C in parallel in the neutral, there is

$$2e - Zi = L_T \frac{di}{dt} + E$$

$$L \frac{di_L}{dt} = E$$

$$C \frac{de}{dt} = i_c$$

where

e = voltage of the incident wave.
Z = surge impedance of the line.
L_T = short-circuit inductance of the transformer.
E = voltage at the neutral.

For calculating the neutral voltage the inherent capacitances of the equivalent circuit of the transformer may be neglected without serious error.

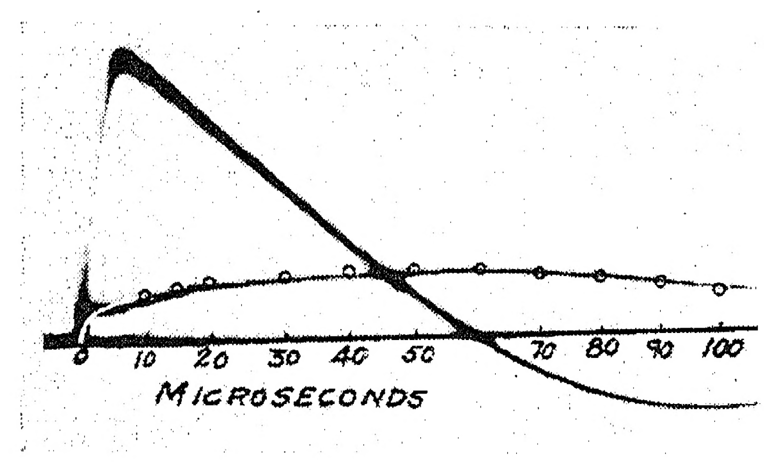

FIG. 17—THYRITE AS A NEUTRAL GROUNDING IMPEDOR

Since R is variable these differential equations cannot be solved directly, but by substituting increments for differentials there results:

$$\Delta i = \frac{2e - Zi - E}{L_T} \Delta t$$

$$\Delta i_L = \frac{E}{L} \Delta t$$

$$\Delta E = \frac{i_c}{C} \Delta t$$

in which voltages and currents are averages over the interval Δt. There is also:

$i_L = \Sigma \Delta i_L$ = current in inductance.
$i = \Sigma \Delta i$ = total current.
$i_R = f(E)$ from the characteristic of thyrite.
$i_c = i - i_R - i_L$ = current in capacitor.
$E = \Sigma \Delta E$ = voltage at neutral.

From these relations a step-by-step tabulation may be compiled to any desired degree of accuracy. The

TABLE IV—STEP-BY-STEP SOLUTION OF FIG. 17

Δt	t	$2e$	E	Δi	i
	0	0	0		0
2				0.80	
	2	350	55		0.80
2				2.08	
	4	550	78		2.88
2				2.72	
	6	620	94		5.60
2				2.80	
	8	610	105		8.40
2				2.68	
	10	595	113		11.08
5				5.97	
	15	525	127		17.05
5				4.89	
	20	460	137		21.94
10				6.80	
	30	325	147		28.74
10				3.08	
	40	200	152		31.82
10				−0.31	
	50	80	151		32.13
10				−3.11	
	60	− 10	148		29.02
10				−5.27	
	70	− 90	140		23.75
10				−6.74	
	80	−140	127		17.01
10				−7.34	
	90	−165	109		9.67
10				−7.00	
	100	−165	76		2.67

trials are adjusted until all the given conditions are satisfied, which ordinarily may be obtained sufficiently accurate in three or four attempts. It is advisable to plot successive points as they are obtained and also to plot a curve of Zi, since extrapolation of the curves provides an excellent means of obtaining the next increment. This also insures against additive errors, since each point is directly dependent upon the preceding one. This method assumes all functions to be linear over the interval Δt, and therefore, if increments are assumed too large, successive points will alternate about their correct value in the region of rapid curvature, resulting in superimposed oscillations that do not exist.

The solution corresponding to the oscillogram of Fig. 17 is given in Table IV. Waves were applied directly from the impulse generator at 620 kilovolts to a transformer having an inductance of 0.368 henrys and with 100 disks of thyrite in the neutral to ground. The plotted circles in Fig. 17 are the solution for the neutral voltage obtained from Table IV, considering only the inductance of the transformer and the thyrite. Since a transmission line, and inductance and capacitance at the neutral were not present, the solution is greatly simplified, and successive points may be obtained fairly accurately on the second trial.

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